Monetary Policy under Bounded Rationality

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Abstract

This paper studies effects of monetary policy under bounded rationality when agents confront uncertainty in model parameters. The model we use in this paper is a version of the New Keynesian model with an IS-Phillips curve as was used in Christiano, Tranbandt, and Walentin (2011). We consider two additional elements of the model that characterize the environment of uncertainty: (i) private agents form forward looking expectations via adaptive learning in the presence of parameter uncertainty, and (ii) the central bank that conducts monetary policy confronts uncertainty about working capital channel. In the presence of parameter uncertainty the central bank adopts two approaches for conducting monetary policy: robust control and feedback control (Bayesian updating algorithm). We study how these two approaches are implemented in an environment of bounded rationality. Also, we evaluate three different policy schemes, policy under rational expectation (perfect information), policy based on the robust control, and policy based on the feedback (Bayesian) control, by examining which policy achieves better outcome for economic stability. Our quantitative analysis shows that the robust control policy achieves the best result under discretion. However, the robust control policy shows the worst performance under commitment. Also, for policies based on perfect information and the feedback control under commitment outperform those under discretion.

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1 Introduction

In modern macroeconomic research, monetary policy as an economic stabilizer has been a main issue. The important goal of the research is to figure out how to design desirable monetary policies under various economic conditions. Through the seminal work of Kydland and Prescott (1977), economics literature became aware of the dynamic consistency problem. That is, commitment policy may be infeasible, because discretionary policy is dynamically consistent. In the work of Taylor (1993), widely known *Taylor principle* is introduced. That is, an increase in inflation brings an increase in the real interest rate.

The performance of the monetary policy has been widely studied in various economic models. In addition, many of recent literatures for monetary economics have been focused on dynamic stochastic general equilibrium (DSGE) models with nominal rigidities and forward looking expectaions of economic agents, since the model can explain the non-neutrality of money with microfounded structure. Typically, Clarida, Gali, and Gertler (1999), MacCallum (1999), Woodford (1999) and Woodford (2001) studied the effects of monetary policy under the *New Keynesian* DSGE model. Most of these works have tried to find the *optimal* monetary policy that minimizes policymaker's loss function, under the assumption of perfect rationality of economic agents. Under this assumption, the central bank can achieve economic stability by considering only the *fundamentals* of the economy.

However, more recently, there has been various works of monetary policy with the model under the economic agents' bounded rationality. The discussion on bounded rationality in the economic model was initiated by Sims (1988) and Chung (1990), and Sargent (1993) who studied conditions and approaches for bounded rationality. Recently, the *Learning* model has received attention for its usefulness in explaining the expectation mechanism under bounded rationality and has been used for optimal monetary policy under bounded rationality by Sargent (1999), Bullard and Mitra (2002), Cho, Williams and Sargent (2002), Evans and Honkapohja (2003), Bullard and Cho (2005), and Evans and Honkapohja (2008) etc. The main question of the learning literature was whether the model can converge to the certain equilibrium (self-confirming equilibrum) via learning and whether the equilibrium can reach the rational expectation equlibrium. Evans and Honkapohja (2001)'s well-known book *Learning and Expectaions in Macroeconomics* summarizes issues and solving techniques of learning models in macroeconomics. From the point of view of monetary policy under learning, related literatures suggest that the policymaker should consider the *expectations* of private agents to stabilize the economy, even if the expectations are not from rational expectation.

In fact, in sync with bounded rationality framework, arguments about the possibilities of *misspecification* of economic models has emerged. To robustly deal with this model misspecification, a new control theory, *Robust Control* has been applied to economics. After the pioneering work of Hansen and Sargent (2001), there has been many related research developing theories and applications of robust control in macroeconomics. Giannoni (2002) showed the solving method to derive robust taylor rules in the forward-looking macroeconomic model when there is the parameter uncertainty, and showed that the robust taylor rules prescribe a stronger response of the interest rate to fluctuations in inflation and output gap than the case in the without uncertainty. Orphanides and Williams (2007) examined robust monetary policy when central bank and private agents possess imperfect information about the structure of the economy, using the structural model of natural rate Phillips curve and unemployment equation. It shows that optimal policy under perfect information can perform poorly if information is imperfect, and a more aggressive response to inflation, and a smaller response to the perceived employment gap would be more efficient in this imperfect information. The robust monetary policy and commitment problem is also has been examined. The celebrated work of Woodford (2009) considers optimal monetary stabilization policy in a classic New Keynesian model with cost-push shock in the Phillips curve, when the central bank has uncertainty about privates' expectations. In the work, by solving multiplier game between central bank and malevolent nature, it was found that a concern for robustness increases the sensitivity of inflation to cost-push shocks under discretionary policy, while it reduces the sensitivity to cost-push shocks under commitment. Also, it was found that the distortions from the discretionary policy become more severe when the central bank allows for the possibility of near-rational expectations of private agents, so that the importance of commitment is increased. Hansen and Sargent (2008) combined techniques and issues of robust control in macroeconomics in a book *Robustness*, and Hansen and Sargent (2011) introduces contents of the book compactly.

In this paper, we analysed learning and robust control problem together in the New Keynesian DSGE model. The model used in the analysis is Christiano, Tranbandt, and Walentin (2011)'s New Keynesian model with working capital channel and input material which are not contained in the classic New Keynesian model. We assumed bounded rationality of central bank and private agents. Central bank does not know the working capital channel coefficient of the model, meanwhile private agents use adaptive learning when they form forward looking expectations. We studied effect of three types of monetary policy, optimal control under perfect information of the parameter, robust control, and feedback control. We examined which policy induces better outcome, and founded that the performance of policies depends on wheter the policy is under commitment or discretion.

Section 2 introduces the structural model and considers the determinacy and learnability condition of the model. Section 3 characterizes the policy problem under the central bank's uncertainty in the model under commitment and discretion. Section 4 conducts quantitative analysis of the former chapters, and section 5 expands the quantitative analysis in the case of a structural parameter varies over time. Finally, Section 6 gives concluding remarks.

2 Model

2.1 CTW (2011) New Keynesian Macroeconomic Model

In this section, we introduce the small-sized New Keynesian DSGE model of Christiano, Tranbandt, and Walentin (2011) (CTW). This model is composed with the IS-Phillips curves with forward looking expectations. The economic system is composed with following two log-linearized equations.

$$\pi_t = \kappa_p[\gamma(1+\phi)x_t + \frac{\psi}{(1-\psi)\beta+\psi}i_t] + \beta E_t\pi_{t+1} \tag{2.1}$$

$$x_t = E_t x_{t+1} - (i_t - E_t\pi_{t+1} - \frac{1}{\gamma}E_t\mu_{z,t+1}) \tag{2.2}$$

In equations (2.1)-(2.2), x_t and π_t denote the output gap and inflation, respectively. i_t stands for nominal interest rate that is used as a policy variable of the central bank. Equation (2.1) represents the relationship between inflation, expected inflation, and the output gap around the steady state. These equations are log linearized versions of equilibrium conditions derived from maximization procedures between competitive final goods firm, monopoliscally competitive intermediate goods firms, and household. See Christiano et al (2011).

In the above model, $\beta, \psi, \phi, \gamma, \kappa_p$ are parameters. β is subjective time discount rate, which takes the value between 0 and 1. $\psi \in [0,1]$ represents the *working capital channel* emphasized by Barth and Ramey (2002). If $\psi = 0$, the intermediate goods firms need not require advanced financing for the cost of labor and input materials. If $\psi = 1$, full amount of the cost must be financed at the *beginning* of the period. ϕ is the Frisch inverse elasticity, the inverse of elasticity of the labor supply. $\gamma \in (0,1]$ denotes the contribution of labor force in the production of intermediate goods. κ_p forms slope of the Phillips curve in the classic New Keynesian model, which is

$$\kappa_p = \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} \tag{2.3}$$

where ξ_p is the degree of price stickiness, which takes the value between 0 and 1. $\hat{\mu}_{t,t+1}$ denotes the difference of technology shock from the steady state between period $t+1$ and t, i.e., $\hat{z}_t - \hat{z}_{t+1}$, where logarithm of z_t is the technology shock occuring in the beginning of period t, and assume that it follows the first order autocorrelation process.

$$\rho_z \log z_t + u_t, \quad u_t : \textit{White noise } N(0, \sigma_z^2). \tag{2.4}$$

2.2 Control under Perfect Information of ψ

From now on, we will represent $\frac{\psi}{(1-\psi)\beta+\psi}$ as κ_ψ, for notational convenience. Also, we assume the monetary policy rule is expectational based rule (EBR), following the suggestion of Evans and Honkaphoja (2003). First, consider the monetary policy under discretion. Following Giannoni (2002)'s specification, let δ be the linear policy rule ($\delta \in \Delta \ \subset \mathbf{R}^n$). Denote $\theta = (\theta_1, \theta_2, .., \theta_m)'$ the finite dimensional vector of structrual parameters, such that $\theta \in \Theta \subset R^m$. Denote vector of endogeneous variables at t such as $\mathcal{F}_t = [\pi_t, x_t, i_t]$. The stochastic process $\mathcal{F}_t$ should satisfy equations (2.1) and (2.2) at all dates t. This can be written as follows

$$\mathcal{G}(\mathcal{F}, \theta) = 0 \tag{2.5}$$

Then loss function of the controller can be denoted by $L_0(\mathcal{F}, \theta)$. So the control problem can be written as

$$\min_{\delta \in \Delta} E[L_0(\mathcal{F}(\delta, \theta), \theta)] \tag{2.6}$$

Assume that the central bank seeks to minimize the following loss function.

$$E[L_0(\mathcal{F}(\delta, \theta), \theta)] = E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2}((\pi_t - \pi^*)^2 + \alpha(x_t - x^*)^2) \tag{2.7}$$

where $\pi^* = x^* = 0$, and $\alpha \in (0,1)$.

Under the model without uncertainty of κ_ψ, by using Clarida, Gali, and Gertler (1999)'s method for deriving optimal discretionary monetary policy, the policy rule can be derived by solving following first order condition

$$\lambda(\pi - \pi^*) + \alpha(x - x^*) = 0 \tag{2.8}$$

where $\lambda = \kappa_p \gamma(1+\phi)$. Then policy under discretion becomes

$$i_t^d = \delta_x E_t x_{t+1} + \delta_\pi E_t \pi_{t+1} + \delta_z \log z_t \tag{2.9}$$

where

$$\delta_x = \frac{\lambda^2 + \alpha}{\lambda^2 + \alpha - \lambda\kappa_p\kappa_\psi}, \delta_\pi = \frac{\lambda^2 + \lambda\beta + \alpha}{\lambda^2 + \alpha - \lambda\kappa_p\kappa_\psi},$$
$$\delta_z = \frac{(\lambda^2 + \alpha)(\rho_z - 1)}{\gamma(\lambda^2 + \alpha - \lambda\kappa_p\kappa_\psi)}.$$

If the policy is under commitment, the policy should be history dependent. Following Taylor (1999)'s work, interest rate policy in this case becomes

$$i_t = \xi_t(\pi_t, \pi_{t-1}, ..., x_t, x_{t-1}, ..., i_{t-1}, i_{t-2}, ...), \tag{2.10}$$

Using Evans and Honkapohja (2008)'s conclusion, commitment policy in our model can be derived from following first order condition

$$\lambda(\pi_t - \pi^*) + \alpha(x_t - x_{t-1}) = 0 \tag{2.11}$$

Consequently, the commitment policy becomes

(2.12), $$i_t^c = \delta_{-1} x_{t-1} + \delta_x E_t x_{t+1} + \delta_\pi E_t \pi_{t+1} + \delta_z \log z_t$$

where $\delta_{-1} = -\frac{\alpha}{\gamma(\lambda^2 + \alpha - \lambda\kappa_p\kappa_\psi)}$. Other parameters are the same as in the discretionary policy.

2.3 Determinacy and Learnability Condition

2.3.1 Under Discretion

Using the equation (2.9), substituting it to (2.2) and substituting this again into (2.1), we get

$$\mathbf{y_t} = \alpha + BE_t\mathbf{y_{t+1}} + c\log z_t \tag{2.13}$$

where

$$\mathbf{y_t} = (\pi_t, x_t)', E_t\mathbf{y}_{t+1} = (E_t\pi_{t+1}, E_tx_{t+1})'$$

$$\alpha = (0,0)', B = \begin{pmatrix} \kappa_p\gamma(1+\phi)(1-\delta_\pi) + \kappa_p\kappa_\psi\delta_\pi + \beta\kappa_p & \gamma(1+\phi)(1-\delta_x) + \kappa_p\kappa_\psi\delta_x \\ 1-\delta_\pi & 1-\delta_x \end{pmatrix},$$

$$c = (\kappa_p((\frac{1}{\gamma}(\rho_z - 1) - \delta_z) + \kappa_\psi\delta_z), \frac{1}{\gamma}(\rho_z - 1) - \delta_z)'.$$

Using Evans and Honkapohja (2001)'s method, the sufficient condition that system (2.13) has a unique deterministic solution can be summarized by the following proposition.

Proposition 1. System (2.13) has an unique deterministic solution if all the eigenvalues of B are in a unit circle. That is,

$$\delta_\pi < \frac{1-\beta+(\kappa_p\kappa_\psi+\beta)\delta_x}{\kappa_p\kappa_\phi}, \tag{2.14}$$

$$\delta_\pi > \frac{-1-\beta+(\kappa_p\kappa_\psi+\beta)\delta_x}{\kappa_p\kappa_\psi}, \tag{2.15}$$

$$(\kappa_p\gamma(1+\phi) - 2\kappa_p\kappa_\psi(\delta-1) + (1+\beta+\kappa_p\kappa_\psi)\delta_x < 2(1+\beta+\kappa_p\kappa_\psi), \tag{2.16}$$

$$\kappa_p\gamma(1+\phi)(\delta_\pi-1)+(1-\beta-\kappa_p\kappa_\psi)\delta_x>0. \tag{2.17}$$

proof : See Appendix A.1.

Thus, parameters $\gamma,\phi,\xi_p,\psi,\beta$ and policy parameters δ_π,δ_x should be located in the region made by equations (2.14)-(2.17). It implies that a central bank may not achieve the rational expectation solution by controlling the policy parameters δ_π,δ_x if parameters of the IS, Phillips curve $\gamma,\phi,\xi_p,\psi,\beta$ are located in the region outside the region made by equations (2.14)-(2.17).

As can be seen in equations (2.1)-(2.2), private agents' forward looking expectations to period $t+1$ influence inflation and the at the period t. In this section, we assume that private agents don't behave under rational expectations but under expectations from bounded rationality, following the works of learning literatures. Private agents do not know the exact values of the parameters of the model. Instead, they estimate the parameters by using given information and use these estimates to form their expectations.

Now we consider the situation where private agents form a forward looking vector $E_t\mathbf{y_{t+1}}$ via adaptive expectation. Let recursive least square (RLS) learning rule be an adaptive expectation procedure. The learnability condition is based on E-stability of Minimum State Variable (MSV) solution. Following Evans and Honkapohja (2001),

$$\mathbf{y_t}=\bar{a}+\bar{k}\log z_t \tag{2.18}$$

where $\bar{a}=(0,0)',\bar{k}=(I-\rho_z B)^{-1}c$. Then, private agents' perceived law of motion (PLM) and the actual law of motion (ALM) of the economy take the following forms:

$$\text{(PLM)}\;\; E_t\mathbf{y_{t+1}} = a_t + k_t\rho_z \log z_t \tag{2.19}$$

$$\text{(ALM)}\;\; \mathbf{y_t} = Ba_t + (\rho B k_t + c)\log z_t \tag{2.20}$$

The T- mapping from PLM to ALM is $T(a,k) = (Ba, \rho Bk + c)'$. Learnability is determined by the following differential equation:

$$\frac{d}{d\tau} = T(a,k) - (a,k) \tag{2.21}$$

and this can be explicitly represented as the following form.

$$\frac{d}{d\tau}(a,k) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} B-I & 0 \\ 0 & \rho_z Bk - I \end{pmatrix} \begin{pmatrix} a \\ k \end{pmatrix} \tag{2.22}$$

According to the method of Evans and Honkapohja (2001), the learnability (E-stability) condition of the system can be summarized by the following proposition:

Proposition 2. The MSV solution $(\bar{a}, \bar{k})$ is learnable if all eigenvalues of $\begin{pmatrix} B-I & 0 \\ 0 & \rho_z Bk - I \end{pmatrix}$ have real parts that are less than 1. That is,

$$2 - \kappa_p\gamma(1+\phi)(1-\delta_\pi) - \kappa_p\kappa_\psi\delta_\pi - \beta - (1-\delta) > 0 \tag{2.23}$$

$$\kappa_p\gamma(1+\phi)(\delta_\pi - 1) + (1-\beta-\kappa_p\kappa_\psi)\delta_x > 0. \tag{2.24}$$

Specifically, private agents' PLM can be written as the following form:

$$E_t\pi_{t+1} = a_{1,t} + k_{1,t}\log z_t \tag{2.25}$$

$$E_t x_{t+1} = a_{2,t} + k_{2,t} \log z_t \tag{2.26}$$

Also, we will assume that private agents update their expectation values by decreasing-gain recursive least square learning rule as follows.

$$\theta_{1,t} = \theta_{1,t-1} + t^{-1} R_t^{-1} X_t (y_{1,t} - \theta'_{1,t-1} X_t)' \tag{2.27}$$

$$\theta_{2,t} = \theta_{2,t-1} + t^{-1} R_t^{-1} X_t (y_{2,t} - \theta'_{2,t-1} X_t)' \tag{2.28}$$

$$R_t = R_{t-1} + t^{-1} (X_t X'_t - R_{t-1}) \tag{2.29}$$

where

$$\theta_{i,t} = (a_{i,t}, k_{i,t})', X_t = (1, \log z_t)', y_{1,t} = \pi_t, y_{2,t} = x_t.$$

2.3.2 Under Commitment

Under the commitment policy (equation (2.11)), the system (2.1)-(2.2) becomes following equation that contains predetermined endogeneous variable.

$$\mathbf{y_t} = A + B E_t \mathbf{y_{t+1}} + C \mathbf{y_{t-1}} + D \log z_t \tag{2.30}$$

where

$$A = (0,0)', B = \begin{pmatrix} \kappa_p(\gamma(1+\phi)(1-\delta_\pi) + \kappa_\psi \delta_\pi & \kappa_p(\gamma(1+\phi)(1-\delta_x) + \kappa_\psi \delta_x \\ 1-\delta_\pi & 1-\delta_x \end{pmatrix},$$

$$C = \begin{pmatrix} 0 & \kappa_p \delta_{-1}(\kappa_\psi - \gamma(1+\phi)) \\ 0 & -\delta_{-1} \end{pmatrix}, D = (\kappa_p((\frac{1}{\gamma}(\rho_z - 1) - \delta_z) + \kappa_\psi \delta_z), \frac{1}{\gamma}(\rho_z - 1) - \delta_z)'.$$

In this case, rational expectation solution is given by (a, b, c) in the following equation.

$$\mathbf{y_t} = a + b\mathbf{y_{t-1}} + c \log z_t \tag{2.31}$$

Following (2.31), private forms expectation values, thus $E_t\mathbf{y_{t+1}} = (I - b)a + b^2\mathbf{y_{t-1}} + (bc + c\rho_z) \log z_t$ Thus equation (2.31) can be represented as follows.

$$\mathbf{y_t} = B(I - b)a + (Bb^2 + C)\mathbf{y_{t-1}} + (Bbc + Bc\rho_z + D) \log z_t \tag{2.32}$$

In this circumstance, RE solution (a, b, c) should satisfy the following three conditions by applying Evans and Honkapohja (2001)'s argument.

$$(I - B - Bb)a = 0 \tag{2.33}$$

$$Bb^2 - b + C = 0 \tag{2.34}$$

$$(I - Bb)C - BC\rho_z = D\rho_z \tag{2.35}$$

by using Uhlig's (1999) method of indeterminate coefficient, we can get (a, b, c). Remark that solution b need not be unique, since b^2 in equation (2.34) implies it can be multiple solution.

The mapping from the PLM to the ALM takes the form

$$T(a, b, c) = ((B_0 + B_1 + B_1 b)a, B_1 b^2 + B_0 b + c, (B_0 c + B_1 bc + B_1 c\rho_z + D\rho_z)) \tag{2.36}$$

Learnability is determined by the following differential equation:

$$\frac{d}{d\tau} = T(a, b, c) - (a, b, c) \tag{2.37}$$

The fixed points of equation (2.37) give MSV solution. Following Evans and Honkapohja (2001), a particular MSV solution $(\bar{a}, \bar{b}, \bar{c})$ is learnable if the MSV fixed point of the equation.(2.37) is locally asymptotically stable at that point. Mathematically, the MSV solution $(\bar{a}, \bar{b}, \bar{c})$ is learnable if all eigenvalues of $DT_b(b) = b' \bigotimes B + I \bigotimes (Bb)$, $DT_c(b, c) = \rho_z \bigotimes B + I \bigotimes (Bb)$, and $B + Bb$ have real parts less than 1.

Let private agents don't know exact solution of (a, b, c) of equation (2.31). Under the Adaptive learning, private agent estimates unknown parameters of the model using current information and updates the estimates when new information is obtained. The private's model in period t takes the form

$$\mathbf{y_t} = a_t + b_t \mathbf{y_{t-1}} + c_t \log z_t \tag{2.38}$$

where $a_t = (a_{1,t}, a_{2,t})'$, $b_t = \begin{pmatrix} b_{11,t} & b_{12,t} \\ b_{21,t} & b_{22,t} \end{pmatrix}$, $c_t = (c_{1,t}, c_{2,t})'$. Under the RLS learning with decreasing gain, the parameters are updated as following fomular.

$$\theta_t = \theta_{t-1} + t^{-1} R_t^{-1} X_{t-1} (\mathbf{y_t} - \theta_{t-1}' X_{t-1})' \tag{2.39}$$

$$R_t = R_{t-1} + t^{-1} (X_{t-1} X_{t-1}' - R_{t-1}) \tag{2.40}$$

where $\theta_t = \begin{pmatrix} a_{1,t} & a_{2,t} \\ b_{11,t} & b_{21,t} \\ b_{12,t} & b_{22,t} \\ c_{1,t} & c_{2,t} \end{pmatrix}$, $X_{t-1} = (1, \pi_{t-1}, x_{t-1}, \log z_t)'$. Thus, private agents' expection of $\mathbf{y}$ from t to $t+1$ takes the from

$$E_t \mathbf{y_{t+1}} = a_t + b_t a_t + b_t^2 \mathbf{y_{t-1}} + (b_t c_t + c_t \rho_z) \log z_t. \tag{2.41}$$

More generally containing the both of cases of discretion and commitment, private agents' subjective model can be rewritten as

$$\mathbf{y_t} = \theta'_{t-1} X_{t-1} + \eta_t \tag{2.42}$$

where error η_t has a mean zero property. That is, the errors are believed to be orthogonal to the regressor X_{t-1}. Thus, with the private agents' subjective expectation, the orthogonality condition can be expressed as

$$E[X_{t-1}(\mathbf{y_t} - \theta'_{t-1} X_{t-1})'] = 0. \tag{2.43}$$

Even if the private agents' model is misspecified, when $t^{-1} \to 0$, this belief induces the self confirming equilibrium (SCE). This convergence in beliefs can be written by the following ordinary differential equations (ODE).

$$\dot{\theta} = R^{-1} \bar{g}(\theta) \tag{2.44}$$

$$\dot{R} = [\bar{M}(\theta) - R] \tag{2.45}$$

where $\bar{g}(\theta) = X_{t-1}(\mathbf{y_t} - \theta'_{t-1} X_{t-1})'$, and $\bar{M}(\theta) = X_{t-1} X'_{t-1}$. This convergence in beliefs is called *mean dynamics* in Cho, Williams, and Sargent (2002).

3 Policy rules of Central Bank

3.1 Parameter Uncertainty of ψ as a Kind of Structural Uncertainty

In the model, we will assume that central bank is unaware of the exact value of parameter ψ (let true ψ is fixed). Central bank doesn't know its distribution, either. Thus, the *Knightian uncertainty* exists in this model. The rationalization of this assumption, although it could be *ad hoc*, can be thought in the following way: In real world, the degree of external financing of the intermediate goods firm (ψ) depends on the firm's financial status and the shock of the financial market. However, this information are the intermediate firm's inside information, so it would be hard for the central bank to know the information, even if the central bank knows the whole *structure* of the economy. This idea is the case of the asymmetric partial information of Svensson and Woodford (2003). In this case, central bank merely knows the fact that $\psi \in [0,1]$, and does not know where the value is located.

In this case, the central bank could take actions in two ways. First, it could be think of the worst-case scenario caused by parameter ψ and derive the policy that can minimize the worst case. Second, using information and structure of the economy that the central bank can use, he could estimate and update the value of ψ. This two policy rules can be called *robust policy*, and *feedback policy*, respectively. In the next two following subsections, we will derive these two policies.

3.2 Robust Control Policy

We will now turn our head to robust control policy under the uncertainty of parameter ψ. Generally, robust control problem under parameter uncertainty can be written as the following minimax problem:

$$\min_{\delta\in\Delta}\max_{\theta\in\Theta} E[L_0(\mathcal{F}(\delta,\theta),\theta)] \tag{3.1}$$

Then, following Giannoni(2002), the pair (δ^*, θ^*) forms Nash equilibrium profile if the pair satisfies the following two conditions simultaneously.

$$\delta^* = \arg\min_{\delta \in \Delta} L(\mathcal{F}(\delta, \theta^*), \theta^*) \tag{3.2}$$

$$\theta^* = \arg\max_{\theta \in \Theta} L(\mathcal{F}(\delta^*, \theta), \theta) \tag{3.3}$$

In our CTW model, under the uncertainty of the parameter ψ (connected to the uncertainty of κ_ψ), this minimax problem can be represented as follows:

$$\min_{\pi_t, x_t} \max_{\kappa_\psi} E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2}((\pi_t - \pi^*)^2 + \alpha(x_t - x^*)^2) \tag{3.4}$$

with constraint (2.1) and (2.2). Then, central bank's robust policy rule can be derived by the following step. First, solve the loss maximization parameter (by malevolent nature) κ_ψ^*. The solution can be represented by the following proposition.

Proposition 3. Let the centeral bank perceive boundary of κ_ψ as $[\kappa_{\psi_{\min}}, \kappa_{\psi_{\max}}]$ and let κ_ψ^* be the malevolent nature's solution. Then κ_ψ^* is

$$\kappa_\psi^* = \kappa_{\psi_{\max}} \quad \text{if} \quad \gamma(1+\phi) > \kappa_{\psi_{\max}} \tag{3.5}$$

$$\kappa_\psi^* = \kappa_{\psi_{\min}} \quad \text{if} \quad \gamma(1+\phi) < \kappa_{\psi_{\max}} \tag{3.6}$$

$proof$. See Appendix A.2.

Second, given κ_ψ^*, derive control policy (as we did in the above section). Then the robust policy under discretion can be derived by

$$i_t^{dr} = \delta_x^r E_t x_{t+1} + \delta_\pi^r E_t \pi_{t+1} + \delta_z^r \log z_t \tag{3.7}$$

where $\delta_x^r = \frac{\lambda^2+\alpha}{\lambda^2+\alpha-\lambda\kappa_p\kappa_\psi^*}, \delta_\pi^r = \frac{\lambda^2+\lambda\beta+\alpha}{\lambda^2+\alpha-\lambda\kappa_p\kappa_\psi^*}, \delta_z^r = \frac{(\lambda^2+\alpha)(\rho-1)}{\gamma(\lambda^2+\alpha-\lambda\kappa_p\kappa_\psi^*)}$. In this case, it is evident that

$$\delta_x^r \geq \delta_x, \;\; \delta_\pi^r \geq \delta_\pi.$$

In Taylor (1998)'s work, it was propsed that the coefficient δ_π should be greater than 1. This means that the interest rate should be raised more than the degree of increase of the inflation rate. This *aggresive* interest rate policy is called *Taylor principle*. In this sense, equation (3.7) implies that central bank reacts more aggressively against expected values of inflation and output gap when he is uncertain about the true value of ψ. It can be summerized that the central bank holds more strict Taylor principle when it executes robust control policy. Proposition 3 also holds in the case of commitment, and the robust control policy under commitment becomes

$$i_t^{cr} = \delta_{-1}^r x_{t-1} + \delta_x^r E_t x_{t+1} + \delta_\pi^r E_t \pi_{t+1} + \delta_z^r \log z_t \tag{3.8}$$

where $\delta_{-1}^r = -\frac{\alpha}{\gamma(\lambda^2+\alpha-\lambda\kappa_p\kappa_\psi^*)}$.

3.3 Feedback Control Policy

After central bank observes private agents' expectation values and state variables $[\pi_{t-1}, x_{t-1}, \log z_t]$, central bank estimates κ_ψ using his known Philips curve (equation (2.1)), via following Kalman filter algorithm.

$$\hat{\kappa}_{\psi,t} = G_t \hat{\kappa}_{\psi,t-1} + P_t F_t'(V_t + F_t P_t F_t')^{-1} e_t \tag{3.9}$$

$$\Sigma_t = P_t - P_t F_t' (V_t + F_t P_t F_t)^{-1} F_t \tag{3.10}$$

where

$$F_t = \kappa_p i_t, V = 0.01, G_t = I,$$

$$P_t = G_t \Sigma_{t-1} G_t' + W, W_t = 0.1,$$

$$e_t = \pi_t - \kappa_t \gamma (1+\phi) x_t - \beta E_t \pi_{t+1}, \text{ and } \theta = \kappa_\psi.$$

Then central bank's feedback policy rule under discretion becomes:

$$i_t^{dk} = \delta_{x,t}^k E_t x_{t+1} + \delta_{\pi,t}^k E_t \pi_{t+1} + \delta_{z,t}^k \log z_t \tag{3.11}$$

where $\delta_{x,t}^k = \frac{\lambda^2+\alpha}{\lambda^2+\alpha-\lambda\kappa_p\hat{\kappa}_{\psi,t}}, \delta_{\pi,t}^k = \frac{\lambda^2+\lambda\beta+\alpha}{\lambda^2+\alpha-\lambda\kappa_p\hat{\kappa}_{\psi,t}}, \delta_{z,t}^k = \frac{(\lambda^2+\alpha)(\rho-1)}{\gamma(\lambda^2+\alpha-\lambda\kappa_p\hat{\kappa}_{\psi,t})}$
Also, feedback policy rule under commitment is

$$i_t^{ck} = \delta_{-1,t}^k x_{t-1} + \delta_{x,t}^k E_t x_{t+1} + \delta_{\pi,t}^k E_t \pi_{t+1} + \delta_{z,t}^k \log z_t \tag{3.12}$$

where $\delta_{-1,t}^k = -\frac{\alpha}{\gamma(\lambda^2+\alpha-\lambda\kappa_p\hat{\kappa}_\psi)}$. In the following section, we conduct numerical analysis on the dynamics of the macroeconomic variables and their stability.

4 Quantitative Analysis I

In the quantitative analysis, we followed the Christiano et al (2011)'s calibration when assigning values for some structural parameter. We set $\beta = 0.99$, $\kappa_p = 0.0858$ (*i.e.*, $\xi_p = 0.75$), $\rho_z = 0.9$, and $\alpha = 0.1$. We set $\phi = 1$ arbitrarily, since with this value, the system is more likely to achieve determinacy. Also we assume that technology shock $\sigma_z = 0.01$. In the case of ψ and γ, we moved the range of two parameters between $\psi \in [0,1]$, $\gamma \in [0.5, 1]$. In the quantitative analysis, we compared three policies (perfect information policy, robust policy, and feedback policy) under discretion and commitment.

β	Discount factor	0.95
ξ_p	Degree of price stickiness	0.75
α	Relative weight for output deviations	0.1
ϕ	Frisch inverse elaticity	1
ρ_z	AR(1) coefficient of technology	0.9
σ_z	Standard deviation of technology shock	0.01

Table 1: Baseline Parameter Values

4.1 Case 1: Under Discretion

Figures 1 shows the results under discretionary policy when ψ is fixed to 0.25 and $\gamma = 0.7$. As it can be seen in Figure 1, inflation and output gap tend to move around near with stationary process (but cannot reach to it), as time goes on, in all three policies. However, it also can be seen that the inflation in the robust policy is generally below that of the perfect information policy and feedback policy. On the other hand, output gap in the robust policy is generally above that of the other policies. Inflation and output gap in the feedback policy are located between the results of the other two policies. Remark that outcomes from adaptive learning converge to specific values by learnability or orthogonality conditions, even if it is not a rational expectations equilibrium. Evans and Honkapohja (2003) and Evans and Honkapohja (2008) show that there would be a large difference in the cooperation between two sectors when central bank uses fundamental based rule (FBR) whereas private agents have bounded rationality. Since these three policies use expectational based rules (EBR), there is not much instability in this outcome. It seems rather peculiar that robust policy can induce better realizations than the perfect information control policy.

This outcome can be explained by two factors. First, this is because the central bank performs discretionary policy and private agents' expectation is bounded rational. Central bank performs *aggressive* monetary policy under the parameter uncertainty. As can be seen from section 3.2, the central bank's policy coefficients δ_x, δ_π are bigger than those of the perfect information policy. This policy induces private agents' belief of future inflation rate and the output gap is lower than the those under the perfect information policy.

In fact, the total loss of three policies vary when the parameter moves. Figure 2 shows the variation of total loss $\frac{1}{2}\sum_{t=1}^{T}(\pi_t^2 + x_t^2)$ when ψ moves (given γ). It reports that the difference of loss of three policies reduces as ψ approaches 1. Next, Figure 3 shows that the total loss of policies when γ moves (given $\psi = 0.25$). Results show that the total loss is decreasing when γ approches 1. In fact, $\psi \to 0$ and $\gamma \to 1$ imply that the Phillips curve reduces to the curve of the classic New Keynesian model.

4.2 Case 2: Under Commitment

The former performance that robust policy has better outcomes than the perfect information control policy (and feedback policy) no longer holds in the commitment policy. Figure 4 shows that dynamics of inflation and output gap is minimized when the central bank performs perfect information policy. Despite some fluctuations in early periods, inflation rate and output gap are moving around zero (steady state), whereas those under robust policy are slightly dislocated. The outcomes under feedback policy are similiar to those of the perfect information policy.

Figure 5 shows that difference of total loss between robust policy and perfect information policy remains nonnegative within the range of $\psi \in [0, 1]$ and the descrepancy is maximized when the true $\psi = 0$. Also, Figure 6 shows that as γ becomes smaller, the total loss under robust policy increases steeply, and the loss under feedback policy is almost similar to that of perfect information policy. These results show that the performance of the robust policy is the worst among the three policies under commitment, whereas that of the perfect information is the best. It also shows that feedback policy is quite effective to catch up the outcome of the perfect information policy.

5 Quantitative Analysis II

5.1 Random Walk of ψ

In this section, we expand the model from section 2 and consider a model that reflects the case where an economy evolves with time varying parameters. This modification may be justified economically as follows. In the real world, the firms' degree of external finance is not constant, but varies depending on the conditions of financial market. We assume that the shock of the financial market is exogenous with continuous state space and the status of the financial market has inertia. Then ψ becomes a time varying paramter ψ_t with its shock is continuous random variable.

However, it is quite difficult to impose stochastic property on ψ directly. This is because, if we construct ψ_t, it is computationally difficult to identify the stochastic character of $\kappa_{\psi,t} = \frac{\psi_t}{(1-\psi_t)\beta+\psi_t}$. Even if we assume that the shock of ψ_t is in a familiar parametric family (e.g. normal distribution), inducing the law of motion of $\kappa_{\psi,t}$ analytically is complicated. Instead, we will impose a stochastic process into $\kappa_{\psi,t}$ directly. This can work since

$$\frac{\partial \kappa_\psi}{\partial \psi} = \frac{\beta}{((1-\psi)\beta+\psi)^2} > 0,$$

the κ_ψ is a monotone increasing function of ψ. In addition, the range of $\kappa_{\psi,t}$ is identical to ψ, that is, [0,1]. Thus, it can be thought that the law of motion of $\kappa_{\psi,t}$ follows the key feature of law of motion of ψ_t. Thus it could be justified that imposing stochastic process of $\kappa_{\psi,t}$ directly as an approximation of ψ_t is not *ad hoc*. By adding certain stochastic properties to key parameters that determine the economic processes, the entire model now harbors a probabilistic uncertainty.

Let's assume that $\kappa_{\psi,t}$ follows truncated random walk process as follows.

$$\kappa_{\psi,t} = \rho_\psi \kappa_{\psi,t-1} + \varepsilon_{\psi,t}, \;\; \varepsilon_{\psi,t} \sim w.n.N(0, \sigma^2_{\psi,t}) \text{ if } 0 \le \kappa_{\psi,t} \le 1 \tag{5.1}$$

$$= 0 \text{ if } \kappa_{\psi,t} < 0$$

$$= 1 \text{ if } \kappa_{\psi,t} > 1.$$

5.2 Case 1: Under Discretion

Simulation studies tell that the general features are not so different from those when ψ is a fixed parameter. Robust policy outperforms perfect information policy, even if ψ_t is a time varying parameter. Figure 7 and Figure 10 are similar to Figure 1 and Figure 3.

This reveals that the effect of *aggresive* policy rule caused by robust control still works in the case of time varying property of $\kappa_{\psi,t}$, since the monetary policies lack commitment and private agents' expectaions are come from bounded rationality.

Another notable feature is that feedback policy is very effective to catch up ψ_t's behavior. In fact, it is well-known that Kalman filter algorithm is effective to estimate time varying parameter when the state space is continuous, the law of motion of the parameter is linear, and the random shock of the parameter is not far from the Gaussian shock. As it can be seen in Figure 8, $\hat{\kappa}_{\psi,t}$ successfully catch up the real behavior of κ_t. Consequently, the coefficients form linear policy function $\delta_{-1,t}$, $\delta_{x,t}$, $\delta_{\pi,t}$, and $\delta_{z,t}$ in the perfect information policy and feedback policy have almost same movements. Figure 9 shows this result. Finally, Figure 10 shows that the total loss of three policies decrease as γ goes to 1, as in the case of fixed ψ.

5.3 Case 2: Under Commitment

As in section 4.2, in the case of policy with commitment, in the Figure 11, the main feature that the robust policy is inferior to that of the perfect information policy and the feedback policy is unchanged, even if ψ_t is time varying.

In addition, as it can be seen from Figure 12 and Figure 13, $\hat{\kappa}_{\psi,t}$ of the feedback policy successfully catches up the real behavior of $\kappa_{\psi,t}$, so the time varying policy coefficients of the feedback policy also catches up those of the perfect information policy. Figure 14 shows consistent results that the total loss of three policies also decrease when $\gamma \to 1$.

6 Conclusion

In this paper, we examined the effects of monetary policy under the bounded rationality and parameter uncertainty. Under the CTW (2011)'s New Keynesian model, private agents form expected values based on bounded rationality, and central bank has uncertainty about the degree of internal financing of intermediate goods firm. We examined which policy induces better outcome between perfect information, robust, and feedback policy.

The rank of performance between three policies depends on whether the policy is under commitment or not. In the discretionary policy, robust control policy shows the best performance among the three policies, within the framework of loss function minimization. On the other hand, robust control policy shows the worst performance among three poilcies with commitment. Also, performance of perfect information policy and the feedback policy with commitment outperform that of discretionary policies.

This result gives following implications. First, the importance of commitment in the aspect of stabilization of the economy. Commitment policy that is history dependent can induce more stable economy with lower inflation rate and output gap near steady state. Second, desirable attitude of central bank to deal with imperfect information of the economy. If central bank conducts commitment policy, feedback action is strongly required; central bank must try to know ψ using available information. However, if central bank wants to conduct discretionary policy, central bank's action to minimize the worst-case scenario induces better outcomes than the perfect information policy, whether central bank intended or not.

The contributions of this paper are in the following. First of all, this paper is the first paper that studied issue of bounded rationality in the CTW's modified New Keynesian structural model. Deriving perfect information, robust, feedback policies under commitment and discretion, and deriving determinacy and learnability condition of the model are innate achievements of this study. Second, the study considered robust control, feedback control, learning, and commitment problem together, that was not done in the previous studies. It is our inventive finding that with imperfect information of economic agents, the rank of performance between robust policy and feedback policy is switched subject to the existence of commitment.

Of course, this paper has some limitations. The policies are not derived by fully dynamic way. Since private agents' forward looking expectation does not come from rational expectation, it is very hard to solve dynamic programming entirely. Thus we derived policies via somewhat static methods as a makeshift. Also, we solved robust control problem in the case of parameter uncertainty, and the Hansen and Sargent (2008)'s canonical form of robust regulator problem still untouched. We remain these for further study.

Appendix A

A.1. Proof of Proposition 1

The characteristic equation of matrix B in the equation (2.5) can be represented as $\det(B - \omega I) = 0$. That is,

$$(A.1.1) \qquad \omega^2 + a_1\omega + a_0$$

where

$$a_1 = -(\kappa_p \gamma(1+\phi)(1-\delta_\pi) + \kappa_p \kappa_\psi \delta_\pi + 1 - \delta_x + \beta),$$
$$a_2 = (1-\delta_x)\beta + \kappa_p \kappa_\psi (\delta_\pi - \delta_x).$$

According to LaSalle (1986), the following two conditions should be satisfied if the roots of equation (A.1.1) (eigenvalues of B) would inside in unit circle.

$$(1)\ |a_0| < 1,$$

$$(2)\ |a_1| < 1 + a_0$$

The inequalities (2.14) and (2.15) are derived from the condition (1), and the inequalities (2.16) and (2.17) are derived from the condition (2).

A.2. Proof of Proposition 2

The characteristic equation of matrix $B - I$ can be represented as $\det(B - (\omega + 1)I) = 0$. That is,

$$(A.2.1) \qquad \omega^2 + b_1\omega + b_1$$

where

$$b_1 = (2 - \kappa_p \gamma(1+\phi)(1-\delta_\pi) - \kappa_p \kappa_\psi \delta\pi - (1-\delta_x) - \beta),$$

$$b_0 = \kappa_p \gamma(1+\phi)(\delta - 1) + (1 - \beta - \kappa_p \kappa_\psi)\delta_x$$

By using quadratic formular, the roots of equation (A.2.1) (eigenvalues of $B - I$) is

$$(A.2.2) \qquad \omega_{1,2} = \frac{-b_1 \pm \sqrt{b_1^2 - 4b_0}}{2}$$

To MSV solutions have learnability (e-stability), the all of eigenvalues should be smaller than 0. Thus, $-b_1 \pm \sqrt{b_1^2 - 4b_0} < 0$ should hold. By using Routh theorm, the necessary and sufficient conditions are $b_1 > 0$ and $b_2 > 0$. When calculate these conditions, equation (2.23) and (2.24) are derived.

A.3. Proof of Proposition 3.

Let $L(\delta^*(\theta), \theta)$ be the total loss of central bank given the optimal policy rule and parameters of system (2.1)-(2.2). Following Lemma 3 of Ginannoni (2002), the worst-case κ_ψ^* (chosen by malevolent nature) is

$$(A.3.1) \qquad \kappa_\psi^* = \kappa_{\psi,\max} \text{ if } \frac{\partial L(\delta^*(\theta), \theta)}{\partial \kappa_\psi} > 0$$

$$(A.3.2) \qquad = \kappa_{\psi,\min} \text{ if } \frac{\partial L(\delta^*(\theta), \theta)}{\partial \kappa_\psi} < 0.$$

The derivative of $L(\delta^*(\theta), \theta)$ with respect to κ_ψ can be represented as

$$(A.3.3) \qquad \frac{\partial L(\delta^*(\theta), \theta)}{\partial \kappa_\psi} = \pi_t(\delta^*(\theta)) \frac{\partial \pi_t(\delta^*(\theta))}{\partial \kappa_\psi} + \alpha x_t(\delta^*(\theta)) \frac{\partial x_t(\delta^*(\theta))}{\partial \kappa_\psi}$$

When we compute each component of right hand side of equation (A.3.3),

$$(A.3.4) \qquad \frac{\partial x_t(\delta^*(\theta))}{\partial \kappa_\psi} = -\frac{\partial i_t^*}{\partial \kappa_\psi} = -\frac{\partial \delta_x^*}{\partial \kappa_\psi} E_t x_{t+1} - \frac{\partial \delta_\pi^*}{\partial \kappa_\psi} E_t \pi_{t+1} - \frac{\partial \delta_z^*}{\partial \kappa_\psi} \log z_t$$

$$= -E_t x_{t+1} \frac{\lambda \kappa_p (\lambda^2 + \alpha)}{(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)^2} - E_t \pi_{t+1} \frac{\lambda \kappa_p (\lambda^2 + \lambda\beta + \alpha)}{(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)^2} - \frac{\lambda \kappa_p (\lambda^2 + \alpha)(\rho_z - 1)}{\gamma(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)} \log z_t$$

$$(A.3.5) \quad \frac{\partial \pi_t(\delta^*(\theta))}{\partial \kappa_\psi} = \lambda \frac{\partial x_t(\delta^*(\theta))}{\partial \kappa_\psi} + \kappa_p \left(i_t^* + \frac{\partial i^* t}{\partial \kappa_\psi} \right) = \frac{\alpha \kappa_p + \lambda \kappa_p^2 (1 - \kappa_\psi)}{\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi} i_t^*.$$

where

$$i_t^* = \left(\frac{\lambda^2 + \alpha}{\gamma(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)} E_t x_{t+1} + \frac{\lambda^2 + \lambda\beta + \alpha}{\gamma(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)} E_t \pi_{t+1} + \frac{(\lambda^2 + \alpha)(\rho_z - 1)}{\gamma(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)} \log z_t \right)$$

and

$$(A.3.6) \;\; \pi_t(\delta^*(\theta)) = (1 - \delta_x^*) E_t x_{t+1} + (1 - \delta_\pi^*) E_t \pi_{t+1} + \left(\frac{1}{\gamma}(\rho_z - 1) - \delta_z^* \right) \log z_t$$

$$= -\frac{\lambda \kappa_p \kappa_\psi}{\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi} E_t x_{t+1} - \frac{\lambda \kappa_p \kappa_\psi + \lambda\beta}{\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi} E_t \pi_{t+1} + \frac{\lambda \kappa_p \kappa_\psi (1 - \rho_z)}{\gamma(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)} \log z_t$$

$(A.3.7)$

$$x_t(\delta^*(\theta)) = (\lambda(1 - \delta_x^*) + \kappa_p + \kappa_\psi \delta_x^*) E_t x_{t+1} + (\lambda(1 - \delta_\pi^*) + \kappa_p \kappa_\psi \delta^* \pi + \beta) E_t \pi_{t+1}$$

$$+ \kappa_p (\lambda (1 + \phi) \left(\frac{1}{\gamma}(\rho_z - 1) - \delta_z^* + \kappa_\psi \delta_z^* \right) \log z_t$$

$$= \frac{\alpha \kappa_p \kappa_\psi}{\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi} E_t x_{t+1} + \frac{\alpha(\kappa_p \kappa_\psi + \beta)}{\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi} E_t \pi_{t+1} + \frac{\kappa_p \kappa_\psi (1 - \rho_z)(\lambda^2 - \lambda^2 - \alpha)}{\gamma(\lambda^2 + \alpha - \lambda \kappa_p \kappa_\psi)} \log z_t$$

when we combine equation (A.3.4)-(A.3.7), the two inequality conditions (3.5) and (3.6) are derived.

A.4. Kalman filter

In this paper, we assume the parameter κ_ψ to be time-varying to reflect the uncertainty of an economy. In this case, using the RLS learning method is inappropriate because it does not recognize the probabilistic property of the parameters. Rather, the Bayesian learning method, in which the previously obtained probability distribution of a parameter is combined with the posterior probability distribution should be used to accurately cover the context.

Kalman (1960) suggested an effective method, referred to as the Kalman filter, to estimate parameter under such parametric uncertainty using Bayesian methodology. In estimating the parameter κ_ψ and coefficients of the private sector $a_{1,t}, a_{2,t}, d_{1,t}, k_{1,t}$ and $k_{2,t}$, this paper uses the Kalman filter algorithm. Instead of Kalman (1960), this paper follows the guidance of Chow (1984) and Meinhold and Sinpurwalla (1983). According to the algorithm, updates of time-varying parameters are performed as follows.

In the central bank's case, the desired parameter $(\kappa_\psi)' = \theta_t$ can be arranged in the form of an observation equation:

$$(A.4.1) \qquad Y_t = F_t\theta_t + v_t$$

$$\text{where } Y_t = (y_{1,t}, y_{2,t}), F = I_2, v_t \sim N(0, I_2)$$

Now the system equation of parameter :

$$(A.4.2) \qquad \theta_t = G_t\theta_{t-1} + w_t$$

where $G_t = I_2, \omega_t \sim N(0, I_2)$

Using Bayes' theorem, the posterior distribution of θ_t after $Y_t = (y_{1,t}, y_{2,t})'$ is observed can be expressed as the product of the prior distribution of θ_t based on the observations up to time $t-1$, and the likelihood function of Y_t. To illustrate:

$$(A.4.3) \qquad P(\theta_t|Y_t) \propto P(Y_t|\theta_t, Y_{t-1}) \times P(\theta_t|Y_{t-1})$$

Meanwhile the conditional probability function of parameter θ_{t-1} at time $t-1$ is expressed as follows:

$$(A.4.4) \qquad (\theta_{t-1}|Y_{t-1}) \sim N(\hat{\theta}_{t-1}, \Sigma_{t-1})$$

Upon this recursive relationship of the parameter's conditional probability distribution, the posterior distribution can be calculated using Bayesian estimation. At time $t-1$ when Y_t is not observed, the conditional probability function of θ_t is $(\theta_t|Y_{t-1}) \sim N(G_t\hat{\theta}_{t-1}, R_t = G_t\Sigma_{t-1}G_t' + W_t)$. And this serves as a prior distribution of the Bayesian estimation at time t. Next, the likelihood function $P(Y_t|\theta_t, Y_{t-1})$ is determined, where the prediction error of Y_t, namely e_t, is as follows.

$$(A.4.5) \qquad e_t = Y_t - \hat{Y}_t = Y_t - F_tG_t\hat{\theta}_{t-1}.$$

Since $F_tG_t\theta_{t-1|t-1}$ is observed in the initial condition and prior times, observing Y_t would be equivalent to observing e_t. Thus $P(Y_t|\theta_t, Y_{t-1}) = P(e_t|\theta_t, Y_{t-1})$

and $(e_t|\theta_t, Y_{t-1}) \sim N(F_t(\theta_t - G_t\hat{\theta}_{t-1}), V_t)$ are hence obtainable. Based on this, the posterior distribution after observation of Y_t can be arranged as follows:

$$(A.4.6) \qquad P(\theta_t|Y_t, Y_{t-1}) = \frac{P(e_t|\theta_t, Y_{t-1}) \times P(\theta_t|Y_{t-1})}{\int_{\theta_t} P(e_t|\theta_t, Y_t)d\theta_t}$$

Equation $(A.4.6)$ shows the typical relationship in Bayes' theorem. However, there is a complication in calculating the right-hand side of the equation, the product of the distributions and the integral of a distribution in parametric space. Thus, in this paper, we utilize the method of Meinhold and Singpurwalla (1983) in calculating the left-hand side, the posterior distribution, $P(\theta_t|Y_t, Y_{t-1})$ Using the property of bivariate normal distributions, the probability vector would be $(X_1, X_2')' \sim N((\mu_1, \mu_2)', \Sigma)$ and under the condition of $X_2 = x_2$, X_1 would be

$$(A.4.7) \qquad (X_1|X_2 = x_2) \sim N(\mu_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2' - \mu_2), \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21})$$

If we transform the probability vector $(X_1, X_2)'$ into $(\theta_t, e_t)'$, then $\mu_1 = G_t\hat{\theta}_{t-1}$, $\sigma_{11} = R_t$, $\mu_2 = 0$, and $\Sigma_{21} = F_tR_t, \Sigma_{22} = V_t + F_tR_tF_t'$. By directly using these values, the posterior distribution $P(\theta_t|Y_t, Y_{t-1})$ can be expressed as follows:

$$(A.4.8) \qquad (\theta_t|Y_t, Y_{t-1}) = (\theta_t|e_t, Y_{t-1}) \sim N(\hat{\theta}_t, \Sigma_t)$$

$$(A.4.9) \qquad \hat{\theta}_t = G_t\hat{\theta}_{t-1} + R_tF_t'(V_t + F_tR_tF_t')^{-1}e_t$$

$$(A.4.10) \qquad \Sigma_t = R_t - R_tF_t'(V_t + F_tR_tF_t')^{-1}F_tR_t$$

The vector $K_t = R_t F_t' (V_t + F_t R_t F_t')^{-1}$ is called a Kalman gain. By setting the initial values $\hat{\theta}_0, \Sigma_0$ and using the same algorithm, we can simulate the expected value of the posterior distribution $\hat{\theta}_t = \hat{\kappa}_\psi$, which will be used by the central bank to determine the interest rate. In this paper, $\sigma_0 = I_1$ and $\hat{\theta}_0$ are set as various values satisfying each of the parameter properties. All results were similar and this paper introduces the results drawn by setting $\psi_0 = 0.25$.

Appendix B: Figures

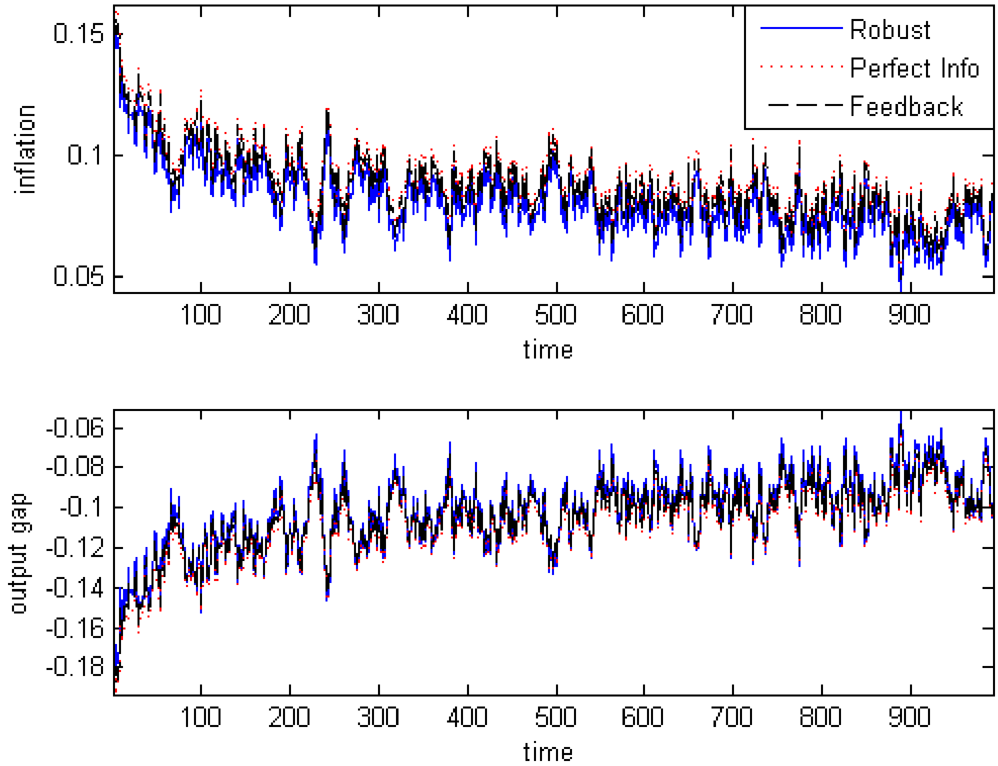

Figure 1: Dynamics of realized inflation π_{t+1} (Top) and the output gap x_{t+1} (Bottom) under discretion ($\psi = 0.25$, $\gamma = 0.7$)

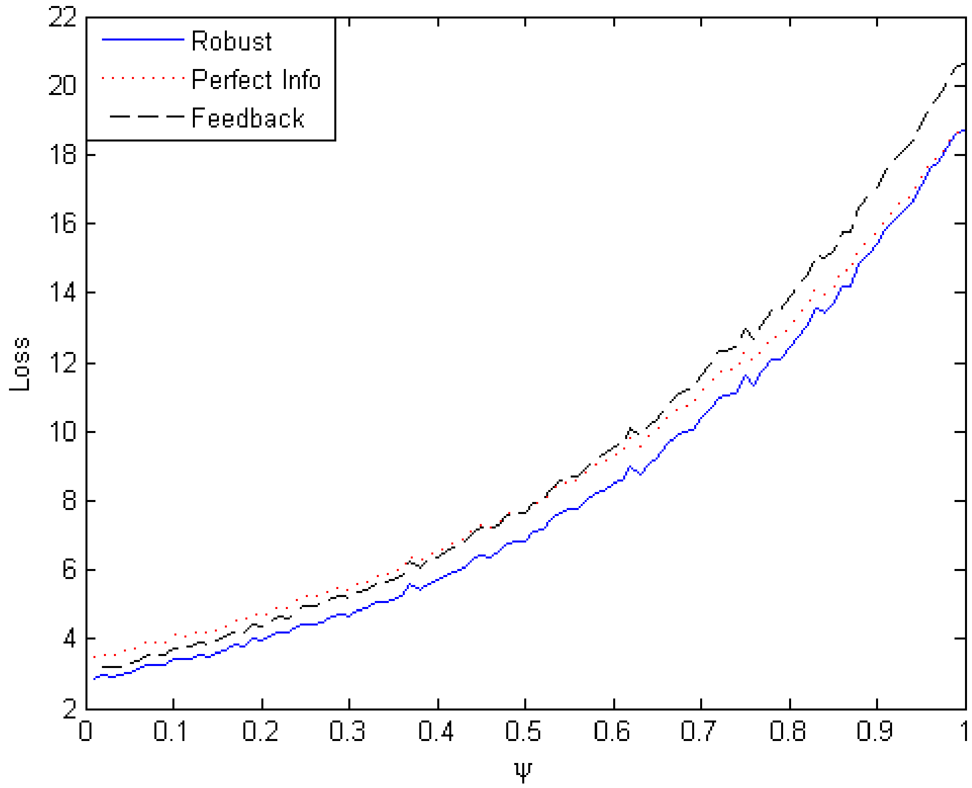

Figure 2: Total loss of policies with $\psi \in [0, 1]$ under discretion $(\gamma = 0.7)$

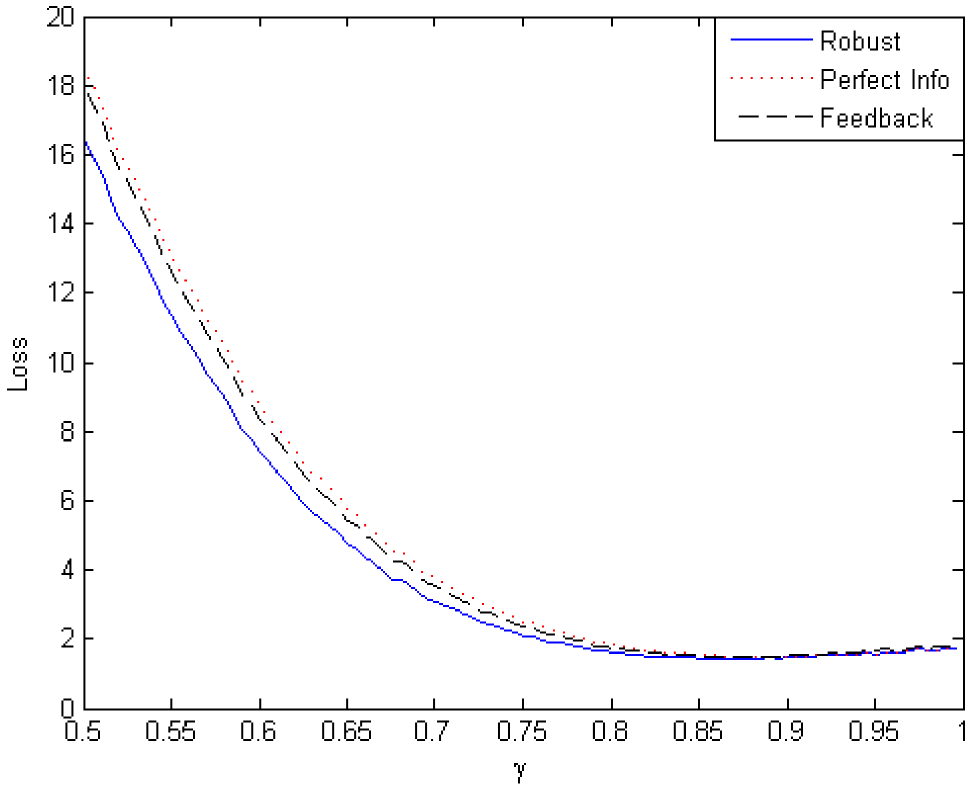

Figure 3: Total loss of policies with $\gamma \in [0.5, 1]$ under discretion

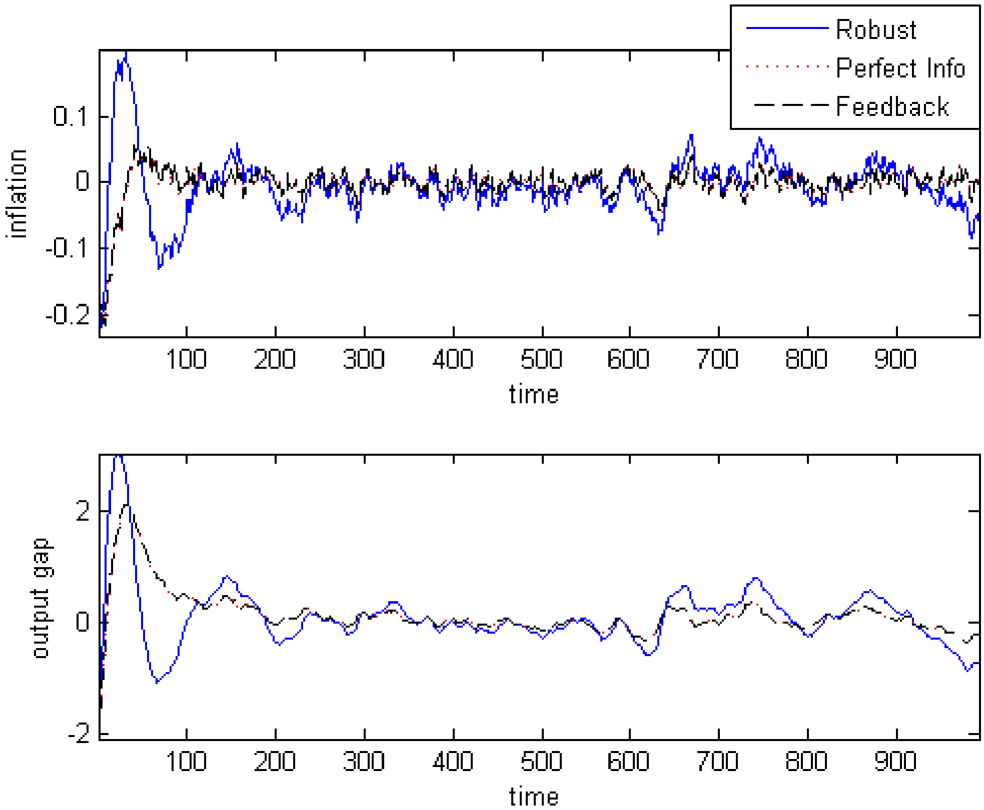

Figure 4: Dynamics of realized inflation π_{t+1} (Top) and the output gap x_{t+1} (Bottom) under commitment $(\psi = 0.25, \gamma = 0.7)$

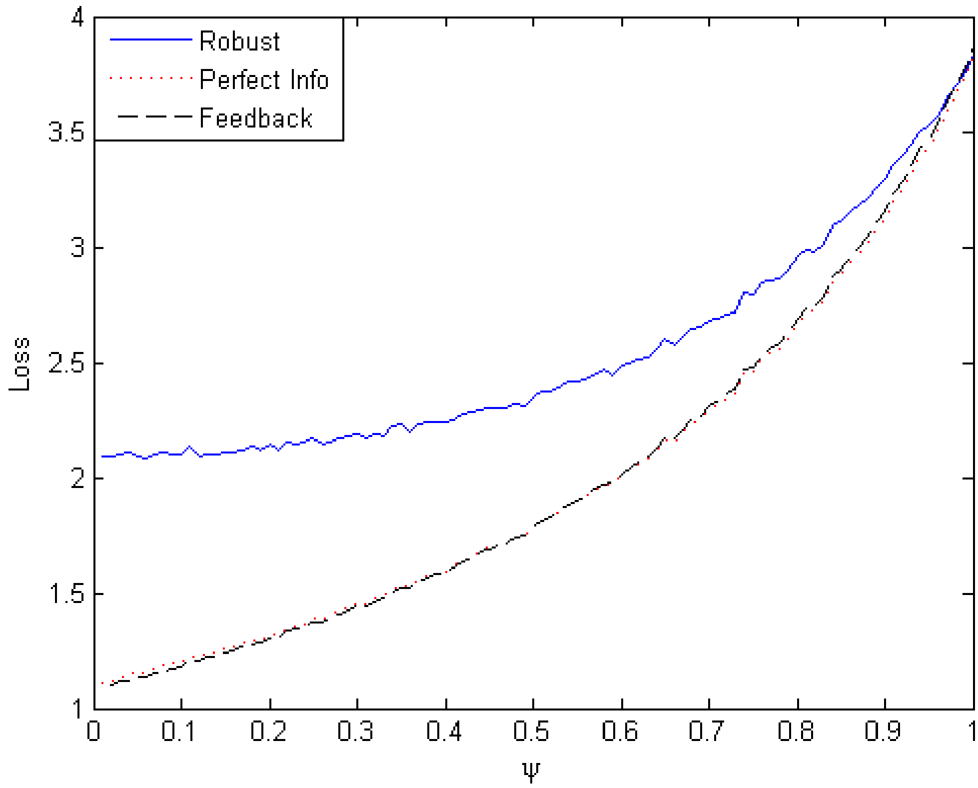

Figure 5: Total loss of policies with $\psi \in [0,1]$ under commitment ($\gamma = 0.7$)

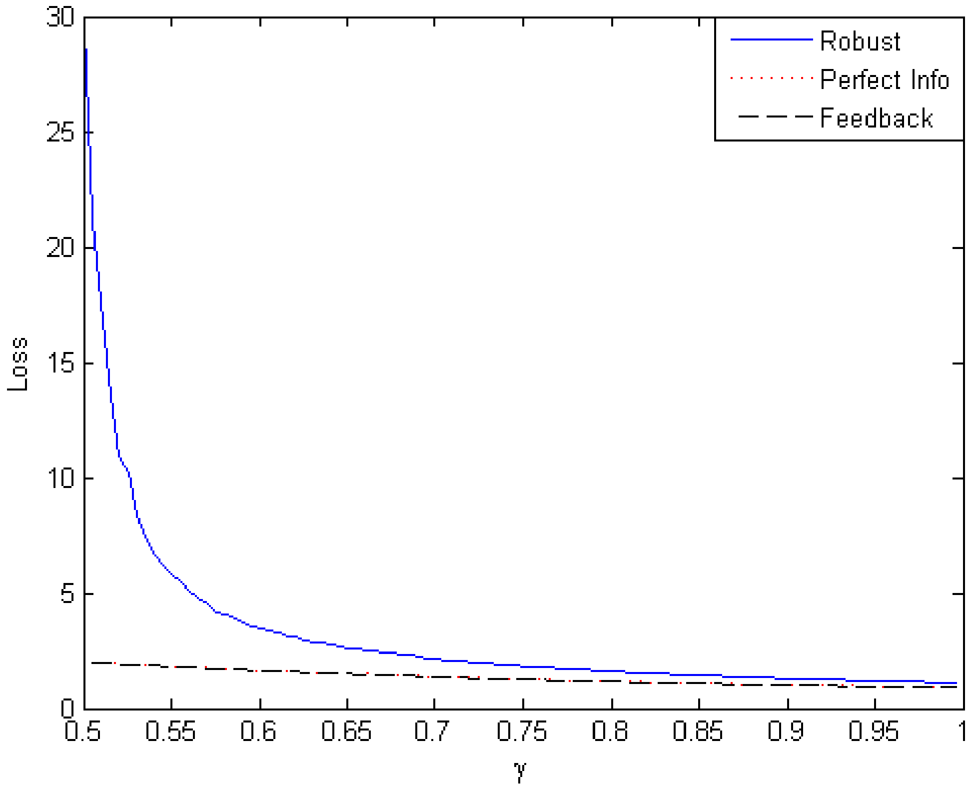

Figure 6: Toal loss of policies with $\gamma \in [0.5, 1]$ under commitment ($\psi = 0.25$)

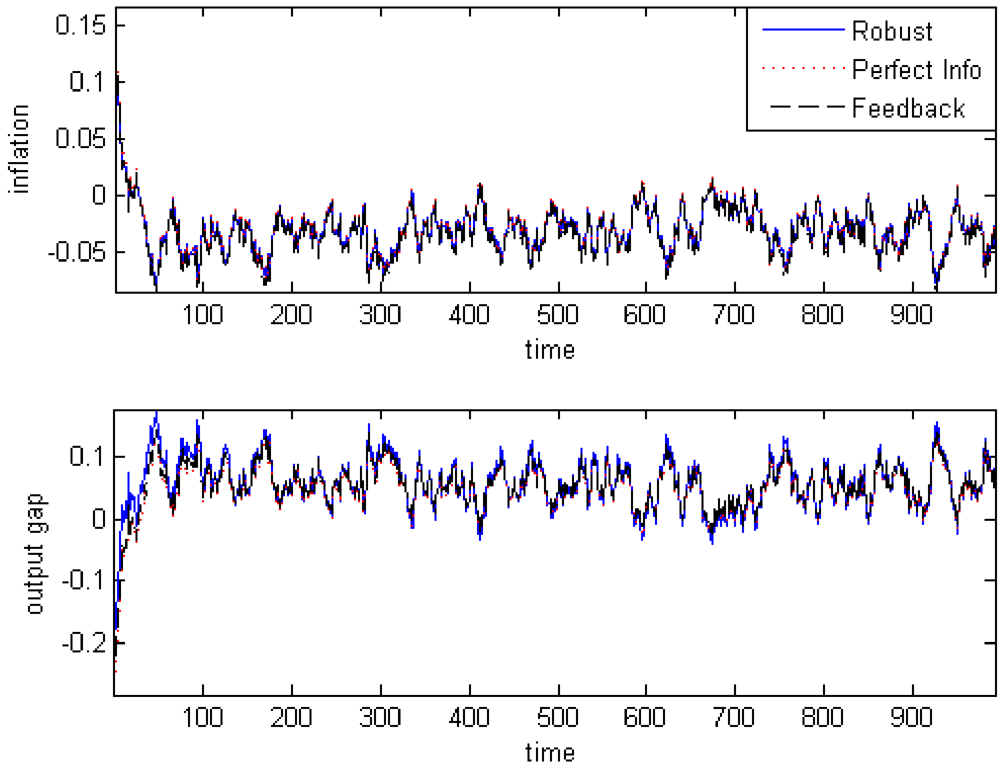

Figure 7: Dynamics of realized inflation π_{t+1} (Top) and the output gap x_{t+1} (Bottom) with time varying $\kappa_{\psi,t}$ under discretion ($\gamma = 0.7$)

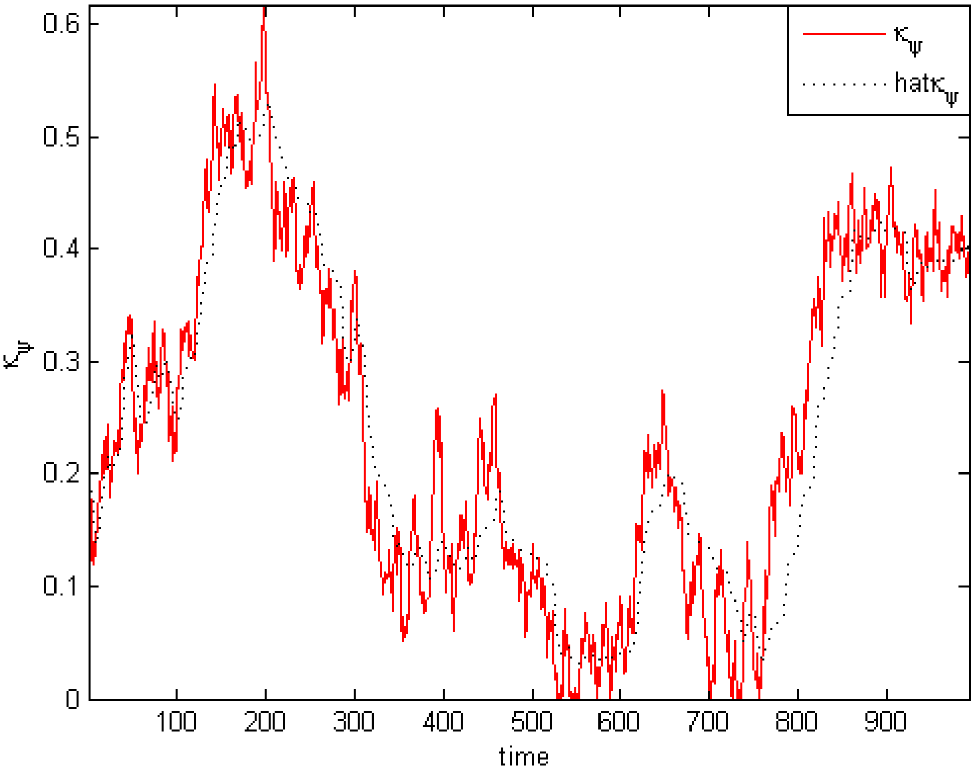

Figure 8: Dynamics of $\kappa_{\psi,t}$ and its estimate $\hat{\kappa}_{\psi,t}$ under discretion

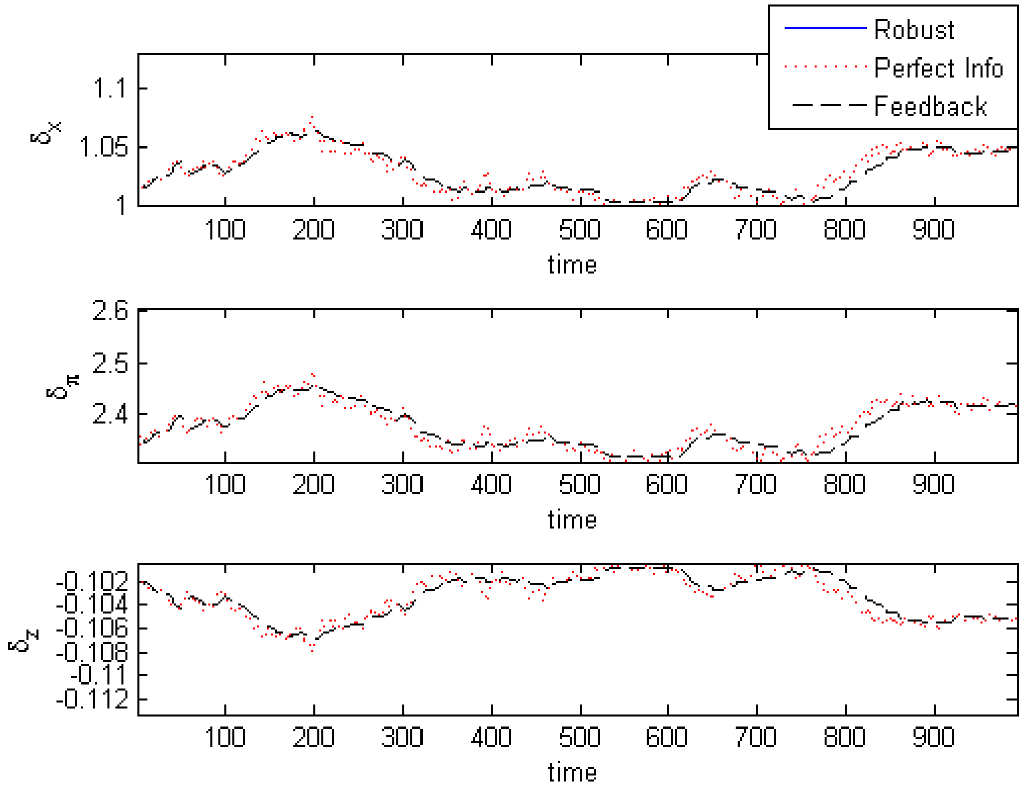

Figure 9: Dynamics of monetary policy coefficient $\delta_{x,t}$ (Top), $\delta_{\pi,t}$ (Middle) and $\delta_{z,t}$ (Bottom) with time varying $\kappa_{\psi,t}$ under discretion

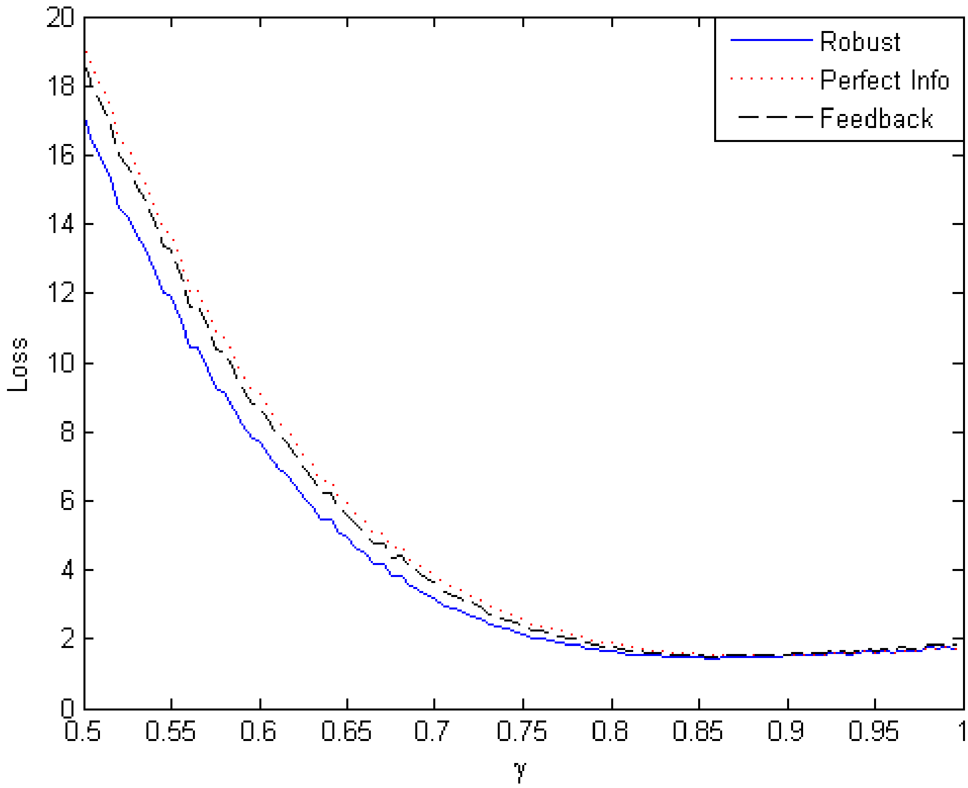

Figure 10: Total loss of policies with $\gamma \in [0.5, 1]$ and time varying $\kappa_{\psi,t}$ under discretion

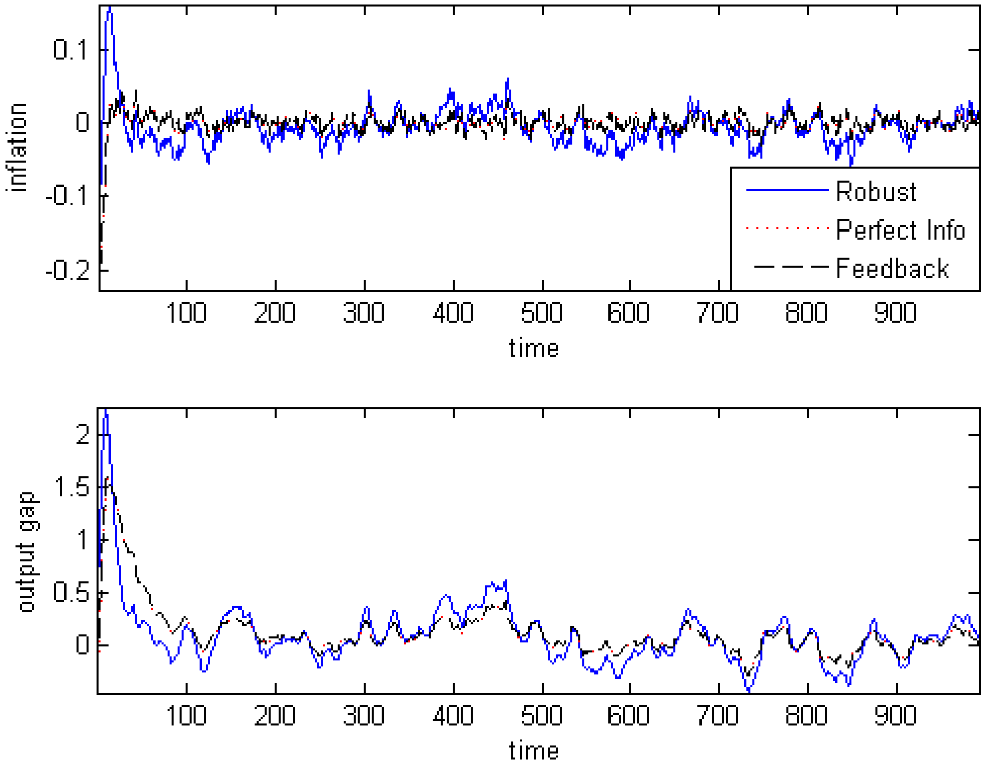

Figure 11: Dynamics of realized inflation π_{t+1} (Top) and the output gap x_{t+1} (Bottom) with time varying $\kappa_{\psi,t}$ under commitment $(\gamma = 0.7)$

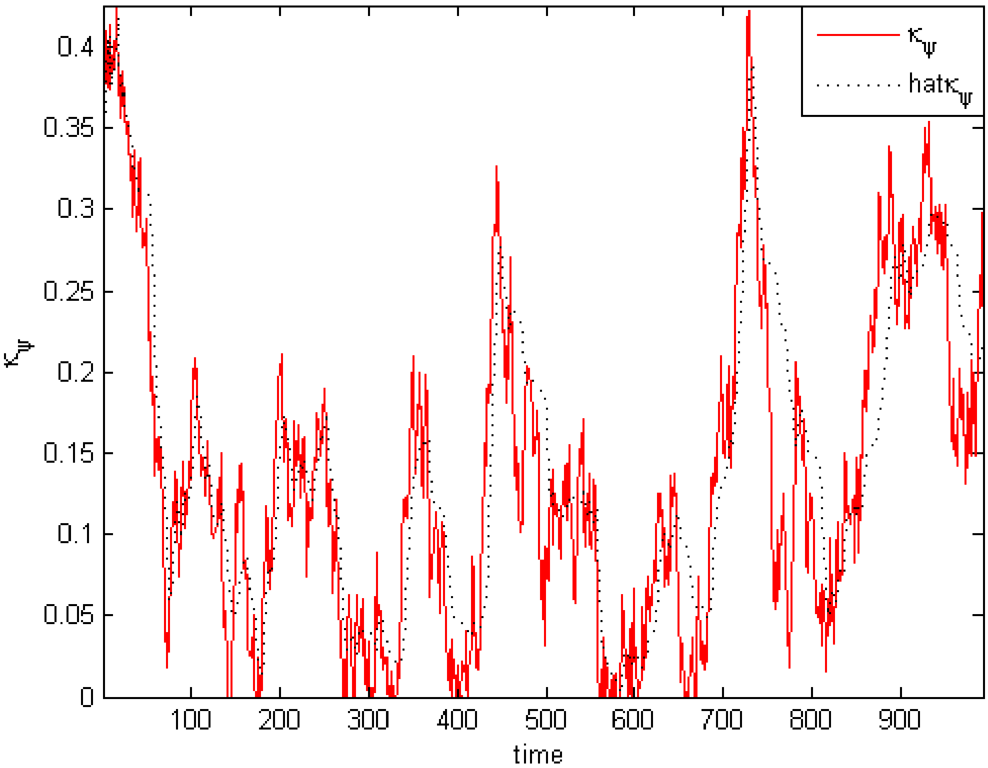

Figure 12: Dynamics of $\kappa_{\psi,t}$ and its estimate $\hat{\kappa}_{\psi,t}$ under commitment

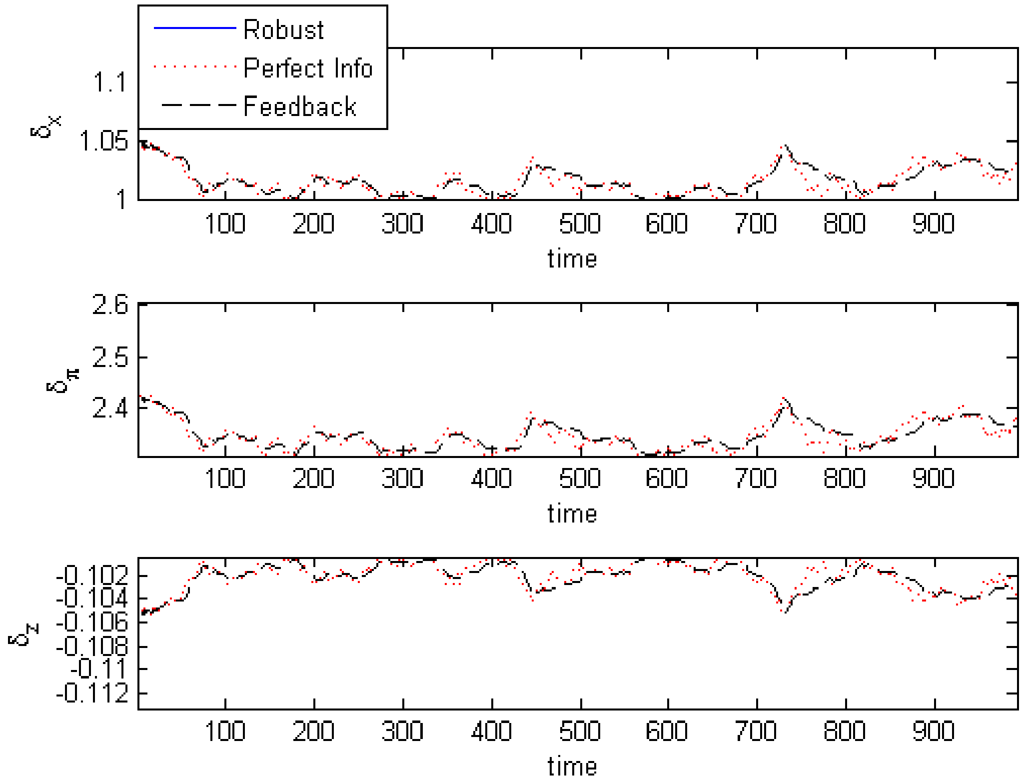

Figure 13: Dynamics of monetary policy coefficient $\delta_{x,t}$ (Top), $\delta_{\pi,t}$ (Middle) and $\delta_{z,t}$ (Bottom) with time varying $\kappa_{\psi,t}$ under commitment

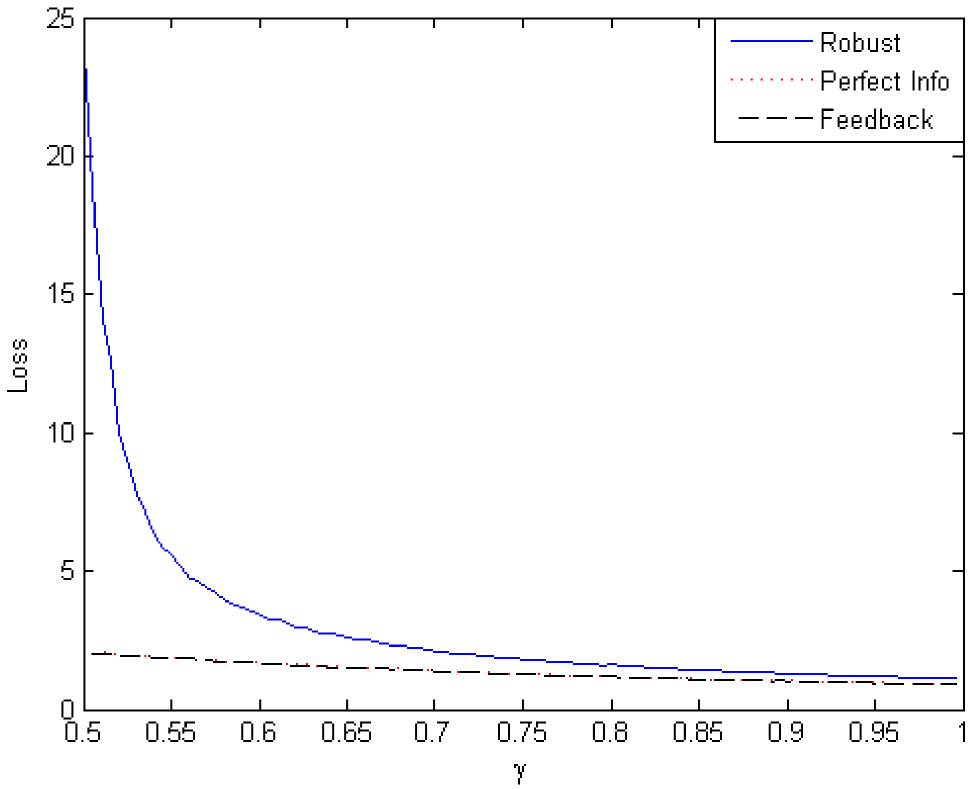

Figure 14: Total loss of policies with $\gamma \in [0.5, 1]$ and with time varying $\kappa_{\psi,t}$ under commitment

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