Multivariate realized stochastic volatility model with leverage.

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Outline

1. Introduction
   1. Models of daily return
   2. Realized volatility
   3. Simultaneous modeling of the daily return and it's realized volatility
2. Multivariate realized SV model.
3. Bayesian estimation using Markov chain Monte Carlo method.
4. Empirical study
5. Conclusion

Motivation

1. Forecasting variance-covariance matrix of the daily stock return series.
2. Using both
    1. high frequency data through realized measures
    2. the traditional daily volatility models
3. Simultaneous modeling of daily returns and multivariate realized measures.

y_t : Observed daily return at t.

1. GARCH (generalized autoregressive conditional heteroskedasticity) model (Engle (1982), Bollerslev (1986))

$$\begin{aligned} y_t &= \sigma_t \varepsilon_t, \\ \sigma_{t+1}^2 &= \omega + \alpha y_t^2 + \beta \sigma_t^2. \qquad (GARCH) \\ (\log \sigma_{t+1}^2 &= \omega + \beta \log \sigma_t^2 + \gamma y_t^2 + \delta y_t). \qquad (EGARCH) \end{aligned}$$

2. SV (stochastic volatility) model (Taylor(1986),)

$$\begin{aligned} y_t &= \exp(h_t/2)\varepsilon_t, \\ h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \end{aligned}$$

Introduction:High frequency data

1. About this two decades, tick by tick trade and quote data is available.
2. Model free variance volatility estimators called Realized measures (RM) (e.g. Andersen, Bollerslev, Diebold and Labys (2001) and Barndorff-Nielsen and Shephard (2002).
3. The study of forecasting model using the RM is also active field. (e.g. Mykland, and Aït-Sahalia (2005), Barndorff-Nielsen and Shephard (2004b), Barndorff-Nielsen, Hansen, Lunde and Shephard (2008), Zhang (2006), Jacod et al. (2009), and so on)

Realized Covariance

$\boldsymbol{y}_{t-1+i/M}$: M times traded observed return vector from $t-1$ to t.
Realized covariance:

$$RC_t = \sum_{i=1}^{M} \boldsymbol{y}_{t-1+i/M}\boldsymbol{y}'_{t-1+i/M}$$

(Barndorff-Nielsen and Shephard (2004)). More sophisticated estimators: (Hayashi and Yoshida (2005), Voev and Lunde (2007), Griffin and Oomen (2010), Christensen et al. (2010), Barndorff-Nielsen et al. (2010).)

Models for realized covariance

Models of RM to predict the daily return covariance or it's proxy, e.g. RM.

1. (univariate) Heterogeneous autoregressive model (Corsi (2009)).
2. Wishart Process (Jin and Maheu (2006), Asai and So (2010)).
3. Factor model (Bannouh, Martens, Oomen and van Dijk (2009)).
4. Log-vech form VARMA, VARFIMA (Chiriac and Voev (2011)).
5. Simultaneous modeling of daily returns and RM (next slide).

Models of daily returns with RM

GARCH models

1. Multiplicative error model (MEM, A model with Range and RM. Engle and Galo(2006))
2. HEAVY model (Simplified MEM, Shephard and Sheppard (2010)) multivariate extension (Noureldin, Shephard and Sheppard (2010))
3. GARCH with measurement equation of RM (Realized GARCH, Hansen, Huang and Shek (2010))
   Bivariate extension with market return (Realized beta GARCH, Hansen, Huang and Shek (2010)).
4. GARCH with high frequency data (Chen, Ghysels, Wang (2009, 2010) HYBRID GARCH)

SV models (Next slide)

Realized SV models

Realized SV (RSV) model:

1. RSV with leverage (Takahashi Omori and Watanabe (2009))
2. Using RQ to estimate the variance of RM error, superposition model. (Dobrev and Szerszen (2010)).
3. Endogeneity between returns and RM errors (Chaussè and Xù (2011), Koopman and Scharth (2011)).
4. General setting (Koopman and Scharth (2011)).
5. GH skew-t distribution (Takahashi, Omori, and Watanabe (2012)).
6. Alternative spacification (Jacquier and Miller (2012))
7. Markov switching (Ishihara and Omori (2008)).
8. Long memory (Shirota, Hidsu and Omori (2012)).
9. Jump (Shirota and Omori (2012)).
10. Wishart specification (Jin and Maheu (2012))
11. Multivariate extension with leverage (This research).

(Univariate Realized SV model)

$$
\begin{aligned}
y_t &= \exp(h_t/2)\varepsilon_t && (1)\\
\log(RM_t) &= \xi + h_t + u_t && (2)\\
h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, && (3)\\
(\varepsilon_t, \eta_t)', &\sim N_2(\mathbf{0}, \Sigma), \quad \Sigma = \begin{pmatrix} 1 & \rho\sigma_\eta \\ \rho\sigma_\eta & \sigma_\eta^2 \end{pmatrix},\\
u_t &\sim N(\mathbf{0}, \sigma_u^2)
\end{aligned}
$$

We extend this model to the multivariate case with dynamic correlation and leverage effects.

Properties

Properties to specify the volatility process of multivariate returns:

- Time varying volatility (volatility clustering),
- Time varying correlation of returns,
- Correlations of volatility,
- Leverage effect (asymmetry),
- Cross asset leverage effect.

Properties to specify the RM mesurement:

- Scale bias from overnight return, and Epps effect.

Modeling Dynamic correlations

Methods to keep positive definiteness

1. Reparameterization:
   1. Fisher transformation (bivariate) (Yu and Meyer (2006)).
   2. Choleski decomposition, AR(1) (Tsay(2005), Asai, McAleer, and Yu(2006), Lopes and Tsay (2012)),
   3. Matrix exponential (Asai, McAleer, and Yu(2006), Ishihara Omori, and Asai(2012)).
2. Wishart Process:
   1. Scale matrix (Philipov and Glickman (2006), Asai and So (2010)).
   2. Conditional Laplace transformation (Gourieroux, Jasiak, Sufana (2009)).
3. Positive definite Lévy Process:
   1. Barndorff-Nielsen and Stelzer (2006), Pigorsch and Stelzer (2006).
4. Factor model:
   1. Mean factor (Pitt and Shephard (1998), Chib, Nardari, and Shephard (2006)).

Matrix exponential and logarithm

For $p \times p$ matrix $\mathbf{A}$, matrix exponential is defined by

$$\exp(\mathbf{A}) \equiv \sum_{s=0}^{\infty} \frac{1}{s!} \mathbf{A}^s,$$

If $\mathbf{A}$ is symmetric, since $\mathbf{A} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}'$, we have

$$\exp(\mathbf{A}) = \mathbf{U} \left(\sum_{s=0}^{\infty} \frac{1}{s!} \mathbf{\Lambda}^s \right) \mathbf{U}' = \mathbf{U} \exp(\mathbf{\Lambda}) \mathbf{U}'.$$

Moreover, there exist the one to one inverse transformation (matrix logarithm) (Chiu, Leonard, and Tsui(1996)).

Multivariate Realized SV (MRSV) model

$\boldsymbol{y}_t$:$p \times 1$, daily return vector of the day t.
$\mathbf{X}_t = \log RC_t$:Log realized covariance of the day t.
$\exp(\mathbf{H}_t)$: $p \times p$, covariance matrix of $\boldsymbol{y}_t$

$$
\begin{aligned}
\boldsymbol{y}_t &= \exp\left(\frac{\mathbf{H}_t}{2}\right)\boldsymbol{\varepsilon}_t \\
\mathbf{X}_t &= \boldsymbol{\Xi} + \mathbf{H}_t + \mathbf{U}_t, \\
\mathbf{H}_{t+1} &= \mathbf{M} + \tilde{\boldsymbol{\Phi}} \odot (\mathbf{H}_{t-1} - \mathbf{M}) + \mathbf{E}_t,
\end{aligned}
$$

$\mathbf{H}_t = [h_{ij,t}]$, $\mathbf{U}_t = [\upsilon_{ij,t}]$, $\mathbf{E}_t = [\eta_{ij,t}]$, $\boldsymbol{\Xi} = [\xi_{ij}]$, $\mathbf{M} = [\mu_{ij}]$, $\tilde{\boldsymbol{\Phi}} = [\phi_{ij}]$,
$i, j = 1, \ldots, p$, are $p \times p$ symmetric matrices.

$\boldsymbol{x}_t$:= vech($\mathbf{X}_t$), $\boldsymbol{h}_t$:= vech($\mathbf{H}_t$), $\boldsymbol{\xi}$:= vech(Ξ), $\boldsymbol{\mu}$:= vech($\mathbf{M}$),
$\boldsymbol{\Phi}$:= diag(vech($\tilde{\boldsymbol{\Phi}}$)).

$$
\begin{aligned}
\boldsymbol{y}_t &= \exp\left(\frac{\mathbf{H}_t}{2}\right)\boldsymbol{\varepsilon}_t, \quad t = 1, \ldots, n, && (4)\\
\boldsymbol{x}_t &= \boldsymbol{\xi} + \boldsymbol{h}_t + \boldsymbol{\upsilon}_t, \quad t = 1, \ldots, n, && (5)\\
\boldsymbol{h}_{t+1} &= \boldsymbol{\mu} + \boldsymbol{\Phi}(\boldsymbol{h}_t - \boldsymbol{\mu}) + \boldsymbol{\eta}_t, \quad t = 1, \ldots, n-1, && (6)\\
& \quad \boldsymbol{h}_1 \sim \mathcal{N}_q\left(\mathbf{0}, \boldsymbol{\Sigma}_0\right), \\
\begin{pmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{\eta}_t \end{pmatrix} &\sim \text{i.i.d.}\mathcal{N}_{p+q}(\mathbf{0}, \boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \begin{pmatrix} \mathbf{I}_p & \boldsymbol{\Sigma}_{\varepsilon\eta} \\ \boldsymbol{\Sigma}_{\eta\varepsilon} & \boldsymbol{\Sigma}_{\eta\eta} \end{pmatrix}, \\
\boldsymbol{\upsilon}_t &\sim \text{i.i.d.}\mathcal{N}_q(\mathbf{0}, \boldsymbol{\Sigma}_{\upsilon\upsilon}),
\end{aligned}
$$

MESV model with leverage

If the equation (4) is dropped, this model is reduced to matrix exponential SV (MESV) model (Ishihara, Omori, and Asai (2011))

$$
\begin{aligned}
\boldsymbol{y}_t &= \exp\left(\frac{\mathbf{H}_t}{2}\right)\boldsymbol{\varepsilon}_t, \\
\mathbf{H}_{t+1} &= \mathbf{M} + \bar{\boldsymbol{\Phi}} \odot (\mathbf{H}_t - \mathbf{M}) + \mathbf{E}_t,
\end{aligned}
$$

We compare MRSV model with MESV models.

Matrix exponential SV model with leverage effects

Technical issues and estimation and inference strategy for MRSV model and MESV model.

- High dimensional parameters and latent variables
  $\rightarrow$ Bayesian estimation using Markov chain Monte Carlo method.

- Model comparisons. We can't compare MRSV with MESV using Bayesian marginal likelihood or information criterion.
  $\rightarrow$ Model comparison based on the predictive performance.

- Interpretation of the parameters is difficult. (e.g. The negative $(1,1)$ element of $\mathbf{\Sigma}_{\varepsilon\eta}$ doesn't imply leverage effect on y_{1t})
  $\rightarrow$ Graphical presentations via simulations, e.g. paths of the standard deviations and correlations from $\exp(\mathbf{H}_t)$, news impact curves, and caluclating the process of principal component.

Matrix exponential: example

$a, \rho \in \mathbf{R}$

$$\mathbf{A} = \begin{pmatrix} a & \rho \\ \rho & a \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} a & \rho & \rho \\ \rho & a & \rho \\ \rho & \rho & a \end{pmatrix}.$$

Then

$$\begin{aligned} \exp(\mathbf{A}) &= \begin{pmatrix} (e^{a+\rho} + e^{a-\rho})/2 & (e^{a+\rho} - e^{a-\rho})/2 \\ (e^{a+\rho} - e^{a-\rho})/2 & (e^{a+\rho} + e^{a-\rho})/2 \end{pmatrix}, \\ \exp(\mathbf{B}) &= \begin{pmatrix} (e^{a+2\rho} + 2e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 \\ (e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} + 2e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 \\ (e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} - e^{a-\rho})/3 & (e^{a+2\rho} + 2e^{a-\rho})/3 \end{pmatrix}. \end{aligned}$$

$$(e^{a+2\rho} + 2e^{a-\rho})/3 \geq (e^{a+\rho} + e^{a-\rho})/2,$$

This shows that the interpretation of the parameters depends on the dimension of $\boldsymbol{y}_t$.

MCMC algorithm for MRSV models

We consider a MCMC algorithm to generate from the posterior

$$\pi(\boldsymbol{\theta}|Y_n) \quad \propto \quad f(Y_n|\boldsymbol{\theta})\pi(\boldsymbol{\theta}).$$

1. Generate $\boldsymbol{h}|\boldsymbol{\beta}, \phi, \boldsymbol{\Sigma}, \boldsymbol{Y}$
    1. *Single-move sampler* (Independent MH)
       Generate $\boldsymbol{h}_t|\boldsymbol{h}_{-t}, \boldsymbol{\beta}, \phi, \boldsymbol{\Sigma}, \boldsymbol{\Sigma}_\upsilon, \boldsymbol{y}, t = 1, \ldots, n$
2. Generate $\boldsymbol{\beta}|\boldsymbol{\Sigma}, \phi, \boldsymbol{\Sigma}_\upsilon, \boldsymbol{h}, Y_n$ (Gibbs sampler).
3. Generate $\boldsymbol{\Sigma}_\upsilon|\boldsymbol{\beta}, \phi, \boldsymbol{h}, Y_n$ (Gibbs sampler).
4. Generate $\boldsymbol{\Sigma}|\boldsymbol{\beta}, \phi, \boldsymbol{\Sigma}_\upsilon, \boldsymbol{h}, Y_n$ (Independent MH).
5. Generate $\phi|\boldsymbol{\beta}, \boldsymbol{\Sigma}, \boldsymbol{\Sigma}_\upsilon, \boldsymbol{h}, Y_n$ (Independent MH).

where $Y_n = (\boldsymbol{y}_1, \boldsymbol{x}_1, \ldots, \boldsymbol{y}_n, \boldsymbol{x}_n)$, $\boldsymbol{\beta} = (\boldsymbol{\mu}, \boldsymbol{\xi})$.
Blocked multimove sampler (Omori and Watanabe (2008)) via simulation smoother can be applied. Considering the computational time and inefficiency of the MCMC, we use single move sampler.

Data set

1. Dataset provided by Noureldin, Shephard and Sheppard (2011), ”Multivariate High-Frequency-Based Volatility (HEAVY) Models”.
2. 1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil. daily return (open to close) and realized covariance matrix.
3. They calculate from NYSE TAQ database.
4. Period
    1. From Feb. 2 2001 to Dec. 31, 2006 (1486 days for estimation)
    2. From Jan. 2 2006 to Mar. 8, 2008 (300 days for prediction window)
5. We exclude financial crisis period from the data set. Because our model is basic one but many abnormal events have appeared, e.g. for Bank of America,
    1. Public fund injection ($ 250 million), Oct. 14, 2008.
    2. Acquisition of Merrill Lynch & Co., Inc., Jan. 1, 2009.
    3. Earnings announcement (net profit $ 40 million), Jan. 16, 2009. However, because of the debt of Merrill Lynch($ 271 million), additional public fund injection ($ 200 million) is announced.
    4. Extinguish $ 450 million Dec. 2, 2009.

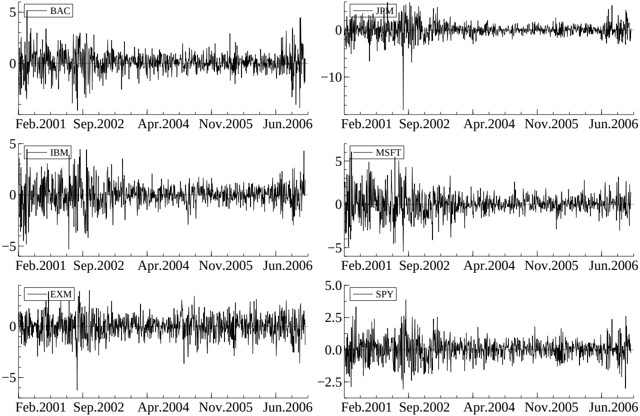


Figure: Daily returns of the series

Result of SV vs RSV

First we fit univariate models
The difference of parameter estimates (one dimensional SV vs one dimensional RSV model)

- 95% credible intervals of parameters except μ get closer in RSV models.
- Leverage effect and autocorrelation parameters are smaller in RSV models.
- 95% credible interval of ξ(log scale bias of RV) include zero.

Univariate SV model

Estimation result of parameters of the univariate SV and RSV
Posterior means and 95% credible intervals

	SV		RSV	
	mean	[95%interval]	mean	[95%interval]
ϕ_1	0.980	[0.965, 0.991]	0.974	[0.960, 0.986]
ϕ_2	0.989	[0.979, 0.997]	0.983	[0.973, 0.993]
ϕ_3	0.990	[0.980, 0.998]	0.979	[0.967, 0.991]
ϕ_4	0.989	[0.979, 0.997]	0.985	[0.974, 0.995]
ϕ_5	0.968	[0.947, 0.984]	0.968	[0.953, 0.982]
$\rho_{1,\varepsilon\eta}$	-0.449	[-0.599, -0.271]	-0.300	[-0.402, -0.198]
$\rho_{2,\varepsilon\eta}$	-0.485	[-0.632, -0.311]	-0.309	[-0.414, -0.204]
$\rho_{3,\varepsilon\eta}$	-0.381	[-0.562, -0.173]	-0.313	[-0.412, -0.210]
$\rho_{4,\varepsilon\eta}$	-0.323	[-0.509, -0.116]	-0.271	[-0.376, -0.162]
$\rho_{5,\varepsilon\eta}$	-0.283	[-0.480, -0.075]	-0.236	[-0.342, -0.126]

1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil.

95% credible intervals of parameters get closer and leverage effect and autocorrelation parameters are smaller in RSV models

Univariate SV model

Estimation result of parameters of the univariate SV and RSV
Posterior means and 95% credible intervals

	SV		RSV	
	mean	[95%interval]	mean	[95%interval]
$e^{\mu_1/2}$	1.077	[0.852, 1.436]	0.993	[0.819, 1.235]
$e^{\mu_2/2}$	1.113	[0.656, 1.645]	1.162	[0.824, 1.574]
$e^{\mu_3/2}$	0.865	[0.458, 1.310]	0.918	[0.701, 1.139]
$e^{\mu_4/2}$	1.036	[0.626, 1.567]	1.035	[0.735, 1.396]
$e^{\mu_5/2}$	0.987	[0.852, 1.136]	0.992	[0.863, 1.133]
ξ_1			0.061	[-0.018, 0.141]
ξ_2			0.082	[0.000, 0.162]
ξ_3			0.052	[-0.030, 0.127]
ξ_4			0.201	[0.124, 0.274]
ξ_5			0.146	[0.069, 0.224]

1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil.
95% credible intervals of parameters of μ doesn't get closer and ξ's (log scale bias of RV) include zero.

Result of MESV vs MRSV

The results of the estimates of the Multivariate models
In the MRSV model,

1. The paths of the volatility and correlations behave sharply.
2. 95% intervals of the news impact curve are narrow.
3. News impact curves on the correlation are constant.
4. The paths of principal component process and their volatilities also behave sharply.

MESV vs MRSV plot 1

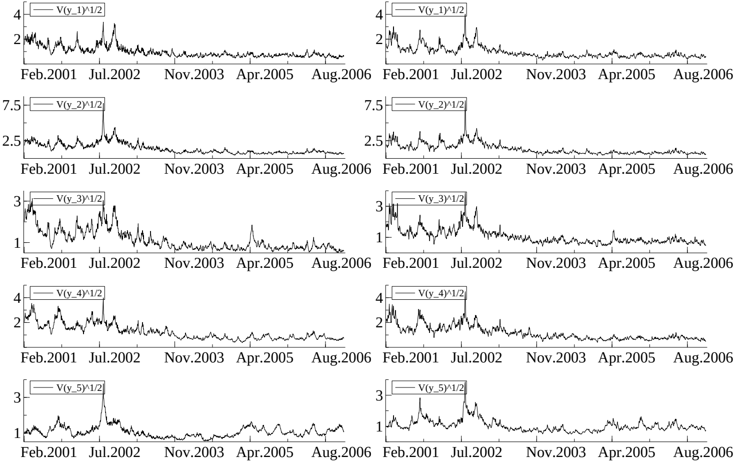


Figure: Estimated path of the standard deviations. MESV (Left), MRSV (Right) 1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil. The paths of the volatility behave sharply.

MESV vs MRSV plot 2

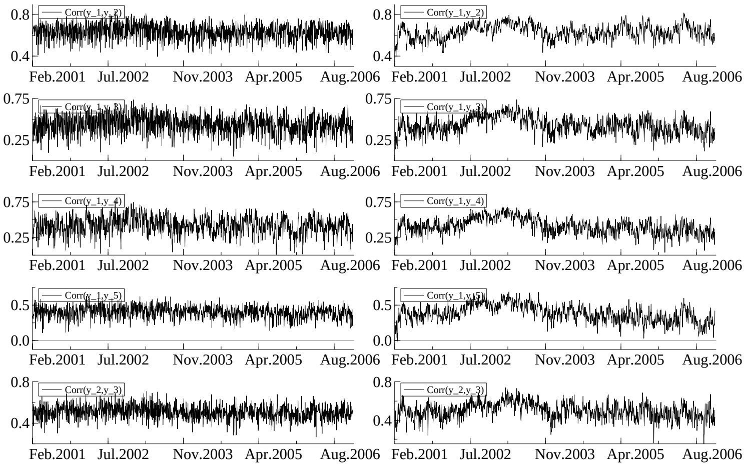


Figure: Estimated path of the correlations 1. MESV(left), MRSV(right) 1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil. In 2002 and 2003 correlations increase.

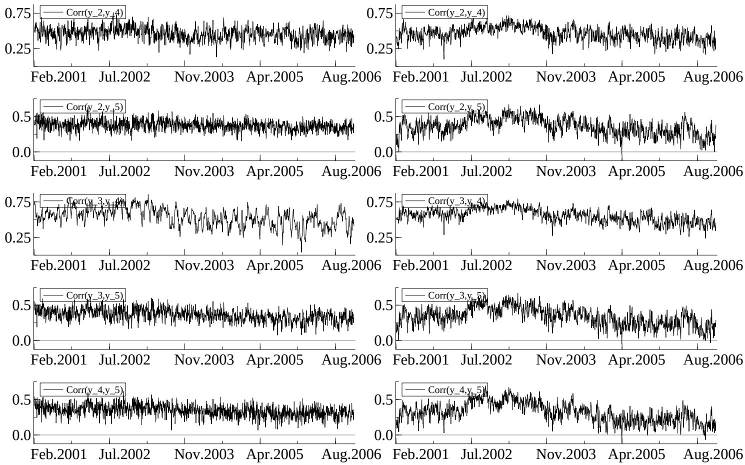


Figure: Estimated path of the correlations 1. MESV(left), MRSV(right) 1:Bank of America, 2:J.P. Morgan, 3:IBM, 4:Micro Soft, 5:Exxon Mobil. In 2002 and 2003 correlations increase.

Conditional vs unconditional news impact curve.

To show the leverage effect we write news impact curves.
News impact curves: the curves of the elements of $\exp(\mathbf{H}_{t+1})$ given changing $\boldsymbol{y}_t$'s.
Two approaches to handle $\boldsymbol{h}_t$'s

1. Conditioning at stationary means $\boldsymbol{h}_t = \boldsymbol{\mu}$ and given points $\boldsymbol{y}_t$. (Asai and McAleer (2006))
2. Integrate out by stationary distribution $h_t^{\dagger} \sim \mathcal{N}_q(\boldsymbol{\mu}, \boldsymbol{\Sigma}_0)$ and $\boldsymbol{y}_t^{\dagger} \sim \mathcal{N}_q(\mathbf{0}, \exp \mathbf{H}_t^{\dagger}))$. (Takahashi, Omori, and Watanabe (2012))

Using the distribution of $\boldsymbol{h}_{t+1}|\boldsymbol{y}_t, \boldsymbol{h}_t, \boldsymbol{\Sigma}, \boldsymbol{\mu}, \boldsymbol{\Phi}$ given by

$$
\begin{aligned}
\boldsymbol{h}_{t+1} &= \boldsymbol{\mu} + \boldsymbol{\Phi}(\boldsymbol{h}_t - \boldsymbol{\mu}) + \boldsymbol{\Sigma}_{\eta\varepsilon} \exp(-\mathbf{H}_t/2)\boldsymbol{y}_t + \eta_t^{\dagger}, \\
\eta_t^{\dagger} &\sim \mathcal{N}_{p(p+1)/2}(\mathbf{0}, \boldsymbol{\Sigma}_{\eta\eta} - \boldsymbol{\Sigma}_{\eta\varepsilon}\boldsymbol{\Sigma}_{\varepsilon\eta}).
\end{aligned}
$$

Conditional news impact curve under MRSV models

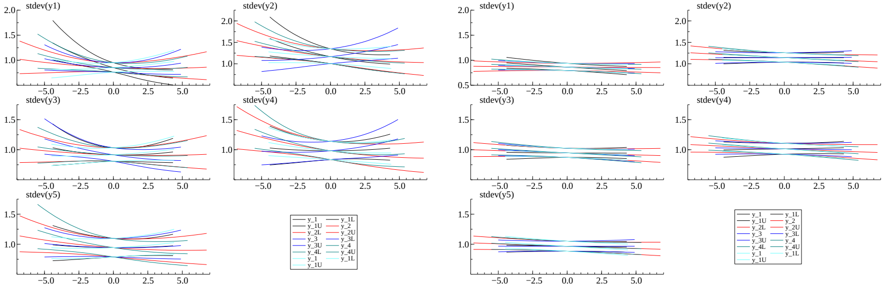
We set $\boldsymbol{h}_t = \boldsymbol{\mu}$ and $\boldsymbol{\eta}_t^\dagger = \mathbf{0}$.
We show the news impact curves, the posterior means of $\exp(\mathbf{H}_{t+1})$ under the changing $\boldsymbol{y}_t$, where

$$\boldsymbol{h}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Sigma}_{\eta\varepsilon} \exp(-\mathbf{M}/2)\boldsymbol{y}_t$$

The domain of the estimated curve is restricted to the range within ± 4 sample standard deviation of actual returns for each y_{it}.
Each y_{it} is moved independently.
We write posterior mean and 95% credible intervals of the news impact curves.

Conditional news impact curves

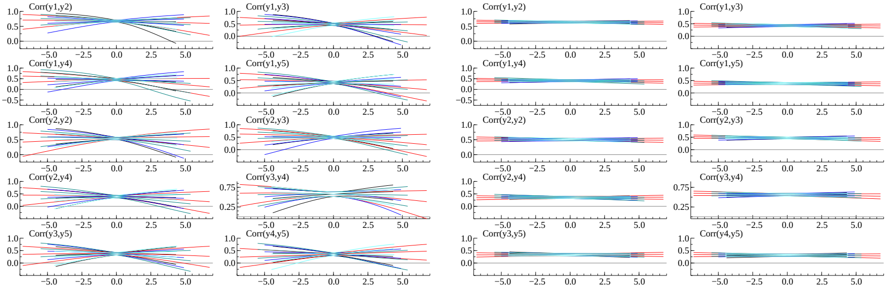
News impact on the standard deviations



Black:Bank of America, Red:J.P. Morgan, Blue:IBM, Green:Micro Soft,
light blue (cyan):Exxon Mobil.

Left panels: MESV model, right panels: MRSV model,
Horizontal axis: the value of $\boldsymbol{y}_t$, vertical axis:Standard derivations of $\boldsymbol{y}_{t+1}$.
The leverage effect of MRSV model get smaller.

News impact on the correlations

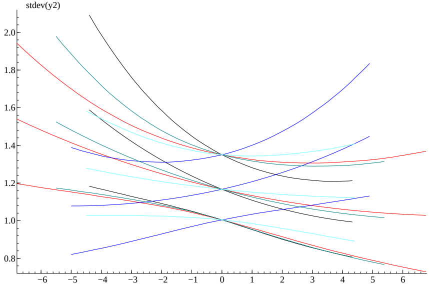


Black:Bank of America, Red:J.P. Morgan, Blue:IBM, Green:Micro Soft, light blue (cyan):Exxon Mobil.

Left panels: MESV model, right panels: MRSV model,
Horizontal axis: the value of y_t, vertical axis:correlations of y_{t+1}.
The correlation of the MRSV model takes constant value. There is no effect from the value of y_t, on the correlations of y_{t+1}.

Conditional news impact curves

The news impact curves on the standard deviations of J.P.Morgan.

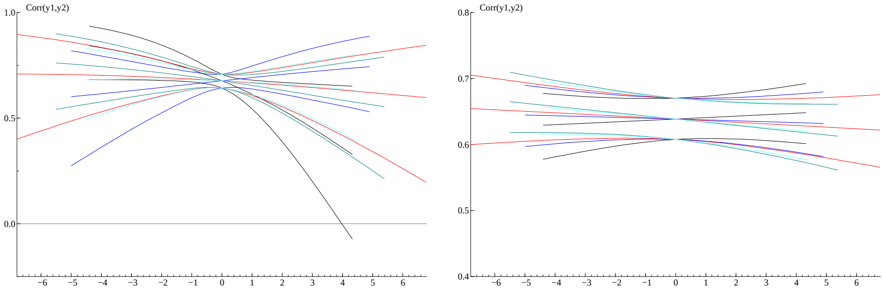


Black:Bank of America, Red:J.P. Morgan, Blue:IBM, Green:Micro Soft, light blue (cyan):Exxon Mobil.

Left panels: MESV model, right panels: MRSV model,
Horizontal axis: the value of y_t, vertical axis:Standard derivations of y_{t+1}.
Leverage effect from the self-series, Micro Soft, and Exxon Mobil exist.
Positive asymmetry seems to exist from IBM in the MESV, but Looks constant in the MRSV.

Conditional news impact curves

The news impact curves on the correlation of Bank of America and J.P. Morgan.



Black:Bank of America, Red:J.P. Morgan, Blue:IBM, Green:Micro Soft, light blue (cyan):Exxon Mobil.

Left panels: MESV model, right panels: MRSV model,
Horizontal axis: the value of y_t, vertical axis:correlations of y_{t+1}.
95% interval of MESV model diverges on the tail and of MRSV model takes constant value.

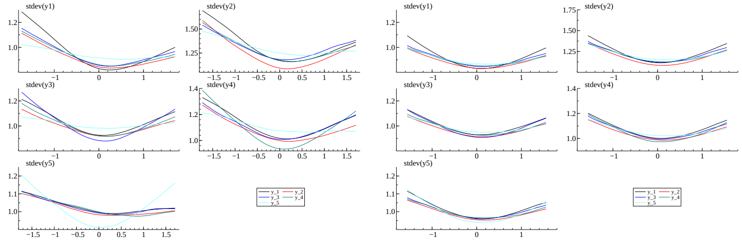
Unconditional news impact curves under MRSV models

1. Generate $h_t^i \sim \mathcal{N}_q(\boldsymbol{\mu}, \boldsymbol{\Sigma}_0^i)$, for each MCMC sample i. and $\boldsymbol{y}_t^i | h_t^i \sim \mathcal{N}_q(\mathbf{0}, \exp(\mathbf{H}_t^i))$
2. Calculate $\exp(\mathbf{H}_{t+1}^i)$ using $\boldsymbol{h}_{t+1}^i = \boldsymbol{\mu}^i + \boldsymbol{\Phi}^i(\boldsymbol{h}_t^i - \boldsymbol{\mu}^i) + \boldsymbol{\Sigma}_{\eta\varepsilon}^i \exp(-\mathbf{H}_t^i/2)\boldsymbol{y}_t^i$.
3. Conduct nonparametric kernel regression: regress standard deviations or correlations calculated from $\exp(\mathbf{H}_{t+1}^i)$ on the element $\boldsymbol{y}_t^i$.

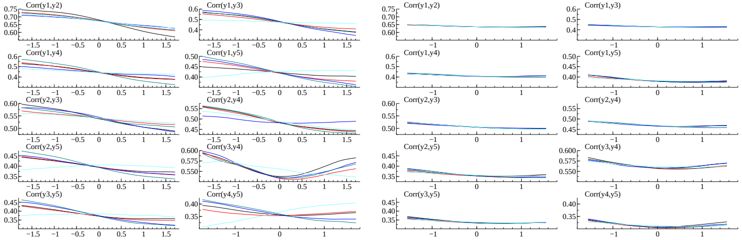
We only show the estimated posterior mean of the unconditional posterior

Unconditional news impact curves

News impact on the standard deviations



News impact on the correlations



Black:Bank of America, Red:J.P. Morgan, Blue:IBM, Green:Micro Soft,

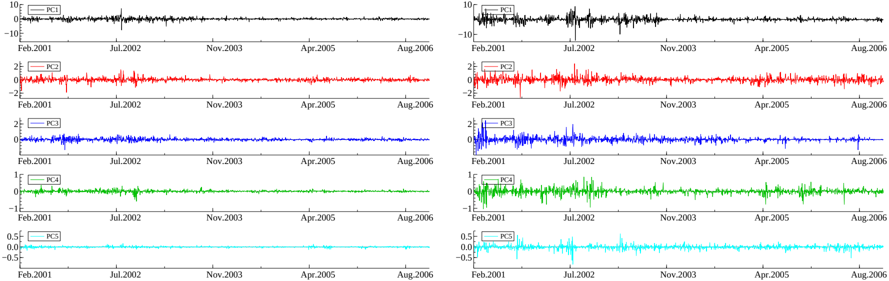
Dynamic principal component based on the MRSV models.

As another application, we make a principal components of $\boldsymbol{y}_t$
Let $\mathbf{H}_t = \mathbf{W}_t \boldsymbol{\Lambda}_t \mathbf{W}_t'$ denote a spectral decomposition of the log volatility matrix where $\mathbf{W}_t \mathbf{W}_t' = \mathbf{I}_p$ and $\boldsymbol{\Lambda}_t$ is a diagonal matrix whose elemets are arranged in descending order. Then the return equation can be transformed as follows

$$\begin{aligned} \mathbf{W}_t' \boldsymbol{y}_t &= \exp(\boldsymbol{\Lambda}_t/2)\boldsymbol{\varepsilon}_t^*, \\ \boldsymbol{\varepsilon}_t^* &= \mathbf{W}_t' \boldsymbol{\varepsilon}_t \sim \mathcal{N}(0, \mathbf{I}_p). \end{aligned}$$

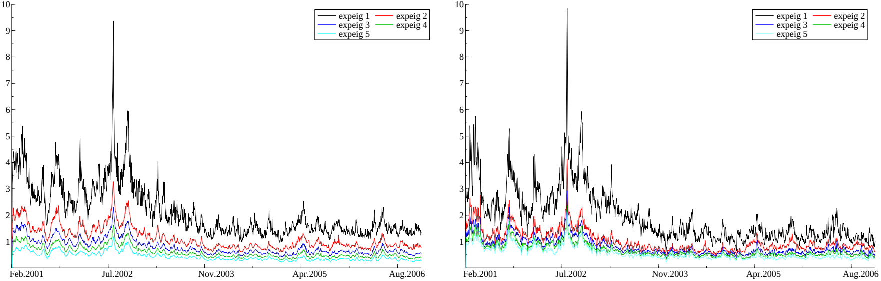
$\boldsymbol{\varepsilon}_t^*$ can be interpreted as the principal component vector, and $\exp(\boldsymbol{\Lambda}_t)$ is it's variance. Further, $\mathbf{W}_t'$ is the dynamic portfolio weight matrix (or loading matrix) to make the principal components.

PC of MESV vs MRSV



Estimated path of the principal components $\mathbf{W}_t' \boldsymbol{y}_t$
MESV(left), MRSV(right), The paths of principal component process also behave sharply.

PC volatilities of MESV vs MRSV



Estimated path of the standard deviations of the principal components, diagonal elements of $\exp(\mathbf{\Lambda}_t/2)$.
MESV(left), MRSV(right)

Prediction & Model selection

Model comparison between MRSV and MESV models

1. Marginal likelihood, DIC, Other information criterion
   (Since data sets are different, it's difficult to use).
2. Prediction
   1. Since predictand variable (volatility) is unobserved, proxy predictand variables (RM's) are used.
   2. Patton (2006), Patton and Sheppard (2010) propose robust loss functions under the proxy variables.

1. Patton (2006), Patton and Sheppard (2010)
$\hat{\Sigma}_t$: Proxy of volatility,
V_t: true unobserved volatility,
$\hat{V}_t$: predictor of true volatility,
If $E_t[\hat{\Sigma}_{t+T}] = V_{t+T}$ then for the predictors $\hat{V}_t^A$, $\hat{V}_t^B$ based on the models A, B,

$$\begin{aligned} & E[L(\hat{\Sigma}_t, \hat{V}_t^A)] \geq (\leq) E[L(\hat{\Sigma}_t, \hat{V}_t^B)] \\ \Leftrightarrow \quad & E[L(V_t, \hat{V}_t^A)] \geq (\leq) E[L(V_t, \hat{V}_t^B)] \end{aligned}$$

holds under the quasi-likelihood loss function.

$$L(\hat{\Sigma}_t, \hat{V}_t) = \log|\hat{V}_t| + \mathrm{tr}(\hat{\Sigma}_t \hat{V}_t^{-1}) - K_t$$

(K_t doesn't depend $\hat{V}_t$)
For the proxy, we use future RM_t.

Prediction & Model selection

Setting:

1. 1486 days for estimation,
2. 300 days for prediction window (from Jan. 2 2006 to Mar. 8, 2008)

- Rolling window with forecast horizon $T = 1, 5, 10, 22$(1 day, 1,week, 2 weeks，1 month).
- The predictor is the mean of the posterior predictive distributions.

Loss functions

Difference of losses between MRSV model and MESV model
Proxy predictand variables : $\exp(\mathbf{X}_{n+T})$,
predictor: the mean of the posterior predictive distribution of $\exp(\mathbf{H}_{n+T})$

Losses						
T	MESV			MRSV		
T	mean	st.er.	[95% interval]	mean	st.er.	[95% interval]
1	-7.58	1.74	[-11.00,-4.17]	-9.52	2.03	[-13.52,-5.53]
5	-5.47	1.37	[-8.13,-2.74]	-6.61	1.50	[-9.48,-3.73]
10	-4.18	1.13	[-6.40,-1.96]	-5.20	1.17	[-7.50,-2.89]
22	-3.01	0.89	[-4.76,-1.26]	-4.18	0.95	[-6.06,-2.30]

T	The difference of losses (MRSV−MESV)		
	mean	st.er.	[95% interval]
1	-1.94	0.61	[-3.14,-0.74]
5	-1.17	0.31	[-1.77,-0.57]
10	-1.01	0.21	[-1.42,-0.61]
22	-1.17	0.14	[-1.44,-0.90]

Mean, standard error, and 95% confidence intervals. The loss of the

Summary and Future works

1. MRSV model with leverage effect.
2. Application to stock return and realized covariance.
3. Model comparison between with and without realized covariance. The MRSV have higher performance.

Future work

1. Application of the principal component vector processes.
2. Extensions:
    1. Simultaneous jumps of $\boldsymbol{y}_t$ and $\boldsymbol{x}_t$, and using bi-power variations.
    2. Superpositioned factor model ($\mathbf{H}_t = \mathbf{H}_{1t} + \mathbf{H}_{2t}$)
    3. Modeling means of daily returns using high frequency data.
3. Comparison with other specifications: HIBRID-MESV, HEAVY type SV, matrix exponential realized GARCH and so on.
4. Prediction ranking consistency based on the Bayes risk with proxy predictand.