Forecasting with Non-spurious Factors in U.S. Macroeconomic Time Series

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Abstract

Time instability in factor loadings can induce an overfitting problem in forecasting analyses since the structural change in factor loadings inflates the number of principal components and thus produces spurious factors. This paper proposes an algorithm to estimate non-spurious factors by identifying the set of observations with stable factor loadings based on the recursive procedure suggested by Inoue and Rossi (2011). I found that 51 out of 132 U.S. macroeconomic time series of Stock and Watson (2005) have stable factor loadings. Although crude principal components provide eight or more factors, there are only one or two non-spurious factors. The forecasts using non-spurious factors significantly improve out-of-sample performance.

JEL Classification Number: C12, C38, E17

Keywords: dynamic factor model, principal components, structural change, spurious factors, out-of-sample forecasts, overfitting

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1 Introduction

In macroeconomic and financial forecasting analyses, accounting for time instability or structural change is an important issue.[1] Forecasting models often exhibit poor out-of-sample performance despite an excellent in-sample fit either when they are not stable over time or when they include irrelevant regressors, i.e. overfitting.[2] Factor models are introduced to avoid this overfitting problem when many predictors are available, however, how to account for time instability in factor models is still an open question. Stock and Watson (2002) and Bates et al. (2012) argue that as far as the additive instability component in factor loadings is $o_p(1)$, a common factor space can still be consistently estimated. These results imply that researchers can use a dynamic factor model in forecasting exercises without worrying too much about time instability. On the other hand, a growing body of empirical research finds that many factor or factor-augmented models yield better performance when coefficients are susceptible to time instability (for example Del Negro and Otrok, 2008; Mumtaz and Surico, 2012). An important ingredient of this discussion is that if factor loadings are subject to time instabilities, then that model can be written as another factor model with a larger number of factors having time invariant loadings. Thus, a newly defined set of factors will include the original factor space as well as the so-called spurious factors (Stock and Watson, 2005). A primary concern is that a model with too many regressors overfits the data and its forecasting performance deteriorates. Also, other literature emphasizes interpretation of the factors as an economically meaningful series in the model. Indeed, many macroeconomists have long considered that few economically meaningful unobservable factors exist in the economy (See Sims and Sargent, 1977, Boivin and Giannoni, 2006, and Yamamoto, 2012, for example). Considering this perspective, one also needs to use an exact number of factors for effective economic analysis.

Keeping this caveat in mind, this paper tackles the question why so many factors exist in the U.S. economy, raised by Stock and Watson (2005) who found that the information criteria proposed by Bai and Ng (2002) fit seven dynamic factors and nine static factors in U.S. macroeconomic time series. To this end, I focus on the fact that time instability in factor loadings inflates the number of principal components and thus produces spurious fac-

[1]See Rossi (2011) for a comprehensive survey of recent discussions on forecasting under instability. Stock and Watson (1996) and Rapach and Wohar (2008), among many others, show major empirical examples.

[2]In practice, the two problems happen at once. Rossi and Sekhposyan (2011) propose a method to decompose the difference of in-sample and out-of-sample forecasting performance into time-instability and overfitting.

tors. I first consider recently proposed structural change tests for factor loadings in dynamic factor models.[3] As far as the author knows, a formal test for structural changes in factor loading was first proposed by Breitung and Eickmeier (2011) (this test is referred to as BE hereafter) in the context of principal components estimation. Their test is based on the structural change tests of Andrews (1993) and Andrews and Ploberger (1994) but uses the factors estimated by principal components under the null hypothesis, that is, they assume the loadings other than those tested are all stable. Indeed, the test can suffer from serious size distortions if some loadings exhibit structural changes. This is because the principal components' factor estimates can include spurious factors whose pseudo-true factor loadings exhibit some instabilities. Hence, conducting the BE test on the factor loading of every observation never provides the set of observations with stable factor loadings. Subsequent literature provides a structural change collectively test for the factor loadings of all observations or a subset thereof. For example, Han and Inoue (2012) propose a test (HI hereafter) that equivalently investigates the stability of the second moments of principal components. Chen, Dolado, and Gonzalo (2012) suggest a stability test (CDG hereafter) for the regression coefficients of the first principal component on the remaining principal components. However, when only a small portion of observations in the hypothesis have unstable factor loadings, both tests exhibit some power loss. Moreover, conducting a collective test on loadings of all possible subsets of observations is infeasible. Hence, what is apparently needed is a consistent estimator of the non-spurious factors.

This paper proposes an algorithm to estimate non-spurious factors by identifying the set of observations with stable factor loadings. The algorithm is based on the recursive procedure suggested by Inoue and Rossi (2011). They proposed a method to identify the set of stable parameters in standard parametric models where all explanatory variables are observed. However, the method proposed in this paper updates the factor estimates in every step to overcome the above problems in existing structural change tests. What makes it feasible is that one can utilize factor estimates which are the closest to the non-spurious factors at every step in the procedure. I then applied this procedure to estimate the non-spurious factors in the U.S. macroeconomic time series used by Stock and Watson (2005). As a result, I found that 51 out of 152 series have stable factor loadings. The factor loadings for the "fast variables" such as housing and financial variables are less likely to be stable than those for the "slow variables." Although crude estimates provide eight or more non-spurious

[3] See Perron (2006) for a review of recent theoretical developments of structural change analyses although it does not cover structural change problems in factor models.

factors, there are only one or two non-spurious factors in the U.S. macroeconomic time series. Most importantly, the non-spurious factors significantly improve out-of-sample forecasting performance over forecasts using all the crude factors, especially with long horizons. This result highlights the usefulness of the non-spurious factors in forecasting analyses.

The rest of the paper is structured as follows. Section 2 provides the model and the existing structural change tests. Section 3 discusses the problems inherent in these tests. Section 4 proposes an algorithm to identify the set of stable observations and investigates its properties via Monte Carlo simulations. Section 5 applies this method to the U.S. macroeconomic time series used by Stock and Watson (2005) and Section 6 concludes.

2 Models and test statistics

Consider a dynamic factor model[4] with r factors applied to the data of the time dimension T and the cross-sectional dimension N. The model consists of the first N_0 ($1 \leq N_0 \leq N$) observations with stable factor loadings so that

$$x_{it} = \lambda_{i0} f_t + u_{it} \qquad \text{for } t = 1, \cdots, T \tag{1}$$

for $i = 1, \cdots, N_0$, whereas the remaining observations may have unstable factor loadings. I allow for any types of time variations of factor loadings considered in the standard literature.[5] For example, consider a one-time structural change at an unknown time T_b common to all i so that:

$$x_{it} = \begin{cases} \lambda_{i1} f_t + u_{it} & \text{for } t = 1, \cdots, T_b \\ \lambda_{i2} f_t + u_{it} & \text{for } t = T_b + 1, \cdots, T \end{cases}, \tag{2}$$

for $i = N_0 + 1, \cdots, N$. Let x_{it} be an observation of the ith individual at time t, f_t be a $r \times 1$ vector of unobservable factors, λ_{ij} ($j = 0, 1, 2$) be an $1 \times r$ vector of associated factor loadings, and u_{it} be idiosyncratic errors. Then, a matrix representation of the above models is available as follows. Let $X = [x_1, \cdots, x_N]$ be a $T \times N$ matrix where $x_i = [x_{i1}, \cdots, x_{iT}]'$ is a $T \times 1$ vector of the ith observation, $\lambda_0 = [\lambda'_{10}, \cdots, \lambda'_{N_0 0}]$ be a $r \times N_0$ matrix of stable factor loadings, and $\lambda_j = [\lambda'_{N_0+1,j}, \cdots, \lambda'_{Nj}]$ be $r \times (N - N_0)$ matrices of the factor loadings at the regime j ($j = 1, 2$). As pointed out in the literature, a model with a structural change in the loadings (1) and (2) can equivalently be written as another factor model with a larger

[4]This is alternatively called a static factor representation of a dynamic factor model.

[5]In other words, any types of time variations are admitted as long as Assumptions 1 and 2 in Appendix A are satisfied.

number of factors such that

$$X = \begin{bmatrix} F_1 & 0_{T_b \times r} \\ F_2 & F_2 \end{bmatrix} \begin{bmatrix} \lambda_0 & \lambda_1 \\ 0_{r\times(N-N_0)} & \lambda_2 - \lambda_1 \end{bmatrix} + u,$$

where $F_1 = [f_1, \cdots, f_{T_b}]'$ is a $T_b \times r$ matrix and $F_2 = [f_{T_b+1}, \cdots, f_T]'$ is a $(T-T_b)\times r$ matrix. Since $F^0 = [F_1', F_2']' = [f_1, \cdots, f_T]'$ are the factors in the original model (1) and (2), I call $F^* = [0_{r\times T_b}, F_2']'$ the spurious factors in line with Stock and Watson's (2005) terminology and F^0 the non-spurious factors. When the loadings have other types of variations, a similar logic goes through and non-spurious factors will show up.

The goal of this paper is to obtain an estimate of the non-spurious factors F^0 with the correct number, or in the most thrift form. This can be achieved by using the principal components of the stable observations $i = 1, \cdots, N_0$, if one can sort the observations and knows N_0. However, researchers have no a priori knowledge on whether one particular observation follows (1) or (2). To this end, I first consider the following class of hypothesis testing procedures which pertains to the factor loadings of one particular observation,

$$H_0 : \lambda_{i1} = \lambda_{i2} \text{ and } H_1 : \lambda_{i1} \neq \lambda_{i2}.$$

For instance, Breitung and Eickmeier (2011) consider structural change tests of Andrews (1993) and Andrews and Ploberger (1994) using the principal components factor estimates. They show that the tests have standard asymptotic distributions and exhibit a good size. They also show that the tests have a good power as T increases. However, these results are obtained under the assumptions considered in Bai (2003) which include a condition that the factor loadings other than those tested are all stable.[6]

Subsequent literature proposes tests that draw an inference on structural changes in the factor loadings of all or a subset of the observations collectively and I call these collective tests. Under the model (2), the hypotheses are formulated as

$$H_0 : \lambda_{i1} = \lambda_{i2} \text{ for all } i \text{ and } H_1 : \lambda_{i1} \neq \lambda_{i2} \text{ for some } i.$$

These hypotheses can also be tested by the BE test by pooling the individual test statistics for observations in the hypothesis. However, this approach will be problematic when the

[6]This is a device commonly used in developing many structural change tests. Kim and Perron (2009) and Perron and Yamamoto (2012) show that the tests evaluated in this framework may lose power when the magnitudes of structural changes are moderate or large, although they behave well under the null hypothesis or under an alternative very close to the null region.

individual test statistics exhibit correlations, that is, when there are cross-sectional correlations in the idiosyncratic errors. Better tests—the HI and CDG tests—are independently proposed by Han and Inoue (2012) and Chen, Dolado, and Gonzalo (2012), respectively. The HI test is based on the second moments of the principal components over time and the CDG test formulates the problem as the stability of the regression of the estimated first principal component on the remaining principal components. The construction of the above three test statistics is explained in Appendix B.

These tests formally specify a one-time structural change in the factor loadings at an unknown timing, however, they are supposed to have non-trivial power even when the factor loadings have other types of variations. For example, the asymptotic properties of the SupWald structural change test when the parameters follow multiple breaks, random walk, and infrequent breaks were studied by Perron and Yamamoto (2012). They show that the tests are consistent and have standard power functions in finite samples. Hence, this paper considers not only a one-time break but also multiple breaks and random walk parameters. In empirical studies, modeling such parameter processes would also be a more popular strategy than only a simple one-time break.

3 Size and power of existing tests

This section investigates the finite sample properties of the above three tests for structural changes in the factor loadings. I consider the following data generating processes with two factors and time-varying factor loadings:

$$\begin{aligned} x_{it} &= \lambda_{it}' f_t + u_{it}, \\ f_{1,t} &= \rho_1 f_{1,t-1} + e_{1,t}, \\ f_{2,t} &= \rho_2 f_{2,t-1} + e_{2,t}, \end{aligned}$$

with $\lambda_{it} = [\lambda_{1it}\ \lambda_{2it}]$. The errors u_{it}, $e_{1,t}$, and $e_{2,t}$ are generated by the independent standard normal distributions, and ρ_1 and ρ_2 are set at 0.4 and 0.0. I assume that the first N_0 loadings are stable so that $\lambda_{jit} = \lambda_{ji0} \sim U[0,b]$ independently for $i = 1, \cdots, N_0$ and for all t. The remaining $N - N_0$ loadings have the following time variations:[7]

[7] The following processes do not include the one described in (2), that is, a one-time break at a date common to all i. It has been found that the BE test does may lose the power when the breaks happen at the same time and their magnitudes are large. This phenomenon was overlooked in the original paper and may require further investigations.

DGP-1) One-time structural change:

$$\lambda_{j,i,t} = \begin{cases} \lambda_{j,i,t-1} + U[0,b] & \text{if } t = T_i^b, \\ \lambda_{j,i,t-1} & \text{otherwise,} \end{cases}$$

where $T_i^b = \lfloor \mu_i T \rfloor$ with $\mu_i \sim U[\varepsilon, 1-\varepsilon]$ and I set ε as equal to the truncation size considered in the structural change tests to ensure that the model exhibits a structural change in the permissible dates.

DGP-2) Multiple structural change:

$$\lambda_{j,i,t} = \begin{cases} \lambda_{j,i,t-1} + U[0,b] & \text{if } t = T_i^{b1}, T_i^{b2}, T_i^{b3}, T_i^{b4}, \\ \lambda_{j,i,t-1} & \text{otherwise,} \end{cases}$$

where $T_i^{bm} = \lfloor \mu_i^m T \rfloor$ with $\mu_i^m \sim U[\varepsilon, 1-\varepsilon]$ and $\mu_i^1 < \mu_i^2 < \mu_i^3 < \mu_i^4$.

DGP-3) Random walk:

$$\lambda_{j,i,t} = \lambda_{j,i,t-1} + v_{j,i,t}, \quad v_{jit} \sim N(0, b/100).$$

I change the magnitude parameter b from 0 to 10 and report the number of factors estimated in the above DGP as well as size and power of the tests at the nominal 5% level based on 5,000 replications. After trying several cases of various sample sizes, I found that the qualitative results remain the same and hence, report the case of $T = 100$ and $N = 100$, whereas I choose N_0 as either $0.3N$, $0.5N$, or $0.7N$. I first consider the number of factors. My conjecture is that this number is overestimated in the presence of breaks since the model now exhibits spurious factors. The number of factors is selected to maximize the information criterion (IC_p2) proposed by Bai and Ng (2002) with the upper bound at 12. The mean, median, and mode of the estimated numbers over the replications are reported. I next consider the properties of the existing structural change tests. I apply the BE test for stable loadings of the first observation to consider the size (labeled "stable loading") and unstable loadings of the $(N_0 + 1)$th observation to evaluate the power (labeled "unstable loadings"). In doing so, I compare the following two methods. The first method estimates the number of factors and the associated factors using all the observations. Since this testing procedure is practical, I call it a "feasible" test. The second method implements the same test, but using the number of factors and the associated factors estimated based solely on the observations with stable factor loadings ($i = 1, \cdots, N_0$). Since this factor estimation procedure utilizes unavailable information, I call it an "infeasible" test. This version is unrealistic, but matches the framework considered by Breitung and Eickmeier (2011) that considers all loadings other than the ones tested as stable. I also investigate the properties of two existing collective tests—Han and Inoue (2012) and Chen, Dolado, and Gonzalo (2012). For either of these, the "feasible" test gives the results for loadings of all N observations, that is, including both N_0 stable and $N - N_0$ unstable loadings. The "infeasible" test is solely for the unstable loadings x_i for $i = N_0 + 1, \cdots, N$. Again, since the latter method uses unavailable information that the tested loadings are all unstable, they are unrealistic. However, the infeasible method is expected to provide a higher power than the feasible method.

The results are summarized in Table 1. First, the number of factors inflates from the true number two as the magnitudes of the breaks increase in any designs of the parameter process. This is consistent with the results of Breitung and Eickmeier (2011) and my conjecture in the previous section. Second, the size of the BE test is close to the nominal level if the factors are estimated based only on the stable observations ("infeasible"), however, it suffers from significant size distortions when the factors are estimated based on all the observations ("feasible"). It implies that the BE test concludes that the truly stable loadings are unstable when the other observations include unstable loadings. Third, the BE test has a good power when the factors are estimated based on the stable observations, however, the feasible test shows a power loss. Finally, I consider the two collective tests. Both tests have a good power when the set only includes unstable factor loadings, however, the power decreases when the portion of the unstable observations is small. These findings are robust to the sample size and parameter process, as shown in every case in Table 1. Therefore, the simulation reveals the difficulties of applying existing methods to obtain the non-spurious factors. What one needs is the information on the set of observations with stable loadings that makes these tests work appropriately.

4 Estimating non-spurious factors

Inoue and Rossi (2011) propose a recursive procedure to estimate the set of stable parameters among the ones attached to the observable variables in linear models with prespecified probability. They use this approach in a dynamic general equilibrium model to find the source of instabilities among a large number of parameters loaded in a macroeconomic model. Their algorithm starts with an a priori assumption that all parameters are stable or in the stable set and then conducts a structural change test for the null hypothesis that all parameters are stable in the stable set. If the test rejects the null, then the algorithm calculates the p-values of the test for every individual parameter in the stable set allowing for the remaining parameters to be unstable. This procedure moves the parameter with the smallest p-value from the stable set to the unstable set. This continues until the structural change test for all the parameters in the stable set becomes insignificant.

This paper proposes a similar algorithm, however, it is different in that the structural change tests for factor loadings suffer from the problems described in the previous section. Hence, the method must utilize the tests for the factor loadings but use the factors closest to the non-spurious factors in every step. This can be implemented by including an estimation of the number of factors and associated factors out of the set of observations with stable factor loadings in every step. This method can avoid the size distortions and power loss of the structural change tests caused by the spurious factors to the extent possible.

To facilitate comparisons of this algorithm with that of Inoue and Rossi (2011), I make minimal changes to the notations and descriptions given below. Let $s \in \{0,1\}^N$ denote a selection vector of the loadings. If $N = 3$ and $s = (0, 1, 0)$, then the loadings of the second observation are time invariant. Let $\lambda(s)$ and $X(s)$ be subsets of λ and X, which are selected by s as stable. I also define s^*, a vector of the true selection vector. Let e_i be the $N \times 1$ vector whose ith element is 1 and 0 otherwise, $1_{N\times1}$ be the $N \times 1$ vector of ones, and $0_{N\times1}$ be the $N \times 1$ vector of zeros. Let $T_T^I(e_i, s)$ be an individual test statistic for stability of the factor loadings of the ith observations using the number of factors and the factors estimated by the principal components of $X(s)$. Let $T_T^C(s)$ be a collective test for stability of all the loadings of $\lambda(s)$ using $X(s)$. Finally, let $pv(e_i, s)$ denote the p-value of the individual test for $H_0(e_i)$ against $H_1(e_i)$ using the statistic $T_T^I(e_i, s)$.

The algorithm

- (Step 0) Initially, let $s_0 = 1_{N\times1}$. Test $H_0^{(0)}(s_0)$ against $H_1^{(0)}(s_0)$ at significance level α using a collective test $T_T^C(s_0)$. If the test does not reject, let $\hat{s} = s_0$. If the test rejects, calculate individual tests $T_T^I(e_i, s_0)$ for $i = 1, \cdots, N$ and order them such that $pv(e_1, s_0) \leq pv(e_2, s_0) \leq \cdots \leq pv(e_N, s_0)$. Without loss of generality, let e_1 identify the loading with the smallest p-value. Continue to Step 1.

- (Step 1) Without loss of generality, let $s_1 = [0, 1_{1\times(N-1)}]'$. Test $H_0^{(1)}(s_1)$ against $H_1^{(1)}(s_1)$ at significance level α using a collective test $T_T^C(s_1)$. If the test does not reject, let $\hat{s} = s_1$. If the test rejects, calculate individual tests $T_T^I(e_i, s_1)$ for $i = 2, \cdots, N$ and order them such that $pv(e_2, s_1) \leq pv(e_3, s_1) \leq \cdots \leq pv(e_N, s_1)$. Without loss of generality, let e_2 identify the loading with the smallest p-value. Continue to Step 2. (...)

- (Step j) Without loss of generality, let $s_j = [0_{1\times j}, 1_{1\times(N-j)}]'$. Test $H_0^{(j)}(s_j)$ against $H_1^{(j)}(s_j)$ at significance level α using a collective test $T_T^C(s_j)$. If the test does not reject, let $\hat{s} = s_j$. If the test rejects, calculate individual tests $T_T^I(e_i, s_j)$ for $i = j, \cdots, N$ and order them such that $pv(e_j, s_j) \leq pv(e_{j+1}, s_j) \leq \cdots \leq pv(e_N, s_j)$. Without loss of generality, let e_j identify the loading with the smallest p-value. Continue to Step $j+1$.
(...)

- (Step $N-1$) Without loss of generality, let $s_{N-1} = [0_{1\times(N-1)}, 1]'$. Test $H_0^{(N-1)}(s_{N-1})$ against $H_1^{(N-1)}(s_{N-1})$ at significance level α using a collective test $T_T^C(s_{N-1})$. If the test does not reject, let $\hat{s} = s_{N-1}$. If the test rejects, let $\hat{s} = 0_{N\times 1}$.[8]

A theoretical justification of the above procedure in line with the discussion in Inoue and Rossi (2011) is discussed in Appendix A. This section investigates finite sample properties of the above algorithm by carrying out Monte Carlo simulations. The main focus is on the coverage ratio of $\hat{s}$ and the number of factors estimated using the stable set $\hat{s}$. I use the same data generating processes as in section 2 with various sample sizes $(T, N) = (100, 100), (150, 100), (150, 100)$, and $(150, 150)$. I consider $N_0 = 0.3N$, $0.5N$,and $0.7N$. The number of replications is 300 and the 95% nominal level is used for the coverage ratio. Table 2 summarizes the results.

The coverage ratio would not be meaningful when there are no breaks and $(b = 0)$ is thus not reported in these cases. It is observed that the coverage ratio approaches $(1 - \alpha)$ as b increases or when b is sufficiently large and T increases. The coverage ratio also improves as N increases. The results are robust with regard to the three parameter processes and the choice of N_0. Overall, I show that the algorithm can identify the set of observations having stable factor loadings with its probability close to the significance level. The estimated number of factors using all the observations (I call these factors "crude factors") is about the true number two when $b = 0$, but increases as the breaks become larger as we have seen in Table 1. However, when I estimate the number of factors using the stable set (I call these factors "non-spurious factors"), the simulated mean, median, and mode are very close to the true number two even in the presence of structural changes. This result is robust with regard to the sample sizes and parameter processes.

[8]In practice, one needs to specify an upper bound of the number of factors $\bar{r}$. When the number of observations in the stable set reaches $\bar{r}$, one can lower $\bar{r}$ to continue the algorithm or can stop to conclude "almost" all the observations have unstable loadings. In either case, the identified set may be too small to estimate non-spurious factors precisely.

Finally, I investigate the out-of-sample forecasting performance using the non-spurious factors. I conjecture that the spurious factors are caused by the factor loading instabilities and are only related to certain observations. Hence, using these spurious factors may help in forecasting these observations, but they cause overfitting for the remaining observations. Therefore, solely using the non-spurious factors can improve forecasting accuracy on average. To investigate this claim, I generate data on the basis of the same data generating processes as in section 2; however, such data is of the size $(T+h, N)$ where h is the specified forecasting horizon. I consider the cases of $b = 5$ and 10. The exercise first estimates the set of observations with stable factor loadings and non-spurious factors out of the estimated stable set with the number estimated by $ICp2$. On the other hand, I estimate factors using all the observations for $t = 1, \cdots, T$ with the number estimated by IC_p2. Based on these estimated factors, I compute the forecasting errors and mean squared errors (MSEs) for all the observations $i = 1, \cdots, N$ over the out-of-sample window:

$$MSE_i = \left(\sum_{t=T+1}^{T+h} e_{it}^2\right)/h \text{ for } i = 1, \cdots, N$$

where

$$e_{it} = x_{it} - \hat{\lambda}_i \hat{f}_T \text{ for } t = T+1, \cdots, T+h,$$

with

$$\hat{\lambda} = \left(\sum_{t=1}^{T} \hat{f}_t \hat{f}_t'\right)^{-1} \left(\sum_{t=1}^{T} \hat{f}_t x_{it}\right).$$

Table 3 compares the averages of MSE_i over all the observations when $\left\{\hat{f}_t\right\}$ are non-spurious factors (NS) and when $\left\{\hat{f}_t\right\}$ are crude factors (crude). The non-spurious factors clearly improve the out-of-sample forecasting MSEs in every case.

5 Non-spurious factors in U.S. macroeconomic time series

Stock and Watson (2005) found that there are seven dynamic factors and nine static factors in U.S. macroeconomic time series using the IC_p2 criterion of Bai and Ng (2002). They raise a question why so many factors exist in the U.S. economy. It is concerned that too many factors may cause an overfitting problem and deteriorate the out-of-sample forecasting performance. Also, the conventional view of macroeconomists would be that there exist only a few economically meaningful factors in the macroeconomy (Sargent and Sims, 1977). In this section, I attempt to answer this question using the approach proposed in this paper. My conjecture is that the factor model best fitting US macroeconomic data sets is one with the factor loadings susceptible to structural changes. Therefore, if one estimates factors by the principal components method after ignoring these structural changes, then spurious factors show up. In this section, I use the 132 macroeconomic time series from January 1959 to December 2003 used by Stock and Watson (2005). The same transformations are applied to induce stationarity and each series is demeaned and divided by its sample standard deviation prior to estimating its principal components. Considering the Great Moderation discussed in existing literature, one is likely to see a change in the common factor structure itself around the mid-1980s, and hence, I also consider a subsample from January 1985 for the analysis (the post-1984 subsample hereafter).

I first estimate the number of factors using the full sample data set assuming that all the factor loadings are stable. The results are presented in Table 4. The estimated numbers are eleven by $ICp1$, eight by $ICp2$, and twelve by $ICp3$ and these numbers are in close agreement with that of Stock and Watson (2005). Applying the recursive procedure to the same full sample data, I find that 51 out of the 132 series have stable factor loadings. Table 8 provides the stabilities of individual series, whereas Table 5 summarizes the results. I find that the slow variables defined in Stock and Watson (2005) are more likely stable (35 out of 67 series: 52.2%) than are the fast variables (16 out of 65 series: 24.6%). I also categorize the series into six groups according to their qualitative nature. According this grouping, I find that the series related to "housing", "money / credit", and "stock price / interest rates / exchange rates" are more likely to be unstable. However, since they are not too concentrated in certain categories, it is deemed that the estimated set of observations with stable factor loadings still captures the entire information in the macroeconomic time series. I then estimate the number of factors using the stable observations and find much smaller numbers: one by $ICp1$, one by $ICp2$, and two by $ICp3$. Therefore, they support the fact that the number of factors in the U.S. macroeconomy is inflated owing to the structural changes in factor loadings. Using the post-1984 subsample, I roughly get the same results as in the full sample estimation. This time, I find 47 observations in the stable set, which consists of all the categories except housing. The number of common factors using the stable set are two by $ICp1$, two by $ICp2$, and twelve by $ICp3$.

The next interesting question is whether one can give reasonable interpretations of the estimated non-spurious factors. As is well known, the principal components are merely consistent estimates of the true factor up to a random rotation, and hence, there is no theoretical background for the obtained individual factor estimates interpretable.[9] However,

[9] See Bai and Ng (2010) and Yamamoto (2012).

I simply follow an argument given in Stock and Watson (2005). Through the forecast error variance decomposition of the full sample data, they find that the first factor explains much of the variation in real variables such as production, capacity utilization, and employment. The second factor is mostly related to financial variables such as interest rates and stock prices. The third factor pertains to inflation. Importantly, among the remaining factors, only the fifth is related to a real variable of long-term unemployment, but the fourth, sixth, and seventh factors are financial factors. Therefore, it is interesting to investigate the relationship between these seven crude factors and the first few non-spurious factors estimated by the proposed procedure. Table 6 shows the correlation coefficients of the seven crude factors with the first two non-spurious factors in the full sample analysis. Strong and significant correlations are found within the first two factors and hence, the first two non-spurious factors (real and financial factors) are interpretable roughly in the same way, again, given the rotation is resolved. More importantly, the remaining crude factors, except for the fifth, are significantly correlated with the second non-spurious factor but not with the first. It is concluded that the most of the “many” crude factors can be generated by the time instabilities of factor loadings of the second non-spurious factor, that represents, the housing and financial factor. When I use the post-1984 subsample, I also observe that many of the crude factors are more correlated to the second non-spurious factor, although we may possibly now interpret the first and second crude factors in the opposite way because of the random rotation.

Finally, I compare the out-of-sample forecasting performance using the crude factors and the non-spurious factors. To this end, I specify the last h months of the sample as the out-of-sample window and estimate the crude factors and non-spurious factors respectively using the fixed in-sample data. Then, I compute the MSEs of the forecasts using the crude factors and using the non-spurious factors respectively for each observation in the out-of-sample window. Table 7 compares the averages of the MSEs over all the observations for the out-of-sample period length $h = 12, 24$, and 48.[10] The number of factors estimated in-sample by the IC_p2 criterion and used for forecasting is given in the parentheses.[11] I find that the NS factor forecasts give larger MSEs than the crude factor forecasts when $h = 12$. However, the NS factor forecasts give lower MSEs for longer horizons $h = 24$ and 48. To see if these differences are significant, I conduct a test for equal predictive accuracy by Diebold and Mariano (1995)

[10]The MSEs of the individual observations are reported in Table 8 in the case of $h = 24$.

[11]The estimated numbers and factors are not exactly the same as in the full sample analysis, since the sample is now h periods shorter.

for the pooled observations.[12] The results are in the middle column of Table 7. It is suggested that the NS factor forecast is significantly more accurate than the crude factor forecast for $h = 24$ and 48 at the 1% level and the null hypothesis of equal accuracy is not rejected for $h = 12$ even at the 10% level. This results are the same when using either full sample and post-1984 subsample data. These support the fact that the large number of spurious factors cause overfitting of U.S. macroeconomic time series and deteriorate forecasting performance. Finally, I present results of a pooled version of the forecast break down test recently proposed by Giacomini and Rossi (2009) for either forecasting methods in the last column of Table 7.[13] The "forecast breakdown" means the situation where the out-of-sample accuracy gets significantly worse than the in-sample fit so that the researchers may not rely on the usual measures of the model fit. The results indicate that the NS factor forecasts never breaks down (they are insignificant at the 10% levels), however, the crude factor forecast may show forecast breakdowns at the 10% significant level using post-1984 sample for $h = 12$ and 24.

6 Conclusions

This paper proposes a method to overcome an overfitting problem of dynamic factor models which are induced by instability in factor loadings. The suggested algorithm estimates the non-spurious factors by identifying the set of observations with stable factor loadings in dynamic factor models. Monte Carlo simulations find that the method yields good coverage ratios, estimates the number of non-spurious factors correctly, and improves out-of-sample forecasting accuracy. Using this approach, I provide strong evidence in answering to the question why so many factors exist in the U.S. economy, raised by Stock and Watson (2005). Most importantly, it significantly improves forecasting performance for the U.S. macroeconomic time series.

[12] The test statistic pools the individual tests by $DM = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} DM_i$, where DM_i is the standard Diebold and Mariano (1995) test for the ith observation using HAC robust standard errors proposed by Andrews' (1991) data dependent method with AR1 approximation. The test is two-sided and a positive significance means that the non-spurious factor forecast is more accurate than the crude factor forecast. Note that the pooled test statistic may over-rejects, since it does not accout for the cross-sectional correlations of the individual test statistics.

[13] Again, the tests applied for individual observations account for serial correlations in the loss using HAC standard errors proposed by Andrews (1991), but the pooled statistic does not consider cross-sectional correlations and they may over-reject the null hypothesis.

Appendix A : A theoretical justification of the procedure

In this appendix, I discuss the asymptotic justification for the fact that the proposed sequential algorithm finds $\hat{s}$ which corresponds s^* with probability $1-\alpha$. Let the following assumptions hold:

Assumption 1. The algorithm employs a collective test that satisfies $T_T^C(s) \xrightarrow{d} D(s)$ if $s = s^*$ as $N, T \to \infty$ and $\sqrt{T}/N \to 0$. It also satisfies $T_T^C(s) \to \infty$ if $\max(|s|, |s^*|) > |s^*|$ as $N, T \to \infty$.

Assumption 2. The algorithm employs an individual test that satisfies $\lim_{N,T\to\infty} P(T_T^I(e_i, s) > T_T^I(e_j, s)) = 1$ for all i and j such that $s^*(i) = 0$ and $s^*(j) = 1$ and for s such that $\min(s - s^*) \geq 0$.

Both assumptions are high-level, but Assumption 1 is straightforwardly shown to hold under the standard regularity conditions of Bai (2003) with both HI and CDG tests. Assumption 2 requires that the individual test is able to order the observations with stable and unstable factor loadings asymptotically. This can allow for the size-distortions and for the tests to be inconsistent, however, involves two requirements. First, there is no alternative representations of the model (1) and (2) that completely offset the breaks in $\lambda_i(1-s)$ that loses all the power of the individual tests. Second, the size distortions can be large, but the test should not explode.

Then, the following theorem holds.

Theorem 1 *Under Assumptions 1 and 2,*

$$\lim P_r(\hat{s} = s^*) = 1 - \alpha \qquad \textit{if } s^* \neq 0_{N\times 1}, \tag{A.1}$$

as $N, T \to \infty$ *and* $\sqrt{T}/N \to 0$ *and*

$$\lim P_r(\hat{s} = s^*) = 1 \qquad \textit{if } s^* = 0_{N\times 1}, \tag{A.2}$$

as $N, T \to \infty$. *Also,*

$$\lim P_r(\hat{s} \neq s^* \ \textit{and} \ |\hat{s}| \geq |s^*|) = 0, \tag{A.3}$$

as $N, T \to \infty$.

The proof closely follows the appendix of Inoue and Rossi (2011) and proceeds as follows. Let $k_\alpha^C(s)$ denote the critical value of $T_T^C(s)$ of the null distribution $D(s)$ at the level of significance α. Let $N^* = N - |s^*|$ be the number of observations with unstable loadings. Because the collective test is consistent by Assumption 1,

$$\lim_{N,T\to\infty} P(T_T^C(s_j) > k_\alpha^C(s_j)) = 1, \tag{A.4}$$

for any s_j such that $\max(|s_j|, |s^*|) > |s^*|$ for $j = 1, 2, \cdots, N-N^*-1$. Because the individual test for an observations with unstable loadings is larger than the test for an observation with stable loadings by Assumption 2,

$$\lim_{N,T\to\infty} P\left(\{i : s^*(i) = 0\} \in I_j\right) = 1, \tag{A.5}$$

for $j = 1, \cdots, N - N^*$, where $I_j = \left\{ i : T_T^I(e_i) \text{ is among the } j \text{ largest } \left\{T_T^I(e_l)\right\}_{l=1}^N \right\}$, if $N - N^* > 0$. In particular, equation (A.5) implies that

$$\lim_{N,T\to\infty} P\left(I_{N-N^*} = \{i : s^*(i) = 0\}\right) = 1. \tag{A.6}$$

Consider now the three cases (a) $N^* = 0$, (b) $N^* > 0$ and $N - \bar{r} > N^*$, and (c) $N^* > 0$ and $N^* = N - \bar{r}$.

(a) If $N^* = 0$, then

$$\lim_{N,T\to\infty} P(T_T^C(s_0) < k_\alpha^C(s_0)) = \lim_{N,T\to\infty} P(T_T^C(s^*) < k_\alpha^C(s^*)) = 1 - \alpha,$$

thus proving equation (A.1) under $H_0^{(0)}(s_0)$. When $N^* = 0$, equation (A.2) does not apply, and equation (A.3) trivially holds.

(b) Because I eliminate only one stable observation in each step, I have $\max(|s_j|, |s^*|) > |s^*| = N - N^*$ for any s_j for $j = 0, 1, \cdots, N^* - 1$. Thus it follows from (A.4) that

$$\lim_{N,T\to\infty} P(T_T^C(s_j) > k_\alpha^C(s_j), \forall j = 0, 1, \cdots, N - N^* - 1) = 1. \tag{A.7}$$

By equations (A.6) and (A.7), we have

$$\lim P(s_{N-N^*} = s^*) = 1. \tag{A.8}$$

Now

$$\begin{aligned}
\lim_{N,T\to\infty,\sqrt{T}/N\to 0} P(\hat{s} &= s^*) \\
&= \lim_{N,T\to\infty,\sqrt{T}/N\to 0} P\left(T_T^C(\hat{s}) < k_\alpha^C(\hat{s})\right. \\
&\qquad \left.\mid \text{The test rejects at } j = 0, 1, \cdots, N - N^* - 1\right) \\
&= \lim_{N,T\to\infty,\sqrt{T}/N\to 0} P\left(T_T^C(s^*) < k_\alpha^C(s^*)\right. \\
&\qquad \left.\mid \text{The test rejects at } j = 0, 1, \cdots, N - N^* - 1 \text{ and } s_{N-N^*} = s^*\right) \\
&= \lim_{N,T\to\infty,\sqrt{T}/N\to 0} P(T_T^C(s^*) < k_\alpha^C(s^*)) \\
&= 1 - \alpha,
\end{aligned}$$

where the third equality follows from equations (A.7) and (A.8) and the last equation from Assumption 1. The equation (A.1) holds. The equation (A.3) follows from (A.4).

(c) $T_T^C(s_j)$ for $j = 0, 1, \cdots, N^*$ all rejects by (A.4), and each of the N^* observations with unstable loadings is selected in these steps with probability approaching one by equation (A.6). Therefore $\hat{s}$ converges almost surely to s^*, and the equations (A.2) and (A.3) hold.

Appendix B : Description of the test statistics

I first estimate the number of factors $\hat{r}$ in the factor model (1) and (2) assuming $\lambda_{1i} = \lambda_{2i}$. Let $\hat{f}_t$ be an $\hat{r} \times 1$ vector of the factor estimates by the principal components method. The three test statistics used in this paper are defined as follows, where I use the truncation parameter $\epsilon = 0.15$ for every test.

1. Breitung and Eickmeier (2011)

   The test statistic is

   $$\sup_{\tau \in [\epsilon, 1-\epsilon]} S_T^{BE}(\tau),$$

   where $S_T^{BE}(\tau)$ is either the LM, LR, or Wald statistic for the null hypothesis $\lambda_{1i} = \lambda_{2i}$. In particular, I use the Wald specification so that $S_T(\tau) = T(SSR^r - SSR(\tau))/SSR(\tau)$ where $SSR(\tau)$ is the sum of squared residuals of the regression of x_{it} on $\hat{f}_t$ assuming $T_b = \lfloor \tau T \rfloor$ and SSR^r is the sum of squared residuals of the same regression assuming $\lambda_{1i} = \lambda_{2i}$. However, using the LM or LR versions does not change the qualitative results in this paper.

2. Han and Inoue (2012)

   The test statistic is

   $$\sup_{\tau \in [\epsilon, 1-\epsilon]} A_T(\tau)' V_T(\tau)^{-1} A_T(\tau),$$

   where

   $$A_T(\tau) \equiv vech\left(\sqrt{T} \frac{1}{\lfloor \tau T \rfloor} \sum_{t=1}^{\lfloor \tau T \rfloor} \hat{f}_t \hat{f}_t' - \frac{1}{T - \lfloor \tau T \rfloor} \sum_{t=\lfloor \tau T \rfloor + 1}^{T} \hat{f}_t \hat{f}_t' \right),$$

   and $V_T(\tau)$ is an estimate of the long-run covariance matrix of $A_T(\tau)$. To account for the serial correlations in $\hat{f}_t$, I use the Newey and West (1994)'s HAC estimator for $V_T(\tau)$.

3. Chen, Dolado, and Gonzalo (2012)

   The test statistic is

   $$\sup_{\tau \in [\epsilon, 1-\epsilon]} S_T^{CDG}(\tau),$$

   where $S_T^{CDG}(\tau)$ is either LM, LR, or Wald statistic for the hypothesis $C_1 = C_2$ where $C_1 = [c_{12}, \ldots, c_{1\hat{r}}]$ and $C_2 = [c_{22}, \ldots, c_{2\hat{r}}]$ in the following model

   $$f_{1t} = \begin{cases} c_{12} f_{2t} + \cdots + c_{1\hat{r}} f_{\hat{r}t}, & \text{for } t \leq T_b \\ c_{22} f_{2t} + \cdots + c_{2\hat{r}} f_{\hat{r}t}, & \text{for } t > T_b \end{cases}$$

   In particular, I use the Wald specification. However, using LM or LR versions does not change the qualitative results in this paper.

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Table 1: Estimated number of factors
and empirical powers of structural change tests in factor loadings

1) $N_0 = 0.7N$:

DGP-1) One-time structural change

	number of factors			individual tests				collective tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
b	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	1.8	2.0	2.0	0.05	0.03	0.05	0.03	0.05	0.04	0.10	0.04
0.5	1.8	2.0	2.0	0.07	0.03	0.16	0.17	0.08	0.26	0.12	0.05
1.0	1.9	2.0	2.0	0.10	0.03	0.45	0.56	0.19	0.61	0.15	0.14
1.5	2.0	2.0	2.0	0.18	0.03	0.63	0.78	0.41	0.87	0.22	0.27
2.0	2.0	2.0	2.0	0.28	0.03	0.74	0.87	0.65	0.95	0.30	0.37
3.0	2.4	2.0	2.0	0.31	0.03	0.84	0.94	0.95	1.00	0.63	0.81
4.0	3.0	3.0	3.0	0.28	0.03	0.88	0.97	1.00	1.00	0.93	0.99
5.0	3.7	4.0	4.0	0.28	0.03	0.90	0.98	1.00	1.00	1.00	1.00
10.0	6.5	6.0	6.0	0.37	0.04	0.95	0.99	1.00	1.00	1.00	1.00

DGP-2) Multiple structural change

	number of factors			individual tests				collective tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
b	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	1.8	2.0	2.0	0.05	0.03	0.06	0.03	0.05	0.04	0.09	0.03
0.5	1.8	2.0	2.0	0.10	0.03	0.26	0.49	0.17	0.58	0.14	0.07
1.0	1.9	2.0	2.0	0.24	0.03	0.51	0.94	0.50	0.88	0.26	0.18
1.5	2.1	2.0	2.0	0.36	0.04	0.62	0.99	0.83	0.97	0.46	0.30
2.0	2.5	3.0	3.0	0.27	0.03	0.67	1.00	0.97	0.99	0.75	0.43
3.0	2.9	3.0	3.0	0.18	0.03	0.84	1.00	1.00	1.00	0.96	0.82
4.0	3.2	3.0	3.0	0.23	0.02	0.92	1.00	1.00	1.00	1.00	0.99
5.0	3.7	4.0	4.0	0.28	0.03	0.93	1.00	1.00	1.00	1.00	1.00
10.0	6.4	6.0	6.0	0.41	0.03	0.97	1.00	1.00	1.00	1.00	1.00

DGP-3) Random walk

	number of factors			individual tests				collective tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
b	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	2.0	2.0	2.0	0.05	0.03	0.06	0.03	0.05	0.05	0.15	0.14
0.5	2.0	2.0	2.0	0.06	0.03	0.61	0.54	0.07	0.17	0.17	0.19
1.0	2.0	2.0	2.0	0.09	0.03	0.80	0.76	0.10	0.60	0.17	0.51
1.5	2.1	2.0	2.0	0.10	0.03	0.84	0.83	0.20	0.88	0.24	0.80
2.0	2.4	2.0	2.0	0.13	0.03	0.85	0.88	0.46	0.97	0.47	0.91
3.0	3.0	3.0	3.0	0.14	0.03	0.85	0.92	0.86	1.00	0.83	0.99
4.0	3.4	3.0	3.0	0.15	0.03	0.86	0.95	0.97	1.00	0.96	0.99
5.0	3.7	4.0	4.0	0.16	0.03	0.86	0.96	0.99	1.00	0.99	1.00
10.0	4.5	4.0	4.0	0.19	0.03	0.93	0.99	1.00	1.00	1.00	1.00

$N_0 = 0.5N$

DGP-1) One-time structural change

	number of factors			individual tests				collective tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
b	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	1.8	2.0	2.0	0.06	0.03	0.05	0.03	0.05	0.05	0.11	0.06
0.5	1.9	2.0	2.0	0.09	0.03	0.15	0.19	0.11	0.23	0.12	0.09
1.0	1.9	2.0	2.0	0.17	0.04	0.36	0.55	0.32	0.61	0.18	0.19
1.5	2.0	2.0	2.0	0.29	0.04	0.58	0.78	0.63	0.87	0.25	0.28
2.0	2.0	2.0	2.0	0.39	0.04	0.69	0.86	0.84	0.96	0.31	0.36
3.0	2.6	3.0	3.0	0.37	0.03	0.81	0.93	0.99	1.00	0.75	0.91
4.0	3.6	4.0	4.0	0.37	0.04	0.83	0.96	1.00	1.00	0.99	1.00
5.0	4.4	4.0	4.0	0.41	0.04	0.87	0.97	1.00	1.00	1.00	1.00
10.0	7.6	8.0	7.0	0.52	0.03	0.94	0.99	1.00	1.00	1.00	1.00

DGP-2) Multiple structural change

	number of factors			individual tests				collective tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
b	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	1.8	2.0	2.0	0.06	0.03	0.06	0.04	0.04	0.05	0.10	0.06
0.5	1.9	2.0	2.0	0.15	0.03	0.18	0.48	0.27	0.53	0.16	0.13
1.0	1.9	2.0	2.0	0.37	0.03	0.37	0.93	0.70	0.87	0.27	0.23
1.5	2.1	2.0	2.0	0.46	0.03	0.55	0.99	0.91	0.97	0.43	0.33
2.0	2.4	2.0	2.0	0.40	0.03	0.67	1.00	0.99	0.99	0.64	0.45
3.0	3.0	3.0	3.0	0.33	0.03	0.80	1.00	1.00	1.00	0.94	0.92
4.0	3.7	4.0	4.0	0.37	0.03	0.86	1.00	1.00	1.00	1.00	1.00
5.0	4.4	4.0	4.0	0.40	0.03	0.88	1.00	1.00	1.00	1.00	1.00
10.0	7.4	7.0	7.0	0.53	0.03	0.97	1.00	1.00	1.00	1.00	1.00

DGP-3) Random walk

	number of factors			individual tests				collective tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
b	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	2.0	2.0	2.0	0.06	0.03	0.06	0.03	0.05	0.04	0.14	0.14
0.5	2.0	2.0	2.0	0.07	0.03	0.62	0.55	0.08	0.17	0.15	0.19
1.0	2.1	2.0	2.0	0.11	0.03	0.75	0.73	0.23	0.72	0.26	0.66
1.5	2.6	3.0	3.0	0.16	0.03	0.76	0.83	0.65	0.96	0.63	0.92
2.0	3.2	3.0	3.0	0.14	0.03	0.75	0.88	0.92	1.00	0.88	0.99
3.0	3.8	4.0	4.0	0.15	0.03	0.77	0.92	1.00	1.00	0.99	1.00
4.0	4.0	4.0	4.0	0.18	0.03	0.81	0.95	1.00	1.00	1.00	1.00
5.0	4.2	4.0	4.0	0.17	0.03	0.85	0.96	1.00	1.00	1.00	1.00
10.0	5.3	5.0	5.0	0.24	0.03	0.90	0.98	1.00	1.00	1.00	1.00

$N_0 = 0.3N$

DGP-1) One-time structural change

b	number of factors			individual tests				joint tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	1.8	2.0	2.0	0.06	0.04	0.04	0.03	0.05	0.06	0.10	0.08
0.5	1.9	2.0	2.0	0.10	0.03	0.13	0.18	0.16	0.22	0.12	0.11
1.0	2.0	2.0	2.0	0.23	0.03	0.33	0.53	0.44	0.60	0.19	0.20
1.5	2.0	2.0	2.0	0.37	0.04	0.53	0.74	0.77	0.88	0.27	0.28
2.0	2.0	2.0	2.0	0.47	0.04	0.69	0.84	0.93	0.96	0.33	0.37
3.0	3.0	3.0	3.0	0.45	0.03	0.77	0.92	1.00	1.00	0.88	0.96
4.0	4.1	4.0	4.0	0.45	0.03	0.81	0.96	1.00	1.00	1.00	1.00
5.0	5.0	5.0	5.0	0.48	0.03	0.85	0.97	1.00	1.00	1.00	1.00
10.0	8.4	8.0	8.0	0.63	0.03	0.94	0.99	1.00	1.00	1.00	1.00

DGP-2) Multiple structural change

b	number of factors			individual tests				joint tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	1.8	2.0	2.0	0.05	0.03	0.05	0.03	0.04	0.05	0.09	0.08
0.5	1.9	2.0	2.0	0.21	0.03	0.15	0.48	0.37	0.50	0.17	0.15
1.0	2.0	2.0	2.0	0.41	0.03	0.34	0.92	0.79	0.87	0.28	0.26
1.5	2.0	2.0	2.0	0.56	0.03	0.53	0.99	0.95	0.98	0.35	0.33
2.0	2.2	2.0	2.0	0.56	0.04	0.67	1.00	0.99	0.99	0.50	0.45
3.0	3.2	3.0	3.0	0.47	0.03	0.76	1.00	1.00	1.00	0.95	0.97
4.0	4.1	4.0	4.0	0.49	0.03	0.82	1.00	1.00	1.00	1.00	1.00
5.0	4.9	5.0	5.0	0.50	0.04	0.86	1.00	1.00	1.00	1.00	1.00
10.0	8.3	8.0	8.0	0.61	0.03	0.96	1.00	1.00	1.00	1.00	1.00

DGP-3) Random walk

b	number of factors			individual tests				joint tests			
				BE : stable loading		BE : unstable loading		HI		CDG	
	mean	median	mode	feasible	infeasible	feasible	infeasible	feasible	infeasible	feasible	infeasible
0.0	2.0	2.0	2.0	0.05	0.03	0.06	0.03	0.06	0.06	0.14	0.13
0.5	2.0	2.0	2.0	0.08	0.02	0.62	0.55	0.11	0.18	0.16	0.21
1.0	2.5	2.0	2.0	0.15	0.03	0.70	0.73	0.56	0.83	0.54	0.79
1.5	3.3	3.0	3.0	0.17	0.03	0.66	0.82	0.94	0.99	0.91	0.98
2.0	3.7	4.0	4.0	0.15	0.03	0.67	0.87	1.00	1.00	0.99	1.00
3.0	4.0	4.0	4.0	0.17	0.03	0.74	0.92	1.00	1.00	1.00	1.00
4.0	4.3	4.0	4.0	0.20	0.04	0.80	0.95	1.00	1.00	1.00	1.00
5.0	4.6	5.0	5.0	0.21	0.03	0.82	0.96	1.00	1.00	1.00	1.00
10.0	5.9	6.0	6.0	0.26	0.03	0.88	0.98	1.00	1.00	1.00	1.00

Table 2: Coverage ratios of the procedure

$N_0 = 0.7N$

DGP-1) One-time structural change

	b	coverage ratio	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	2.04	2.00	2.00	1.83	2.00	2.00
	1	0.710	2.09	2.00	2.00	1.90	2.00	2.00
	3	0.830	2.03	2.00	2.00	2.47	2.00	2.00
	5	0.944	2.29	2.00	2.00	3.71	4.00	4.00
	10	0.909	3.48	2.00	1.00	6.48	6.00	6.00
N=100, T=150	0	-	2.10	2.00	2.00	1.98	2.00	2.00
	1	0.720	2.11	2.00	2.00	1.99	2.00	2.00
	3	0.887	1.99	2.00	2.00	2.78	3.00	3.00
	5	0.965	2.07	2.00	2.00	4.16	4.00	4.00
	10	0.993	2.60	2.00	2.00	7.34	7.00	7.00

	b	coverage ratio	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.04	2.00	2.00	1.98	2.00	2.00
	1	0.709	2.03	2.00	2.00	1.99	2.00	2.00
	3	0.847	2.03	2.00	2.00	2.74	3.00	3.00
	5	0.944	2.07	2.00	2.00	4.18	4.00	4.00
	10	0.919	3.29	2.00	2.00	7.13	7.00	7.00
N=150, T=150	0	-	2.08	2.00	2.00	2.00	2.00	2.00
	1	0.727	2.08	2.00	2.00	2.00	2.00	2.00
	3	0.889	1.99	2.00	2.00	2.95	3.00	3.00
	5	0.963	2.13	2.00	2.00	4.50	4.00	4.00
	10	0.994	2.31	2.00	2.00	7.69	8.00	8.00

DGP-2) Multiple structural change

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	1.99	2.00	2.00	1.85	2.00	2.00
	1	0.751	2.03	2.00	2.00	1.89	2.00	2.00
	3	0.968	2.20	2.00	2.00	2.92	3.00	3.00
	5	0.993	2.25	2.00	2.00	3.70	4.00	4.00
	10	0.946	2.87	2.00	1.00	6.37	6.00	6.00
N=100, T=150	0	-	2.08	2.00	2.00	1.98	2.00	2.00
	1	0.795	2.09	2.00	2.00	2.00	2.00	2.00
	3	0.983	1.99	2.00	2.00	3.01	3.00	3.00
	5	0.998	2.13	2.00	2.00	4.20	4.00	4.00
	10	1.000	2.41	2.00	2.00	7.20	7.00	7.00

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.01	2.00	2.00	1.97	2.00	2.00
	1	0.751	2.03	2.00	2.00	1.99	2.00	2.00
	3	0.967	2.06	2.00	2.00	2.99	3.00	3.00
	5	0.994	2.08	2.00	2.00	4.10	4.00	4.00
	10	0.937	2.89	2.00	2.00	7.14	7.00	7.00
N=150, T=150	0	-	2.15	2.00	2.00	2.00	2.00	2.00
	1	0.803	2.10	2.00	2.00	2.01	2.00	2.00
	3	0.982	2.06	2.00	2.00	3.02	3.00	3.00
	5	0.997	2.07	2.00	2.00	4.52	5.00	5.00
	10	1.000	2.01	2.00	2.00	7.57	8.00	8.00

DGP-3) Random walk

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	2.26	2.00	2.00	2.00	2.00	2.00
	1	0.705	2.18	2.00	2.00	2.01	2.00	2.00
	3	0.806	2.22	2.00	2.00	3.02	3.00	3.00
	5	0.891	2.47	2.00	2.00	3.73	4.00	4.00
	10	0.955	2.95	2.00	2.00	4.50	4.00	4.00
N=100, T=150	0	-	2.30	2.00	2.00	2.00	2.00	2.00
	1	0.726	2.20	2.00	2.00	2.28	2.00	2.00
	3	0.914	2.39	2.00	2.00	3.92	4.00	4.00
	5	0.957	2.62	2.00	2.00	4.40	4.00	4.00
	10	0.987	3.16	2.00	2.00	5.79	6.00	6.00

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.28	2.00	2.00	2.00	2.00	2.00
	1	0.706	2.48	2.00	2.00	2.03	2.00	2.00
	3	0.830	2.30	2.00	2.00	3.40	3.00	3.00
	5	0.905	2.32	2.00	2.00	3.96	4.00	4.00
	10	0.962	2.52	2.00	2.00	4.89	5.00	5.00
N=150, T=150	0	-	2.24	2.00	2.00	2.00	2.00	2.00
	1	0.728	2.30	2.00	2.00	2.37	2.00	2.00
	3	0.917	2.23	2.00	2.00	4.00	4.00	4.00
	5	0.964	2.43	2.00	2.00	4.65	5.00	5.00
	10	0.988	2.54	2.00	2.00	6.07	6.00	6.00

$N_0 = 0.5N$

DGP-1) One-time structural change

	b	coverage ratio	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	2.00	2.00	2.00	1.81	2.00	2.00
	1	0.518	2.35	2.00	2.00	1.94	2.00	2.00
	3	0.745	2.07	2.00	2.00	2.63	3.00	3.00
	5	0.911	2.95	2.00	2.00	4.41	4.00	4.00
	10	0.752	5.67	6.00	1.00	7.61	8.00	8.00
N=100, T=150	0	-	2.04	2.00	2.00	1.98	2.00	2.00
	1	0.549	2.22	2.00	2.00	1.99	2.00	2.00
	3	0.843	2.07	2.00	2.00	3.12	3.00	3.00
	5	0.945	2.34	2.00	2.00	5.04	5.00	5.00
	10	0.986	3.21	2.00	1.00	8.52	8.00	8.00

	b	coverage ratio	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.02	2.00	2.00	1.97	2.00	2.00
	1	0.518	2.28	2.00	2.00	1.99	2.00	2.00
	3	0.765	2.23	2.00	2.00	3.04	3.00	3.00
	5	0.895	2.46	2.00	2.00	5.03	5.00	5.00
	10	0.793	4.71	2.00	1.00	8.41	8.00	9.00
N=150, T=150	0	-	2.09	2.00	2.00	2.00	2.00	2.00
	1	0.546	2.19	2.00	2.00	2.00	2.00	2.00
	3	0.843	2.04	2.00	2.00	3.33	3.00	3.00
	5	0.947	2.17	2.00	2.00	5.30	5.00	5.00
	10	0.987	2.75	2.00	2.00	8.88	9.00	9.00

DGP-2) Multiple structural change

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	1.88	2.00	2.00	1.83	2.00	2.00
	1	0.528	6.04	2.00	2.00	1.92	2.00	2.00
	3	0.933	4.39	2.00	1.00	2.97	3.00	3.00
	5	0.984	3.39	1.00	1.00	4.43	4.00	4.00
	10	0.812	4.12	2.00	1.00	7.45	7.00	7.00
N=100, T=150	0	-	2.08	2.00	2.00	1.98	2.00	2.00
	1	0.632	6.80	4.00	12.00	2.00	2.00	2.00
	3	0.981	2.58	2.00	1.00	3.19	3.00	3.00
	5	0.997	2.22	2.00	2.00	4.97	5.00	5.00
	10	0.999	2.45	1.00	1.00	8.33	8.00	8.00

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.06	2.00	2.00	1.98	2.00	2.00
	1	0.546	5.65	2.00	2.00	2.01	2.00	2.00
	3	0.954	2.99	2.00	1.00	3.21	3.00	3.00
	5	0.959	2.54	2.00	1.00	4.97	5.00	5.00
	10	0.851	3.16	2.00	2.00	8.32	8.00	8.00
N=150, T=150	0	-	2.06	2.00	2.00	2.00	2.00	2.00
	1	0.665	5.34	2.00	2.00	2.01	2.00	2.00
	3	0.981	2.07	2.00	2.00	3.36	3.00	3.00
	5	0.998	2.11	2.00	2.00	5.30	5.00	5.00
	10	1.000	2.35	2.00	2.00	8.90	9.00	9.00

DGP-3) Random walk

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	2.40	2.00	2.00	2.00	2.00	2.00
	1	0.520	2.26	2.00	2.00	2.14	2.00	2.00
	3	0.780	2.64	2.00	2.00	3.79	4.00	4.00
	5	0.879	2.90	2.00	2.00	4.19	4.00	4.00
	10	0.941	4.43	2.00	2.00	5.31	5.00	5.00
N=100, T=150	0	-	2.18	2.00	2.00	2.00	2.00	2.00
	1	0.639	2.24	2.00	2.00	3.14	3.00	3.00
	3	0.900	2.52	2.00	2.00	4.35	4.00	4.00
	5	0.954	2.76	2.00	2.00	5.36	5.00	5.00
	10	0.985	4.58	2.00	2.00	6.78	7.00	7.00

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.16	2.00	2.00	2.00	2.00	2.00
	1	0.527	2.22	2.00	2.00	2.30	2.00	2.00
	3	0.803	2.38	2.00	2.00	3.98	4.00	4.00
	5	0.896	2.48	2.00	2.00	4.50	4.00	4.00
	10	0.939	3.08	2.00	2.00	5.88	6.00	6.00
N=150, T=150	0	-	2.28	2.00	2.00	2.00	2.00	2.00
	1	0.680	2.20	2.00	2.00	3.47	3.00	4.00
	3	0.914	2.35	2.00	2.00	4.51	4.00	4.00
	5	0.959	2.37	2.00	2.00	5.67	6.00	6.00
	10	0.987	3.24	2.00	2.00	7.20	7.00	7.00

$N_0 = 0.3N$

DGP-1) One-time structural change

DGP1

	b	coverage ratio	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	2.01	2.00	2.00	1.81	2.00	2.00
	1	0.316	3.24	2.00	2.00	1.99	2.00	2.00
	3	0.626	3.65	2.00	2.00	3.00	3.00	3.00
	5	0.731	7.10	12.00	12.00	4.93	5.00	5.00
	10	0.519	8.39	9.00	12.00	8.39	8.00	8.00
N=100, T=150	0	-	2.06	2.00	2.00	1.97	2.00	2.00
	1	0.350	3.60	2.00	2.00	2.00	2.00	2.00
	3	0.743	3.15	2.00	2.00	3.58	4.00	4.00
	5	0.899	4.65	2.00	1.00	5.57	6.00	6.00
	10	0.951	5.50	4.00	1.00	9.36	9.00	9.00

DGP1

	b	coverage ratio	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.03	2.00	2.00	1.97	2.00	2.00
	1	0.318	2.95	2.00	2.00	2.00	2.00	2.00
	3	0.653	2.95	2.00	2.00	3.54	4.00	4.00
	5	0.705	5.56	2.00	12.00	5.56	6.00	6.00
	10	0.558	7.61	9.00	12.00	9.30	9.00	9.00
N=150, T=150	0	-	2.11	2.00	2.00	2.00	2.00	2.00
	1	0.359	3.22	2.00	2.00	2.00	2.00	2.00
	3	0.770	2.31	2.00	2.00	3.89	4.00	4.00
	5	0.904	3.66	2.00	1.00	5.91	6.00	6.00
	10	0.968	4.41	2.00	1.00	9.89	10.00	10.00

DGP-2) Multiple structural change

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	1.91	2.00	2.00	1.81	2.00	2.00
	1	0.289	8.23	12.00	12.00	1.98	2.00	2.00
	3	0.702	10.22	12.00	12.00	3.21	3.00	3.00
	5	0.818	7.89	12.00	12.00	4.90	5.00	5.00
	10	0.600	7.51	9.00	12.00	8.31	8.00	8.00
N=100, T=150	0	-	2.07	2.00	2.00	1.98	2.00	2.00
	1	0.296	11.04	12.00	12.00	2.00	2.00	2.00
	3	0.893	6.82	8.00	12.00	3.63	4.00	4.00
	5	0.983	4.47	1.00	1.00	5.51	6.00	6.00
	10	0.993	3.72	1.00	1.00	9.22	9.00	9.00

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.03	2.00	2.00	1.96	2.00	2.00
	1	0.297	8.18	12.00	12.00	2.00	2.00	2.00
	3	0.797	8.49	12.00	12.00	3.58	4.00	4.00
	5	0.769	5.17	2.00	1.00	5.53	6.00	6.00
	10	0.670	4.96	2.00	1.00	9.30	9.00	9.00
N=150, T=150	0	-	2.03	2.00	2.00	2.00	2.00	2.00
	1	0.309	10.59	12.00	12.00	2.00	2.00	2.00
	3	0.959	4.32	2.00	1.00	3.88	4.00	4.00
	5	0.985	2.97	1.00	1.00	5.89	6.00	6.00
	10	0.989	2.68	1.00	1.00	9.74	10.00	10.00

DGP-3) Random walk

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=100, T=100	0	-	2.32	2.00	2.00	2.00	2.00	2.00
	1	0.355	2.40	2.00	2.00	2.54	2.00	2.00
	3	0.715	4.52	2.00	2.00	4.01	4.00	4.00
	5	0.826	5.37	2.00	2.00	4.57	5.00	5.00
	10	0.862	6.79	4.00	2.00	5.91	6.00	6.00
N=100, T=150	0	-	2.38	2.00	2.00	2.00	2.00	2.00
	1	0.573	2.48	2.00	2.00	3.72	4.00	4.00
	3	0.880	3.53	2.00	2.00	4.96	5.00	5.00
	5	0.942	4.44	2.00	2.00	5.96	6.00	6.00
	10	0.972	6.27	4.00	2.00	7.62	8.00	8.00

	b	coverage	number of factors (stable) mean	median	mode	number of factors (all) mean	median	mode
N=150, T=100	0	-	2.32	2.00	2.00	2.00	2.00	2.00
	1	0.383	2.36	2.00	2.00	2.91	3.00	3.00
	3	0.762	3.28	2.00	2.00	4.26	4.00	4.00
	5	0.870	3.73	2.00	2.00	5.11	5.00	5.00
	10	0.886	4.86	2.00	2.00	6.55	7.00	7.00
N=150, T=150	0	-	2.34	2.00	2.00	2.00	2.00	2.00
	1	0.622	2.29	2.00	2.00	3.93	4.00	4.00
	3	0.899	2.95	2.00	2.00	5.26	5.00	5.00
	5	0.950	3.31	2.00	2.00	6.30	6.00	6.00
	10	0.984	4.36	2.00	2.00	8.10	8.00	8.00

Table 3. Out-of-sample MSEs

$b = 5$

		N_0=0.7		N_0=0.5		N_0=0.3	
		NS	crude	NS	crude	NS	crude
DGP1	h=1	9.08	10.84	15.09	18.92	19.45	23.40
	h=12	11.93	14.73	17.41	22.08	22.03	27.22
	h=24	12.68	15.59	18.24	22.87	23.20	28.50
	h=48	13.42	16.41	18.77	23.52	24.62	30.05
DGP2	h=1	17.39	22.15	32.17	42.11	39.51	45.41
	h=12	22.65	29.62	39.19	51.38	54.69	64.40
	h=24	24.60	31.56	42.24	54.59	60.13	70.45
	h=48	27.13	34.15	45.42	58.07	66.23	76.78
DGP3	h=1	8.29	9.39	11.25	13.15	14.83	17.46
	h=12	10.06	11.43	13.73	16.56	16.73	19.85
	h=24	10.26	11.59	14.36	17.28	17.30	20.46
	h=48	10.75	12.08	14.82	17.70	18.56	21.79

$b = 10$

		N_0=0.7		N_0=0.5		N_0=0.3	
		NS	crude	NS	crude	NS	crude
DGP1	h=1	28.51	36.18	40.42	51.44	58.34	75.69
	h=12	32.16	41.89	49.03	63.06	73.11	93.45
	h=24	33.00	42.58	53.72	67.85	76.66	97.27
	h=48	35.09	44.84	56.20	70.37	80.96	101.80
DGP2	h=1	65.58	85.90	89.10	118.01	136.02	149.25
	h=12	74.37	102.65	128.71	173.73	193.88	222.48
	h=24	82.31	111.52	140.40	187.63	215.90	242.79
	h=48	92.31	122.55	156.89	204.05	238.06	265.85
DGP3	h=1	12.24	14.71	17.77	21.99	23.89	28.56
	h=12	13.89	16.84	21.11	26.77	29.32	36.28
	h=24	14.78	17.96	21.85	27.55	31.01	38.22
	h=48	15.71	18.94	23.31	29.08	33.42	40.78

Table 4. Number of common factors in U.S. macroeconomic time series

1) Full sample

# of factors	ICp1 NS	ICp1 crude	ICp2 NS	ICp2 crude	ICp3 NS	ICp3 crude
1	-0.0929	-0.1463	-0.0909	-0.1441	-0.0984	-0.1534
2	-0.0902	-0.1904	-0.0862	-0.1861	-0.1011	-0.2046
3	-0.0880	-0.2204	-0.0820	-0.2141	-0.1044	-0.2418
4	-0.0802	-0.2551	-0.0723	-0.2467	-0.1020	-0.2836
5	-0.0657	-0.2796	-0.0558	-0.2690	-0.0931	-0.3152
6	-0.0461	-0.2966	-0.0342	-0.2839	-0.0789	-0.3394
7	-0.0272	-0.3089	-0.0133	-0.2941	-0.0655	-0.3588
8	-0.0011	-0.3147	0.0148	-0.2978	-0.0448	-0.3718
9	0.0244	-0.3162	0.0422	-0.2972	-0.0248	-0.3804
10	0.0515	-0.3181	0.0713	-0.2970	-0.0032	-0.3895
11	0.0769	-0.3206	0.0987	-0.2974	0.0168	-0.3991
12	0.1023	-0.3194	0.1261	-0.2941	0.0368	-0.4051

2) Post-1984 subsample

# of factors	ICp1 NS	ICp1 crude	ICp2 NS	ICp2 crude	ICp3 NS	ICp3 crude
1	-0.3474	-0.2698	-0.3428	-0.2646	-0.3589	-0.2850
2	-0.4089	-0.3318	-0.3998	-0.3215	-0.4319	-0.3622
3	-0.4073	-0.3674	-0.3936	-0.3520	-0.4417	-0.4130
4	-0.3967	-0.3934	-0.3785	-0.3728	-0.4427	-0.4542
5	-0.3887	-0.4146	-0.3659	-0.3889	-0.4461	-0.4906
6	-0.3862	-0.4313	-0.3589	-0.4004	-0.4551	-0.5225
7	-0.3814	-0.4511	-0.3496	-0.4151	-0.4619	-0.5575
8	-0.3669	-0.4648	-0.3305	-0.4236	-0.4589	-0.5864
9	-0.3567	-0.4792	-0.3157	-0.4329	-0.4601	-0.6160
10	-0.3486	-0.4870	-0.3031	-0.4355	-0.4636	-0.6390
11	-0.3434	-0.4931	-0.2933	-0.4365	-0.4698	-0.6603
12	-0.3455	-0.5007	-0.2909	-0.4390	-0.4834	-0.6831

Note: The colored cell indicate the number which minimizes the information criterion.

Table 5. Number of observations with stable factors by category

		total	stable observations: full sample	stable observations: post-1984
	slow variables	67	35	32
	fast variables	65	16	15
a)	income / consumption / employment	39	20	18
b)	production / new orders / inventories	25	11	8
c)	housing	10	1	0
d)	money / credit	11	3	1
e)	stock price / interest rates / exchange rates	26	6	7
f)	consumer price / producer price	21	10	13
	total	132	51	47

Note: The categories of "slow variables" and "fast variables" follow Stock and Watson (2005).
The categories from a) to f) are defined by the author and provided in Table 8.

Table 6. Correlation coefficients among non-spurious and crude factors

1) Full sample

		crude factors							
		f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
NS	f_1	-0.913***	0.172***	0.093**	-0.032	0.235***	0.077*	-0.044	-0.049
factors	f_2	-0.174***	-0.597***	-0.321***	-0.392***	-0.115***	-0.216***	0.229***	0.310***

2) Post-1984 subsample

		crude factors							
		f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8
NS	f_1	0.461***	0.658***	0.577***	-0.001	0.006	-0.004	-0.004	-0.012
factors	f_2	-0.784***	0.432***	0.126**	-0.150**	0.219***	0.184***	0.160**	-0.109*

Note: "***", "**", and "*" show that the test rejects the null of zero correlation coefficient at the 1%, 5%, and 10% significant levels.

Table 7. Comparisons of the out-of-sample forecast MSEs

1) Full sample

h	MSE		Diebold	Giacomini-Rossi	
	NS	crude	-Mariano	NS	crude
12	1.167 (1)	0.977 (11)	-0.73	-0.26	0.91
24	0.941 (1)	1.144 (11)	5.42 ***	-0.40	1.28
48	1.497 (1)	1.583 (9)	4.78 ***	0.06	0.46

2) Post-1984 subsample

h	MSE		Diebold	Giacomini-Rossi	
	NS	crude	-Mariano	NS	crude
12	1.105 (2)	0.985 (12)	-1.31	0.70	1.77 *
24	0.908 (2)	1.287 (11)	5.84 ***	0.30	1.66 *
48	1.462 (2)	1.727 (9)	6.32 ***	0.26	0.83

Note: 1. The number of factors estimated in-sample and used for forecasting in the parenthesis.
2. Diebold-Mariano test and Giacomini and Rossi tests are two sided and "***", "**", "*" denotes 1%, 5%, and 10% significant.

Table 8. Stability of the factor loadings for individual observations

1) Full sample

category		stability	MSE(NS)	MSE(crude)	description
s	a	1	0.571	0.976	Personal income (AR, bil. chain 2000 $)
s	a	0	0.731	1.027	Personal income less transfer payments (AR, bil. chain 2000 $)
s	a	1	0.786	0.741	Real Consumption (AC) A0m224/gmdc
s	a	1	1.288	0.910	Manufacturing and trade sales (mil. Chain 1996 $)
s	a	1	1.237	1.147	Sales of retail stores (mil. Chain 2000 $)
s	b	0	1.546	1.535	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX
s	b	1	1.683	1.471	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
s	b	1	1.531	1.333	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS
s	b	1	1.235	1.168	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
s	b	0	0.793	0.713	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
s	b	1	1.703	1.727	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
s	b	1	1.753	1.407	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
s	b	0	1.058	1.230	INDUSTRIAL PRODUCTION INDEX - MATERIALS
s	b	0	0.790	0.884	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
s	b	1	1.325	1.303	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
s	b	0	1.489	1.288	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)
s	b	0	1.467	1.631	INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES
s	b	1	0.852	0.882	INDUSTRIAL PRODUCTION INDEX - FUELS
s	b	1	1.610	2.342	NAPM PRODUCTION INDEX (PERCENT)
s	b	0	1.209	0.990	Capacity Utilization (Mfg)
s	a	0	1.733	2.425	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
s	a	1	1.389	2.142	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
s	a	1	1.582	1.138	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
s	a	1	2.263	1.845	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)
s	a	1	1.078	1.124	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)
s	a	0	0.981	0.909	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
s	a	1	0.891	1.031	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
s	a	1	1.246	1.289	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
s	a	0	0.969	0.753	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
s	a	0	0.709	0.586	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
s	a	0	1.114	1.016	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
s	a	1	1.164	1.100	Average weekly initial claims, unemploy. insurance (thous.)
s	a	0	2.205	1.526	EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE
s	a	0	1.774	1.402	EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING
s	a	1	0.048	0.047	EMPLOYEES ON NONFARM PAYROLLS - MINING
s	a	1	0.420	0.117	EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION
s	a	0	2.217	2.468	EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING
s	a	0	1.715	2.065	EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS
s	a	0	2.506	2.221	EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS
s	a	1	2.429	1.837	EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING
s	a	1	2.357	1.552	EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES
s	a	0	2.718	2.463	EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE
s	a	0	1.694	1.066	EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE
s	a	0	1.765	1.517	EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES
s	a	1	1.492	1.620	EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT
s	a	1	0.986	0.414	Employee hours in nonag. establishments (AR, bil. hours)
s	a	0	0.648	0.572	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR
s	a	1	0.784	0.809	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR
s	a	0	0.521	0.499	Average weekly hours, mfg. (hours)
s	a	0	1.554	2.178	NAPM EMPLOYMENT INDEX (PERCENT)
f	c	0	0.163	0.148	HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA
f	c	1	0.684	1.124	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
f	c	0	0.182	0.194	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
f	c	0	0.305	0.382	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
f	c	0	0.234	0.214	HOUSING STARTS:WEST (THOUS.U.)S.A.
f	c	0	0.338	0.296	HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)
f	c	0	0.413	0.880	HOUSES AUTHORIZED BY BUILD. PERMITS:NORTHEAST(THOU.U.)S.A
f	c	0	0.200	0.098	HOUSES AUTHORIZED BY BUILD. PERMITS:MIDWEST(THOU.U.)S.A.
f	c	0	0.752	0.985	HOUSES AUTHORIZED BY BUILD. PERMITS:SOUTH(THOU.U.)S.A.
f	c	0	0.217	0.201	HOUSES AUTHORIZED BY BUILD. PERMITS:WEST(THOU.U.)S.A.
f	b	0	1.356	2.062	PURCHASING MANAGERS' INDEX (SA)
f	b	0	1.586	2.349	NAPM NEW ORDERS INDEX (PERCENT)
f	b	0	0.253	0.387	NAPM VENDOR DELIVERIES INDEX (PERCENT)
f	b	0	0.861	1.316	NAPM INVENTORIES INDEX (PERCENT)
f	b	1	1.311	1.189	Mfrs' new orders, consumer goods and materials (bil. chain 1982 $)
f	b	1	2.215	2.194	Mfrs' new orders, durable goods industries (bil. chain 2000 $)
f	b	1	0.719	0.811	Mfrs' new orders, nondefense capital goods (mil. chain 1982 $)
f	b	0	1.705	2.064	Mfrs' unfilled orders, durable goods indus. (bil. chain 2000 $)
f	b	0	1.319	1.408	Manufacturing and trade inventories (bil. chain 2000 $)
f	b	0	1.090	0.944	Ratio, mfg. and trade inventories to sales (based on chain 2000 $)

Note: "stability=0" denotes unstable factor loadings and "1" denotes stable factor loadings.

1) Full sample (continued)

f	d	0	5.408	5.427	MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)
f	d	0	3.987	3.991	MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,
f	d	1	4.026	4.039	MONEY STOCK: M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL$,SA)
f	d	0	1.390	1.348	MONEY SUPPLY - M2 IN 1996 DOLLARS (BCI)
f	d	0	7.032	7.096	MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)
f	d	0	9.552	9.559	DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)
f	d	0	6.914	6.914	DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)
f	d	1	0.722	0.783	COMMERCIAL & INDUSTRIAL LOANS OUSTANDING IN 1996 DOLLARS (BCI)
f	d	0	5.359	5.605	WKLY RP LG COM'L BANKS:NET CHANGE COM'L & INDUS LOANS(BIL$,SAAR)
f	d	1	0.336	0.381	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
f	d	0	0.736	1.110	Ratio, consumer installment credit to personal income (pct.)
f	e	1	1.532	2.609	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
f	e	1	1.577	2.750	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
f	e	0	0.238	0.949	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
f	e	1	3.311	3.788	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
f	e	0	0.296	0.566	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)
f	e	0	0.305	0.614	Cmmercial Paper Rate (AC)
f	e	0	0.351	0.822	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
f	e	0	0.387	0.816	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
f	e	0	0.375	0.758	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
f	e	0	0.664	0.917	INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)
f	e	0	0.760	0.938	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
f	e	0	0.707	0.802	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
f	e	0	0.889	0.896	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
f	e	0	0.179	0.377	cp90-fyff
f	e	0	0.117	0.137	fygm3-fyff
f	e	0	0.138	0.231	fygm6-fyff
f	e	0	0.360	0.460	fygt1-fyff
f	e	0	0.614	0.611	fygt5-fyff
f	e	0	0.849	0.850	fygt10-fyff
f	e	0	1.215	1.265	fyaaac-fyff
f	e	0	1.383	1.524	fybaac-fyff
f	e	0	1.306	1.388	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
f	e	0	0.969	1.137	FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)
f	e	1	0.928	0.920	FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)
f	e	1	0.783	0.845	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
f	e	1	2.494	2.356	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
f	f	0	2.171	2.250	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
f	f	1	2.234	2.289	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
f	f	0	2.184	2.233	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
f	f	0	5.118	5.124	PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
f	f	0	1.066	1.072	SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)
f	f	1	1.095	1.096	INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)
f	f	1	0.872	0.969	NAPM COMMODITY PRICES INDEX (PERCENT)
s	f	1	1.349	1.571	CPI-U: ALL ITEMS (82-84=100,SA)
s	f	1	1.114	1.208	CPI-U: APPAREL & UPKEEP (82-84=100,SA)
s	f	0	3.783	3.855	CPI-U: TRANSPORTATION (82-84=100,SA)
s	f	0	0.299	0.312	CPI-U: MEDICAL CARE (82-84=100,SA)
s	f	0	2.690	2.850	CPI-U: COMMODITIES (82-84=100,SA)
s	f	1	0.616	0.640	CPI-U: DURABLES (82-84=100,SA)
s	f	1	0.501	0.621	CPI-U: SERVICES (82-84=100,SA)
s	f	0	1.899	2.145	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
s	f	1	1.977	2.169	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
s	f	1	1.342	1.540	CPI-U: ALL ITEMS LESS MEDICAL CARE (82-84=100,SA)
s	f	0	2.200	2.320	PCE,IMPL PR DEFL:PCE (1987=100)
s	f	1	0.764	0.803	PCE,IMPL PR DEFL:PCE; DURABLES (1987=100)
s	f	0	3.006	3.136	PCE,IMPL PR DEFL:PCE; NONDURABLES (1996=100)
s	f	0	4.686	4.687	PCE,IMPL PR DEFL:PCE; SERVICES (1987=100)
s	a	1	0.236	0.647	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
s	a	1	0.223	0.473	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
s	a	0	0.277	0.625	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
f	a	0	1.724	2.068	U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)

Note: "stability=0" denotes unstable factor loadings and "1" denotes stable factor loadings.

2) Post-1984 subsample

category		stability	MSE(NS)	MSE(crude)	
s	a	1	0.330	4.201	Personal income (AR, bil. chain 2000 $)
s	a	0	0.432	5.263	Personal income less transfer payments (AR, bil. chain 2000 $)
s	a	0	0.682	1.045	Real Consumption (AC) A0m224/gmdc
s	a	1	0.947	0.817	Manufacturing and trade sales (mil. Chain 1996 $)
s	a	0	1.160	1.202	Sales of retail stores (mil. Chain 2000 $)
s	b	1	1.269	1.685	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX
s	b	1	1.559	2.163	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
s	b	1	1.496	2.116	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS
s	b	1	1.264	1.594	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
s	b	0	0.806	0.823	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
s	b	0	1.739	2.183	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
s	b	0	1.561	2.206	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
s	b	1	0.772	0.936	INDUSTRIAL PRODUCTION INDEX - MATERIALS
s	b	0	0.600	0.834	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
s	b	1	0.937	0.590	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
s	b	1	1.161	1.314	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)
s	b	0	1.513	2.090	INDUSTRIAL PRODUCTION INDEX - RESIDENTIAL UTILITIES
s	b	0	0.928	0.852	INDUSTRIAL PRODUCTION INDEX - FUELS
s	b	0	1.324	1.846	NAPM PRODUCTION INDEX (PERCENT)
s	b	0	0.932	0.871	Capacity Utilization (Mfg)
s	a	1	1.901	3.178	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
s	a	0	1.306	2.219	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
s	a	1	1.312	1.306	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
s	a	0	1.982	1.993	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)
s	a	1	0.960	1.143	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)
s	a	0	1.092	0.907	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
s	a	1	0.847	0.937	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
s	a	1	1.215	1.246	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
s	a	1	1.169	0.926	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
s	a	1	0.950	0.627	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
s	a	0	1.099	1.168	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
s	a	1	0.976	1.053	Average weekly initial claims, unemploy. insurance (thous.)
s	a	1	1.422	1.761	EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE
s	a	0	1.093	1.216	EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING
s	a	0	0.047	0.047	EMPLOYEES ON NONFARM PAYROLLS - MINING
s	a	0	0.236	0.215	EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION
s	a	0	1.450	1.641	EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING
s	a	0	1.115	1.379	EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS
s	a	0	1.703	1.475	EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS
s	a	0	1.976	2.218	EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING
s	a	1	1.973	1.955	EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES
s	a	0	2.572	3.085	EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE
s	a	0	1.430	1.163	EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE
s	a	0	1.466	1.371	EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES
s	a	0	1.469	1.409	EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT
s	a	1	0.725	0.738	Employee hours in nonag. establishments (AR, bil. hours)
s	a	0	0.632	2.055	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR
s	a	1	0.779	0.824	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFAR
s	a	0	0.502	1.814	Average weekly hours, mfg. (hours)
s	a	0	1.225	1.612	NAPM EMPLOYMENT INDEX (PERCENT)
f	c	0	0.224	0.176	HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA
f	c	0	0.416	0.120	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
f	c	0	0.244	0.197	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
f	c	0	0.357	0.198	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
f	c	0	0.297	0.202	HOUSING STARTS:WEST (THOUS.U.)S.A.
f	c	0	0.437	0.195	HOUSING AUTHORIZED: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)
f	c	0	0.255	0.083	HOUSES AUTHORIZED BY BUILD. PERMITS:NORTHEAST(THOU.U.)S.A
f	c	0	0.357	0.151	HOUSES AUTHORIZED BY BUILD. PERMITS:MIDWEST(THOU.U.)S.A.
f	c	0	0.709	0.216	HOUSES AUTHORIZED BY BUILD. PERMITS:SOUTH(THOU.U.)S.A.
f	c	0	0.307	0.211	HOUSES AUTHORIZED BY BUILD. PERMITS:WEST(THOU.U.)S.A.
f	b	0	1.078	1.536	PURCHASING MANAGERS' INDEX (SA)
f	b	0	1.345	1.933	NAPM NEW ORDERS INDEX (PERCENT)
f	b	0	0.176	0.260	NAPM VENDOR DELIVERIES INDEX (PERCENT)
f	b	0	0.738	0.714	NAPM INVENTORIES INDEX (PERCENT)
f	b	0	1.063	0.912	Mfrs' new orders, consumer goods and materials (bil. chain 1982 $)
f	b	0	2.113	2.210	Mfrs' new orders, durable goods industries (bil. chain 2000 $)
f	b	1	0.761	1.225	Mfrs' new orders, nondefense capital goods (mil. chain 1982 $)
f	b	0	1.538	1.606	Mfrs' unfilled orders, durable goods indus. (bil. chain 2000 $)
f	b	0	1.096	1.407	Manufacturing and trade inventories (bil. chain 2000 $)
f	b	0	0.962	0.982	Ratio, mfg. and trade inventories to sales (based on chain 2000 $)

Note: "stability=0" denotes unstable factor loadings and "1" denotes stable factor loadings.

2) Post-1984 subsample (continued)

f	d	0	5.405	5.632	MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)
f	d	0	3.986	4.058	MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,
f	d	0	4.030	4.089	MONEY STOCK: M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL$,SA)
f	d	0	1.582	1.445	MONEY SUPPLY - M2 IN 1996 DOLLARS (BCI)
f	d	0	7.065	7.280	MONETARY BASE, ADJ FOR RESERVE REQUIREMENT CHANGES(MIL$,SA)
f	d	0	9.553	9.575	DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL$,SA)
f	d	0	6.916	6.926	DEPOSITORY INST RESERVES:NONBORROWED,ADJ RES REQ CHGS(MIL$,SA)
f	d	0	0.757	2.916	COMMERCIAL & INDUSTRIAL LOANS OUSTANDING IN 1996 DOLLARS (BCI)
f	d	0	6.021	11.657	WKLY RP LG COM'L BANKS:NET CHANGE COM'L & INDUS LOANS(BIL$,SAAR)
f	d	1	0.325	0.325	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
f	d	0	0.717	2.779	Ratio, consumer installment credit to personal income (pct.)
f	e	1	1.428	2.063	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
f	e	0	1.454	2.254	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
f	e	0	0.245	0.358	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
f	e	1	3.309	3.447	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
f	e	1	0.218	0.307	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)
f	e	0	0.235	0.396	Cmmercial Paper Rate (AC)
f	e	1	0.269	0.512	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
f	e	1	0.326	0.608	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
f	e	1	0.351	0.683	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
f	e	0	0.750	1.134	INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)
f	e	0	0.882	1.130	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
f	e	0	0.822	0.931	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
f	e	0	1.034	1.135	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
f	e	0	0.163	0.301	cp90-fyff
f	e	0	0.223	0.111	fygm3-fyff
f	e	0	0.151	0.151	fygm6-fyff
f	e	0	0.314	0.490	fygt1-fyff
f	e	0	0.629	0.600	fygt5-fyff
f	e	0	0.889	0.788	fygt10-fyff
f	e	0	1.280	0.924	fyaaac-fyff
f	e	0	1.393	1.150	fybaac-fyff
f	e	0	1.429	1.423	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
f	e	0	1.054	1.173	FOREIGN EXCHANGE RATE: SWITZERLAND (SWISS FRANC PER U.S.$)
f	e	1	0.926	0.916	FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)
f	e	0	0.834	0.836	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
f	e	0	2.601	2.529	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
f	f	1	2.261	2.330	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
f	f	1	2.326	2.403	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
f	f	0	2.325	2.269	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
f	f	0	5.149	5.125	PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
f	f	1	1.072	1.118	SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)
f	f	1	1.102	1.158	INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)
f	f	1	0.887	0.638	NAPM COMMODITY PRICES INDEX (PERCENT)
s	f	0	1.796	1.725	CPI-U: ALL ITEMS (82-84=100,SA)
s	f	1	1.682	1.629	CPI-U: APPAREL & UPKEEP (82-84=100,SA)
s	f	1	4.277	4.196	CPI-U: TRANSPORTATION (82-84=100,SA)
s	f	1	0.297	0.300	CPI-U: MEDICAL CARE (82-84=100,SA)
s	f	1	3.266	3.321	CPI-U: COMMODITIES (82-84=100,SA)
s	f	1	0.627	0.614	CPI-U: DURABLES (82-84=100,SA)
s	f	1	0.500	0.490	CPI-U: SERVICES (82-84=100,SA)
s	f	1	2.389	2.200	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
s	f	1	2.451	2.390	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
s	f	0	1.709	1.728	CPI-U: ALL ITEMS LESS MEDICAL CARE (82-84=100,SA)
s	f	0	2.625	2.345	PCE,IMPL PR DEFL:PCE (1987=100)
s	f	0	0.821	0.768	PCE,IMPL PR DEFL:PCE; DURABLES (1987=100)
s	f	0	3.660	3.733	PCE,IMPL PR DEFL:PCE; NONDURABLES (1996=100)
s	f	0	4.693	4.750	PCE,IMPL PR DEFL:PCE; SERVICES (1987=100)
s	a	1	0.242	0.452	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
s	a	1	0.200	0.436	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
s	a	1	0.280	0.423	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NO
f	a	1	1.728	2.831	U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)

Note: “stability=0” denotes unstable factor loadings and “1” denotes stable factor loadings.