Multivariate realized stochastic volatility model with leverage.

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1 Introduction
   1 Models of daily return
   2 Realized volatility
   3 Simultaneous modeling of the daily return and it's realized volatility
2 Multivariate realized SV model.
3 Bayesian estimation using Markov chain Monte Carlo method.
4 Empirical study
5 Conclusion
Introduction

Motivation

1. Forecasting variance-covariance matrix of the daily stock return series.

2. Using both
   1. high frequency data through realized measures
   2. the traditional daily volatility models

3. Simultaneous modeling of daily returns and multivariate realized measures.
Univariate models

\( y_t \): Observed daily return at \( t \).

1. **GARCH** (generalized autoregressive conditional heteroskedasticity) model (Engle (1982), Bollerslev (1986))

\[
\begin{align*}
y_t &= \sigma_t \varepsilon_t, \\
\sigma_{t+1}^2 &= \omega + \alpha y_t^2 + \beta \sigma_t^2. \quad (GARCH) \\
(\log \sigma_{t+1}^2) &= \omega + \beta \log \sigma_t^2 + \gamma y_t^2 + \delta y_t. \quad (EGARCH)
\end{align*}
\]

2. **SV** (stochastic volatility) model (Taylor(1986), )

\[
\begin{align*}
y_t &= \exp(h_t/2)\varepsilon_t, \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t,
\end{align*}
\]
About this two decades, tick by tick trade and quote data is available. Model free variance volatility estimators called Realized measures (RM) (e.g. Andersen, Bollerslev, Diebold and Labys (2001) and Barndorff-Nielsen and Shephard (2002)).

The study of forecasting model using the RM is also active field. (e.g. Mykland, and Aït-Sahalia (2005), Barndorff-Nielsen and Shephard (2004b), Barndorff-Nielsen, Hansen, Lunde and Shephard (2008), Zhang (2006), Jacod et al. (2009), and so on)
$y_{t-1+i/M}$: $M$ times traded observed return vector from $t - 1$ to $t$.
Realized covariance:

$$RC_t = \sum_{i=1}^{M} y_{t-1+i/M} y'_{t-1+i/M}$$

(Barndorff-Nielsen and Shephard (2004)). More sophisticated estimators: (Hayashi and Yoshida (2005), Voev and Lunde (2007), Griffin and Oomen (2010), Christensen et al. (2010), Barndorff-Nielsen et al. (2010).)
Models of RM to predict the daily return covariance or its proxy, e.g. RM.

1. (univariate) Heterogeneous autoregressive model (Corsi (2009)).
2. Wishart Process (Jin and Maheu (2006), Asai and So (2010)).
3. Factor model (Bannouh, Martens, Oomen and van Dijk (2009)).
4. Log-vech form VARMA, VARFIMA (Chiriac and Voev (2011)).
5. Simultaneous modeling of daily returns and RM (next slide).
GARCH models

1. Multiplicative error model (MEM, A model with Range and RM. Engle and Gallo (2006))

2. HEAVY model (Simplified MEM, Shephard and Sheppard (2010)) multivariate extension (Noureldin, Shephard and Sheppard (2010))

3. GARCH with measurement equation of RM (Realized GARCH, Hansen, Huang and Shek (2010)) Bivariate extension with market return (Realized beta GARCH, Hansen, Huang and Shek (2010)).

4. GARCH with high frequency data (Chen, Ghysels, Wang (2009, 2010) HYBRID GARCH)

SV models (Next slide)
Realized SV models

Realized SV (RSV) model:

1. RSV with leverage (Takahashi Omori and Watanabe (2009))
2. Using RQ to estimate the variance of RM error, superposition model. (Dobrev and Szerszen (2010)).
3. Endogeneity between returns and RM errors (Chaussè and Xù (2011), Koopman and Scharth (2011)).
4. General setting (Koopman and Scharth (2011)).
5. GH skew-t distribution (Takahashi, Omori, and Watanabe (2012)).
6. Alternative specification (Jacquier and Miller (2012))
7. Markov switching (Ishihara and Omori (2008)).
8. Long memory (Shirota, Hidsu and Omori (2012)).
9. Jump (Shirota and Omori (2012)).
10. Wishart specification (Jin and Maheu (2012))
11. Multivariate extension with leverage (This research).
Realized SV model Takahashi Omori and Watanabe (2009)

(Univariate Realized SV model)

\[
y_t = \exp(h_t/2)\varepsilon_t
\]

\[
\log(RM_t) = \xi + h_t + u_t
\]

\[
h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t,
\]

\[
(\varepsilon_t, \eta_t)', \sim N_2(0, \Sigma), \quad \Sigma = \begin{pmatrix}
1 & \rho\sigma_\eta \\
\rho\sigma_\eta & \sigma_\eta^2
\end{pmatrix},
\]

\[
u_t \sim N(0, \sigma_u^2)
\]

We extend this model to the multivariate case with dynamic correlation and leverage effects.
Properties to specify the volatility process of multivariate returns:

- Time varying volatility (volatility clustering),
- Time varying correlation of returns,
- Correlations of volatility,
- Leverage effect (asymmetry),
- Cross asset leverage effect.

Properties to specify the RM measurement:

- Scale bias from overnight return, and Epps effect.
Modeling Dynamic correlations

Methods to keep positive definiteness

1. Reparameterization:
   1. Fisher transformation (bivariate) (Yu and Meyer (2006)).
   2. Choleski decomposition, AR(1) (Tsay(2005), Asai, McAleer, and Yu(2006), Lopes and Tsay (2012)),

2. Wishart Process:
   1. Scale matrix (Philipov and Glickman (2006), Asai and So (2010)).
   2. Conditional Laplace transformation (Gourieroux, Jasiak, Sufana (2009)).

3. Positive definite Lévy Process:

4. Factor model:
   1. Mean factor (Pitt and Shephard (1998), Chib, Nardari, and Shephard (2006)).
For $p \times p$ matrix $A$, matrix exponential is defined by

$$
\exp(A) \equiv \sum_{s=0}^{\infty} \frac{1}{s!} A^s,
$$

If $A$ is symmetric, since $A = U \Lambda U'$, we have

$$
\exp(A) = U \left( \sum_{s=0}^{\infty} \frac{1}{s!} \Lambda^s \right) U' = U \exp(\Lambda) U'.
$$

Moreover, there exist the one to one inverse transformation (matrix logarithm) (Chiu, Leonard, and Tsui(1996)).
Multivariate Realized SV (MRSV) model

$y_t: p \times 1$, daily return vector of the day $t$.

$X_t = \log R C_t$: Log realized covariance of the day $t$.

$\exp(H_t)$: $p \times p$, covariance matrix of $y_t$

\[
y_t = \exp\left(\frac{H_t}{2}\right) \varepsilon_t
\]
\[
X_t = \Xi + H_t + U_t,
\]
\[
H_{t+1} = M + \tilde{\Phi} \odot (H_{t-1} - M) + E_t,
\]

$H_t = [h_{ij,t}], U_t = [v_{ij,t}], E_t = [\eta_{ij,t}], \Xi = [\xi_{ij}], M = [\mu_{ij}], \tilde{\Phi} = [\phi_{ij}],$

$i, j = 1, \ldots, p$, are $p \times p$ symmetric matrices.
\( x_t := \text{vech}(X_t), \quad h_t := \text{vech}(H_t), \quad \xi := \text{vech}(\Xi), \quad \mu := \text{vech}(M), \quad \Phi := \text{diag}(\text{vech}(\tilde{\Phi})). \)

\[
\begin{align*}
  y_t &= \exp\left(\frac{H_t}{2}\right) \varepsilon_t, \quad t = 1, \ldots, n, \\
  x_t &= \xi + h_t + \nu_t, \quad t = 1, \ldots, n, \\
  h_{t+1} &= \mu + \Phi(h_t - \mu) + \eta_t, \quad t = 1, \ldots, n - 1, \\
  h_1 &\sim \mathcal{N}_q(0, \Sigma_0), \\
  \left( \begin{array}{c} 
  \varepsilon_t \\
  \eta_t 
  \end{array} \right) &\sim \text{i.i.d.} \mathcal{N}_{p+q}(0, \Sigma), \quad 
  \Sigma = \left( \begin{array}{cc} 
  I_p & \Sigma_{\varepsilon\eta} \\
  \Sigma_{\eta\varepsilon} & \Sigma_{\eta\eta} 
  \end{array} \right), \\
  \nu_t &\sim \text{i.i.d.} \mathcal{N}_q(0, \Sigma_{\nu\nu}),
\end{align*}
\]
If the equation (4) is dropped, this model is reduced to matrix exponential SV (MESV) model (Ishihara, Omori, and Asai (2011))

\[
y_t = \exp\left(\frac{H_t}{2}\right) \varepsilon_t, \\
H_{t+1} = M + \bar{\Phi} \odot (H_t - M) + E_t,
\]

We compare MRSV model with MESV models.
Technical issues and estimation and inference strategy for MRSV model and MESV model.

- High dimensional parameters and latent variables → Bayesian estimation using Markov chain Monte Carlo method.

- Model comparisons. We can’t compare MRSV with MESV using Bayesian marginal likelihood or information criterion. → Model comparison based on the predictive performance.

- Interpretation of the parameters is difficult. (e.g. The negative \((1, 1)\) element of \(\Sigma_{\epsilon\eta}\) doesn’t imply leverage effect on \(y_{1t}\)) → Graphical presentations via simulations, e.g. paths of the standard deviations and correlations from \(\exp(H_t)\), news impact curves, and calculating the process of principal component.
Matrix exponential: example

\( a, \rho \in \mathbb{R} \)

\[
A = \begin{pmatrix}
a & \rho \\
\rho & a
\end{pmatrix}, \quad B = \begin{pmatrix}
a & \rho & \rho \\
\rho & a & \rho \\
\rho & \rho & a
\end{pmatrix}.
\]

Then

\[
\exp(A) = \begin{pmatrix}
(e^{a+\rho} + e^{a-\rho})/2 & (e^{a+\rho} - e^{a-\rho})/2 \\
(e^{a+\rho} - e^{a-\rho})/2 & (e^{a+\rho} + e^{a-\rho})/2
\end{pmatrix},
\]

\[
\exp(B) = \begin{pmatrix}
(e^{a+2\rho} + 2e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 \\
(e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} + 2e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 \\
(e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} + 2e^{a-\rho})/3
\end{pmatrix}.
\]

\[
(e^{a+2\rho} + 2e^{a-\rho})/3 \geq (e^{a+\rho} + e^{a-\rho})/2,
\]

This shows that the interpretation of the parameters depends on the dimension of \( y_t \).
We consider a MCMC algorithm to generate from the posterior

\[ \pi(\theta|Y_n) \propto f(Y_n|\theta)\pi(\theta). \]

1. Generate \( h|\beta, \phi, \Sigma, Y \)
   - *Single-move sampler* (Independent MH)
   - Generate \( h_t|h_{-t}, \beta, \phi, \Sigma, \Sigma_v, y, t = 1, \ldots, n \)

2. Generate \( \beta|\Sigma, \phi, \Sigma_v, h, Y_n \) (Gibbs sampler).

3. Generate \( \Sigma_v|\beta, \phi, h, Y_n \) (Gibbs sampler).

4. Generate \( \Sigma|\beta, \phi, \Sigma_v, h, Y_n \) (Independent MH).

5. Generate \( \phi|\beta, \Sigma, \Sigma_v, h, Y_n \) (Independent MH).

where \( Y_n = (y_1, x_1, \ldots, y_n, x_n) \), \( \beta = (\mu, \xi) \).

Blocked multimove sampler (Omori and Watanabe (2008)) via simulation smoother can be applied. Considering the computational time and inefficiency of the MCMC, we use single move sampler.
Dataset provided by Noureldin, Shephard and Sheppard (2011), "Multivariate High-Frequency-Based Volatility (HEAVY) Models".

1: Bank of America, 2: J.P. Morgan, 3: IBM, 4: Microsoft, 5: Exxon Mobil. Daily return (open to close) and realized covariance matrix.

They calculate from NYSE TAQ database.

Period

1. From Feb. 2 2001 to Dec. 31, 2006 (1486 days for estimation)
2. From Jan. 2 2006 to Mar. 8, 2008 (300 days for prediction window)

We exclude financial crisis period from the data set. Because our model is basic one but many abnormal events have appeared, e.g. for Bank of America,

   However, because of the debt of Merrill Lynch ($271 million), additional public fund injection ($200 million) is announced.
Figure: Daily returns of the series
First we fit univariate models
The difference of parameter estimates (one dimensional SV vs one dimensional RSV model)

- 95% credible intervals of parameters except $\mu$ get closer in RSV models.
- Leverage effect and autocorrelation parameters are smaller in RSV models.
- 95% credible interval of $\xi$ (log scale bias of RV) include zero.
### Univariate SV model

Estimation result of parameters of the univariate SV and RSV

Posterior means and 95% credible intervals

<table>
<thead>
<tr>
<th></th>
<th>SV</th>
<th>RSV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
<td>[95%interval]</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.980</td>
<td>[0.965, 0.991]</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.989</td>
<td>[0.979, 0.997]</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>0.990</td>
<td>[0.980, 0.998]</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>0.989</td>
<td>[0.979, 0.997]</td>
</tr>
<tr>
<td>$\phi_5$</td>
<td>0.968</td>
<td>[0.947, 0.984]</td>
</tr>
<tr>
<td>$\rho_{1,\varepsilon\eta}$</td>
<td>-0.449</td>
<td>[-0.599, -0.271]</td>
</tr>
<tr>
<td>$\rho_{2,\varepsilon\eta}$</td>
<td>-0.485</td>
<td>[-0.632, -0.311]</td>
</tr>
<tr>
<td>$\rho_{3,\varepsilon\eta}$</td>
<td>-0.381</td>
<td>[-0.562, -0.173]</td>
</tr>
<tr>
<td>$\rho_{4,\varepsilon\eta}$</td>
<td>-0.323</td>
<td>[-0.509, -0.116]</td>
</tr>
<tr>
<td>$\rho_{5,\varepsilon\eta}$</td>
<td>-0.283</td>
<td>[-0.480, -0.075]</td>
</tr>
</tbody>
</table>


95% credible intervals of parameters get closer and leverage effect and autocorrelation parameters are smaller in RSV models.
### Univariate SV model

Estimation result of parameters of the univariate SV and RSV

Posterior means and 95% credible intervals

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<td></td>
<td>mean [95%interval]</td>
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</tr>
<tr>
<td>$e^{\mu_1/2}$</td>
<td>1.077 [0.852, 1.436]</td>
<td>0.993 [0.819, 1.235]</td>
</tr>
<tr>
<td>$e^{\mu_2/2}$</td>
<td>1.113 [0.656, 1.645]</td>
<td>1.162 [0.824, 1.574]</td>
</tr>
<tr>
<td>$e^{\mu_3/2}$</td>
<td>0.865 [0.458, 1.310]</td>
<td>0.918 [0.701, 1.139]</td>
</tr>
<tr>
<td>$e^{\mu_4/2}$</td>
<td>1.036 [0.626, 1.567]</td>
<td>1.035 [0.735, 1.396]</td>
</tr>
<tr>
<td>$e^{\mu_5/2}$</td>
<td>0.987 [0.852, 1.136]</td>
<td>0.992 [0.863, 1.133]</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>0.061 [-0.018, 0.141]</td>
<td></td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>0.082 [ 0.000, 0.162]</td>
<td></td>
</tr>
<tr>
<td>$\xi_3$</td>
<td>0.052 [-0.030, 0.127]</td>
<td></td>
</tr>
<tr>
<td>$\xi_4$</td>
<td>0.201 [ 0.124, 0.274]</td>
<td></td>
</tr>
<tr>
<td>$\xi_5$</td>
<td>0.146 [ 0.069, 0.224]</td>
<td></td>
</tr>
</tbody>
</table>


95% credible intervals of parameters of $\mu$ doesn’t get closer and $\xi$’s (log scale bias of RV) include zero.
The results of the estimates of the Multivariate models In the MRSV model,

1. The paths of the volatility and correlations behave sharply.
2. 95% intervals of the news impact curve are narrow.
3. News impact curves on the correlation are constant.
4. The paths of principal component process and their volatilities also behave sharply.
Figure: Estimated path of the standard deviations. MESV (Left), MRSV (Right)
Figure: Estimated path of the correlations 1. MESV(left), MRSV(right) 1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil. In 2002 and 2003 correlations increase.
Figure: Estimated path of the correlations 1. MESV(left), MRSV(right) 1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil. In 2002 and 2003 correlations increase.
To show the leverage effect we write news impact curves.

News impact curves: the curves of the elements of $\exp(H_{t+1})$ given changing $y_t$’s.

Two approaches to handle $h_t$’s

1. Conditioning at stationary means $h_t = \mu$ and given points $y_t$. (Asai and McAleer (2006))

2. Integrate out by stationary distribution $h_t^\dagger \sim \mathcal{N}_q(\mu, \Sigma_0)$ and $y_t^\dagger \sim \mathcal{N}_q(0, \exp(H_t^\dagger))$. (Takahashi, Omori, and Watanabe (2012))

Using the distribution of $h_{t+1}|y_t, h_t, \Sigma, \mu, \Phi$ given by

$$h_{t+1} = \mu + \Phi(h_t - \mu) + \Sigma_{\eta \varepsilon} \exp(-H_t/2)y_t + \eta_t^\dagger,$$

$$\eta_t^\dagger \sim \mathcal{N}_{p(p+1)/2}(0, \Sigma_{\eta \eta} - \Sigma_{\eta \varepsilon} \Sigma_{\varepsilon \eta}).$$
We set $h_t = \mu$ and $\eta_t^\dagger = 0$.
We show the news impact curves, the posterior means of $\exp(H_{t+1})$ under the changing $y_t$, where

$$h_{t+1} = \mu + \sum \eta \exp(-M/2)y_t$$

The domain of the estimated curve is restricted to the range within ±4 sample standard deviation of actual returns for each $y_{it}$.
Each $y_{it}$ is moved independently.
We write posterior mean and 95% credible intervals of the news impact curves.
Conditional news impact curves

News impact on the standard deviations


- Left panels: MESV model, right panels: MRSV model,

- Horizontal axis: the value of $y_t$, vertical axis: Standard derivations of $y_{t+1}$.

- The leverage effect of MRSV model get smaller.
Conditional news impact curves

News impact on the correlations


Left panels: MESV model, right panels: MRSV model,
Horizontal axis: the value of $y_t$, vertical axis: correlations of $y_{t+1}$.
The correlation of the MRSV model takes constant value. There is no effect from the value of $y_t$, on the correlations of $y_{t+1}$.
The news impact curves on the standard deviations of J.P.Morgan.


Left panels: MESV model, right panels: MRSV model, Horizontal axis: the value of $y_t$, vertical axis: Standard derivations of $y_{t+1}$.

Leverage effect from the self-series, Micro Soft, and Exxon Mobil exist. Positive asymmetry seems to exist from IBM in the MESV, but looks constant in the MRSV.
Conditional news impact curves

The news impact curves on the correlation of Bank of America and J.P. Morgan.


Left panels: MESV model, right panels: MRSV model, horizontal axis: the value of $y_t$, vertical axis: correlations of $y_{t+1}$.

95% interval of MESV model diverges on the tail and of MRSV model takes constant value.
Unconditional news impact curves under MRSV models

1. Generate $h^i_t \sim \mathcal{N}_q(\mu, \Sigma^i_0)$, for each MCMC sample $i$. and $y^i_t|h^i_t \sim \mathcal{N}_q(0, \exp(H^i_t))$

2. Calculate $\exp(H^i_{t+1})$ using $h^i_{t+1} = \mu^i + \Phi^i(h^i_t - \mu^i) + \Sigma^i_{\eta\epsilon} \exp(-H^i_t/2)y^i_t$.

3. Conduct nonparametric kernel regression: regress standard deviations or correlations calculated from $\exp(H^i_{t+1})$ on the element $y^i_t$.

We only show the estimated posterior mean of the unconditional posterior
Unconditional news impact curves

News impact on the standard deviations

News impact on the correlations


Left panels: MESV model, right panels: MRSV model,
Horizontal axis: the value of $y_t$,
Vertical axis: Standard deviations of $y_t$ + 1.

News impact curves on the correlations are constant.
As another application, we make a principal components of \( y_t \). Let \( H_t = W_t \Lambda_t W'_t \) denote a spectral decomposition of the log volatility matrix where \( W_t W'_t = I_p \) and \( \Lambda_t \) is a diagonal matrix whose elements are arranged in descending order. Then the return equation can be transformed as follows

\[
W'_t y_t = \exp(\Lambda_t/2) \varepsilon^*_t, \\
\varepsilon^*_t = W'_t \varepsilon_t \sim N(0, I_p).
\]

\( \varepsilon^*_t \) can be interpreted as the principal component vector, and \( \exp(\Lambda_t) \) is its variance. Further, \( W'_t \) is the dynamic portfolio weight matrix (or loading matrix) to make the principal components.
Estimated path of the principal components $W_t' y_t$
MESV(left), MRSV(right), The paths of principal component process also behave sharply.
Estimated path of the standard deviations of the principal components, diagonal elements of $\exp(\Delta_t/2)$.
MESV (left), MRSV (right)
Model comparison between MRSV and MESV models

1. Marginal likelihood, DIC, Other information criterion (Since data sets are different, it’s difficult to use).

2. Prediction
   1. Since predictand variable (volatility) is unobserved, proxy predictand variables (RM’s) are used.

\( \hat{\Sigma}_t \): Proxy of volatility,
\( V_t \): true unobserved volatility,
\( \hat{V}_t \): predictor of true volatility,

If \( E_t[\hat{\Sigma}_{t+T}] = V_{t+T} \) then for the predictors \( \hat{V}_{t}^A, \hat{V}_{t}^B \) based on the models \( A, B \),

\[
E[L(\hat{\Sigma}_t, \hat{V}_t^A)] \geq (\leq) E[L(\hat{\Sigma}_t, \hat{V}_t^B)]
\]

\[
\Leftrightarrow E[L(V_t, \hat{V}_t^A)] \geq (\leq) E[L(V_t, \hat{V}_t^B)]
\]

holds under the quasi-likelihood loss function.

\[
L(\hat{\Sigma}_t, \hat{V}_t) = \log |\hat{V}_t| + \text{tr}(\hat{\Sigma}_t \hat{V}_t^{-1}) - K_t
\]

\( (K_t \text{ doesn’t depend } \hat{V}_t) \)

For the proxy, we use future \( RM_t \).
Setting:

1. 1486 days for estimation,
2. 300 days for prediction window (from Jan. 2 2006 to Mar. 8, 2008)
   • Rolling window with forecast horizon $T = 1, 5, 10, 22$ (1 day, 1 week, 2 weeks, 1 month).
   • The predictor is the mean of the posterior predictive distributions.
Loss functions

Difference of losses between MRSV model and MESV model

Proxy predictand variables: $\exp(X_{n+T})$, predictor: the mean of the posterior predictive distribution of $\exp(H_{n+T})$

<table>
<thead>
<tr>
<th>$T$</th>
<th>MESV mean</th>
<th>st.er.</th>
<th>[95% interval]</th>
<th>MRSV mean</th>
<th>st.er.</th>
<th>[95% interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-7.58</td>
<td>1.74</td>
<td>[-11.00, -4.17]</td>
<td>-9.52</td>
<td>2.03</td>
<td>[-13.52, -5.53]</td>
</tr>
<tr>
<td>5</td>
<td>-5.47</td>
<td>1.37</td>
<td>[-8.13, -2.74]</td>
<td>-6.61</td>
<td>1.50</td>
<td>[-9.48, -3.73]</td>
</tr>
<tr>
<td>10</td>
<td>-4.18</td>
<td>1.13</td>
<td>[-6.40, -1.96]</td>
<td>-5.20</td>
<td>1.17</td>
<td>[-7.50, -2.89]</td>
</tr>
<tr>
<td>22</td>
<td>-3.01</td>
<td>0.89</td>
<td>[-4.76, -1.26]</td>
<td>-4.18</td>
<td>0.95</td>
<td>[-6.06, -2.30]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T$</th>
<th>The difference of losses (MRSV−MESV) mean</th>
<th>st.er.</th>
<th>[95% interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1.94</td>
<td>0.61</td>
<td>[-3.14, -0.74]</td>
</tr>
<tr>
<td>5</td>
<td>-1.17</td>
<td>0.31</td>
<td>[-1.77, -0.57]</td>
</tr>
<tr>
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<td>-1.01</td>
<td>0.21</td>
<td>[-1.42, -0.61]</td>
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<tr>
<td>22</td>
<td>-1.17</td>
<td>0.14</td>
<td>[-1.44, -0.90]</td>
</tr>
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</table>

Mean, standard error, and 95% confidence intervals. The loss of the
Summary and Future works

1. MRSV model with leverage effect.
2. Application to stock return and realized covariance.
3. Model comparison between with and without realized covariance. The MRSV have higher performance.

Future work

1. Application of the principal component vector processes.
2. Extensions:
   1. Simultaneous jumps of $y_t$ and $x_t$, and using bi-power variations.
   2. Superpositioned factor model ($H_t = H_{1t} + H_{2t}$)
   3. Modeling means of daily returns using high frequency data.
3. Comparison with other specifications: HIBRID-MESV, HEAVY type SV, matrix exponential realized GARCH and so on.
4. Prediction ranking consistency based on the Bayes risk with proxy predictand.