Fitting Observed Inflation Expectations

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Abstract

We compare the fit of two variants of a standard Christiano et al. (2005)/ Smets and Wouters (2003) -type DSGE model, one where agents have perfect information about the value of the policymaker's inflation target, and one where they need to infer this value from changes in interest rates as in Erceg and Levin (2003). We find that a standard set of macro variables is unable to discriminate among the two models. Observed inflation expectations provide strong evidence as to which model fits the data best: the perfect information.

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1 Introduction

This paper uses inflation expectations as an observable in the estimation of a DSGE model, along with a standard set of macro variables. Observed inflation expectations are rarely included among the observables in estimating DSGE models, but arguably contain information that is valuable in discriminating across models. We compare the fit of two variants of a standard DSGE model with with several nominal and real rigidities along the lines of Christiano et al. (2005), Smets and Wouters (2003), and Smets and Wouters (2007), where the different lies in the agents' information set. In the first model (Perfect Information) agents have perfect information about the Central Bank's inflation target, while in the second model (Imperfect Information) agents need to infer the target from the behavior of interest rates, as in (Erceg and Levin (2003)). We find that a standard set of macro variables over a standard estimation period (the post-Volcker disinflation period: 1982Q2-2008Q2) is unable to discriminate among the two models. Observed inflation expectations instead provide strong evidence as to which model fits the data best. This is perhaps the least plausible of the two models: the Perfect Information one. We provide evidence that the relative failure of the Imperfect Information model to fit observed inflation expectations is due to the fact that this model imposes much more stringent cross-equation restriction on the law of motion of the perceived inflation target than the Perfect Information model.

There are several reasons for including measured inflation expectations among the set of observables in the estimation of DSGE models. First, as our study shows, inflation expectations help discriminate across models, especially when these models differ in the way agents form expectations. Yet observed expectations are rarely formally used in previous literature, even when comparing rational expectations with learning models (e.g., Milani (2007); a recent paper by Ormeno (2009) is an exception). In fact, we know very little on the extent to which DSGE models can accurately describe the behavior of observed inflation expectations.[1] Second, inflation

[1]Recent literature has used survey measures of inflation expectations in the limited information estimation of models of inflation dynamics. Roberts (1997) estimates a reduced-form New Keyne-

expectations are allegedly important in determining the term structure of interest rates. While this paper makes no attempt to explain the term structure directly since we use a linear model, the model comparison exercise done here can be helpful indirectly in investigating how one should formulate expectation formation.[2] Models that have a hard time generating observed inflation expectations may not be too helpful in understanding the term structure of interest rates. A third reasons to add observed expectations (for inflation as well as other variables) to the econometrician's information set is that agents in the real economy have a richer information set than the econometrician using a standard set of macro variables. Including measured expectations among the observables is a way to exploit such information set.[3] This information can be exploited for both forecasting and estimating latent variables, such as shocks. We show for instance that the estimated process for the inflation target changes whether we include or not inflation expectations among the observables.

There are several issues with using measured expectations as observables in DSGE models, which we discuss in section 4: data revisions, timing, choice of the expectation measures. This paper shows that the results are robust to different choices of measurement and timing assumptions, but does not really address many of these difficult issues. By pointing out the information content from measured expectations, we hope we have shown that it is worthwhile for future research to

sian Phillips curve using survey expectations from the Michigan and Livingston surveys and finds that they are important in explaining inflation. In particular, he finds that measures of inflation expectations have more explanatory power than past inflation. Adam and Padula (2002) estimate a structural New Keynesian Phillips curve with SPF inflation forecasts and obtain plausible structural estimates of nominal rigidities, independently of the measure of marginal costs used (see also Nunes (2009)). None of this papers however studies the extent to which New Keynesian models can explain the dynamics of inflation expectations.

[2]There are attempts to use DSGE models to explain the terms structure, e.g. Rudebusch and Swanson (2008).

[3]Following the FAVAR methodology (Bernanke et al. (2005)) there are some attempts to combine factor and DSGE models with the goal of incorporating as much of the available data as possible (Boivin and Giannoni (2006), Giannone et al. (2008)). We take a different route and incorporate this information by adding agent's expectations to the list of observables.

address these issues more thoroughly than we have. Also, there are several other mechanism of expectation formations, notably learning, that we do not consider in this paper. It is interesting to ask whether learning models provide a better description of observed inflation expectations than rational expectation models (Ormeno (2009) contains some preliminary results on this question).

Our results, while negative for the Imperfect Information model, are not necessarily in contrast with Erceg and Levin (2003)'s. Erceg and Levin (2003) focus on the Great Disinflation ($81-85$), while we are interesting in assessing which model best describes the evolution of inflation expectation in the post deflation period – a period where allegedly the policy regime has not changed. Section 5.6 discusses model comparison using data that include the the Great Disinflation period. We find that this period provides some evidence in support of the Imperfect Information model, in agreement with Erceg and Levin (2003).

In addition to Erceg and Levin (2003), several papers introduce imperfect information about the central bank's inflation target in a monetary DSGE model. Andolfatto et al. (2008) show that imperfect information about the central bank's inflation target can generate small sample rejection of rational expectations. They conduct a Monte Carlo experiment using a calibrated new Keynesian model with infrequent shifts in the monetary authority's inflation target and find rejections of rational expectations in samples of the same size as available measures of inflation expectations.[4] Keen (2009) shows that a calibrated new Keynesian DSGE model with imperfect information about the inflation target implies a response to a monetary expansion that is in accord with VAR studies (see also Melecky et al. (2008)). These papers do not discuss the model's ability to explain observed measures of inflation expectation. Perhaps closer to our paper, Schorfheide (2005) estimates a New Keynesian DSGE model with imperfect information using US data from 1960 to 1997. The paper finds that while the model with full information provides a bet-

[4]Several papers have documented that inflation expectations as measured by surveys like Michigan, Livingston and SPF fail to be consistent with rational expectations in terms of unbiasness and serially uncorrelated forecast errors – e.g., Lloyd (1999) and Roberts (1997). Rich (1989) provides evidence in the opposite direction.

ter fit over the whole sample, the model under imperfect information outperforms for the period of the Volker disinflation. While measures of inflation expectations are not included as observables in his dataset, he compares the time series of inflation expecations generated by the two models and finds that the two models imply inflation expectations – one-year and ten-years average – that are roughly similar over the sample. Finally, Aruoba and Schorfheide (2009) study the distortionary properties of inflation and the optimal long-run inflation rate using an estimated DSGE model with a time-varying inflation target. Their dataset includes a measure on long-run inflation expectations obtained by combining one-year and ten-years SPF inflation forecasts with a one-sided bandpass filter on inflation that removes short and medium term cycles. Their study does not focus on the model's ability to explain inflation expectations.

The next section briefly discusses the econometric framework for evaluating how a model estimated to fit a baseline set of time series – here, the standard macro variables – fares in fitting an additional time series – here, inflation expectations. This is a straightforward application of Bayesian updating, which is routinely done in the DSGE estimation literature in the time series dimension, to the cross-sectional dimension. Section 3 describes the model, with particular emphasis on the difference between perfect and imperfect information. Section 5 discusses our findings.

2 Predictive Checks in the Cross-Section

Let $y^i_{1,T} = \{y^i_t\}^T_{t=1}$ define time series i. A natural question in the DSGE model estimation literature is the following: How does a model that is estimated to fit time series $y^1_{1,T}$ through $y^J_{1,T}$ fare in fitting time series $y^{J+1}_{1,T}$ through $y^{J+K}_{1,T}$? In this paper, for instance, we ask how the Christiano et al. (2005)/Smets and Wouters (2003) model, which allegedly fits standard macro time series well, fare in describing observed inflation expectations. The same question can be posed for asset prices, the yield curve, and several other time series.

Let $Y^0_{1,T}$ and $Y^1_{1,T}$ denote $\{y^1_{1,T}, \ldots, y^J_{1,T}\}$ and $\{y^{J+1}_{1,T}, \ldots, y^{J+K}_{1,T}\}$, respectively. One can of course compute the marginal likelihood for series $y^1_{1,T}$ though $y^{K+J}_{1,T}$:

$$p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i)$$

where $\mathcal{M}_i$ is the model under consideration While the quantity $p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i)$ is certainly of interest, it may not necessarily address the researcher's question. This is for two reasons. First, by construction, the marginal likelihood depends on the prior chosen:

$$p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i) = \int p(Y^0_{1,T}, Y^1_{1,T}|\theta, \mathcal{M}_i)p(\theta|\mathcal{M}_i)d\theta, \tag{1}$$

where $p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i)$ denotes the likelihood function for model $\mathcal{M}_i$, θ the vector of DSGE model parameters, and $p(\theta|\mathcal{M}_i)$ the prior chosen for θ. Prior elicitation for some of the DSGE model parameters can be challenging, and the choice of prior – not surprisingly given the above definition – affects the marginal likelihood computation and therefore the outcome of model comparisons (see DelNegro and Schorfheide (2008)). The researcher who is interested in knowing how well the model fits the time series $y^{J+1,T}$ through $y^{J+K,T}$ may want to use as a prior the posterior obtained from estimating the model on time series $y^{1,T}$ through $y^{J,T}$. This posterior – $p(\theta|Y^0_{1,T}, \mathcal{M}_i)$ – will be far less dependent on the initial prior $p(\theta|\mathcal{M}_i)$ chosen. In our case, the exercise would be to use the posterior obtained from fitting standard macro time series in order to evaluate the model's ability to fit expectations. The object of interest would then be:

$$p(Y^1_{1,T}|Y^0_{1,T}, \mathcal{M}_i) = \int p(Y^1_{1,T}|\theta, Y^0_{1,T}, \mathcal{M}_i)p(\theta|Y^0_{1,T}, \mathcal{M}_i)d\theta. \tag{2}$$

In expression (2) the set of time series $Y^0_{1,T}$ represents the training sample in Bayesian parlance, and $p(\theta|Y^0_{1,T}, \mathcal{M}_i)$ is the training sample prior, whence the title of the section. While training sample priors are often used in Bayesian macroeconometrics along the time series dimension (e.g., using $Y^0_{-P,0}$ as a training sample and then estimating the model over $Y^0_{1,T}$), here we apply the approach to the cross sectional dimension. The second reason why we may be interested in $p(Y^1_{1,T}|Y^0_{1,T}, \mathcal{M}_i)$, rather than in $p(Y^T|\mathcal{M}_i)$, is that $p(Y^T|\mathcal{M}_i)$ provides information on how well model $\mathcal{M}_i$

fits both $Y^0_{1,T}$ and $Y^1_{1,T}$, while the researcher may want to disentangle the goodness of fit of one set of time series versus the other. The quantity $p(Y^1_{1,T}|Y^0_{1,T}, \mathcal{M}_i)$ tells us how well model $\mathcal{M}_i$ fits $Y^1_{1,T}$ only, conditional on the parameter distribution delivering the best possible fit for $Y^0_{1,T}$. This quantity easily obtains as the ratio of two objects we know how to compute, $p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i)$ and $p(Y^0_{1,T}|\mathcal{M}_i)$, since:

$$p(Y^1_{1,T}|Y^0_{1,T}, \mathcal{M}_i) = \frac{p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i)}{p(Y^0_{1,T}|\mathcal{M}_i)}. \tag{3}$$

3 Model

The economy is described by a medium-scale New Keynesian model with price and wage rigidities, capital accumulation, investment adjustment costs, variable capital utilization, and habit formation. The model is based on work of Smets and Wouters (2003), Smets and Wouters (2007), and Christiano et al. (2005). The specific version is taken from DelNegro et al. (2007), except for the monetary policy rule, which we subsequently describe in detail. For brevity we only present the log-linearized equilibrium conditions and refer the reader to the above referenced papers for the derivation of these conditions from assumptions on preferences and technologies.

Monetary Policy: Perfect versus Imperfect Information The central bank follows a standard feedback rule:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)\left(\psi_1 \pi_t - \psi_1 \pi^*_t + \psi_2 \dot{y}_t\right) + \sigma_r \epsilon_{R,t}, \tag{4}$$

where $\hat{y}_t$ captures some measure of economic activity in log-deviations from its steady state (in the baseline specification $\dot{y}_t$ coincides with the growth rate of output $\widehat{y}_t + \widehat{z}_t - \widehat{y}_{t-1}$), and $\epsilon_{R,t}$ is an i.i.d. shock. The inflation target π^*_t, defined in log-deviations from its non-stochastic steady state π^*, evolves according to

$$\pi^*_t = \rho_{\pi^*} \pi^*_{t-1} + \sigma_P \epsilon_{P,t}, \tag{5}$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{P,t}$ is an i.i.d. shock. We follow Erceg and Levin (2003) and model π^*_t as following a stationary process (although our prior for ρ_{π^*} will make sure the this process is highly persistent). This choice is also motivated by the fact that in our sample long-term (10 year ahead) inflation expectations have moved very little, especially in the last ten years. We view this as evidence against the random walk assumption for our post-84 sample.

Under perfect information, agents observe π_t^*. Under imperfect information they need to infer the inflation target from the observed interest rate behavior (see Erceg and Levin (2003)). Call $\tilde{\pi}_t$ the residual in the feedback rule, defined as:

$$\tilde{\pi}_t = (\rho_r R_{t-1} + (1-\rho_r)(\psi_1 \pi_t + \psi_2 \widehat{y}_t) - R_t)/(1-\rho_r)\psi_1. \tag{6}$$

Agents solve a signal extraction problem using

$$\tilde{\pi}_t = \pi_t^* + \sigma_T \epsilon_{R,t} \tag{7}$$

as the measurement equation (where $\sigma_T = \frac{\sigma_r}{(1-\rho_R)\psi_1}$) and (5) as the transition equation. The law of motion of $\pi^*_{t+1|t}$ is obtained using the steady state Kalman filter

$$\pi^*_{t+1|t} = \rho_{\pi^*} \pi^*_{t|t-1} + \rho_{\pi^*} K \left(\tilde{\pi}_t - \pi^*_{t|t-1} \right), \tag{8}$$

where $K = \frac{V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})}{1+V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})}$ is the steady state Kalman gain coefficient and $\sigma_T^2 V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})$ is the steady state uncertainty regarding the inflation target. V solves:

$$V = \rho_{\pi_*}^2 \left[V - V\,(V+1)^{-1}\,V \right] + (\frac{\sigma_P}{\sigma_T})^2.$$

We also consider the alternative law of motion for inflation target π_t^* proposed in Gurkaynak et al. (2005):

$$\pi_t^* = \rho_{\pi^*} \pi^*_{t-1} + \chi \pi_{t-1} + \sigma_P \epsilon_{P,t}. \tag{9}$$

As above agents know the policy rule and the evolution of the unobserved inflation target. The forecast of the unobserved inflation target $\pi^*_{t+1|t}$ (10) now becomes: steady state Kalman filter

$$\pi^*_{t+1|t} = \rho_{\pi_*} \pi^*_{t|t-1} + \rho_{\pi_*} K \left(\tilde{\pi}_t - \pi^*_{t|t-1} \right) + \chi \pi_t \tag{10}$$

where K is defined as before.

Firms. The economy is populated by a continuum of firms that combine capital and labor to produce differentiated intermediate goods. These firms have access to the same Cobb-Douglas production function with capital elasticity α and total factor productivity Z_t. Total factor productivity is assumed to be non-stationary, and its growth rate $z_t = \ln(Z_t/Z_{t-1})$ follows the autoregressive process:

$$z_t = (1-\rho_z)\gamma + \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}. \tag{11}$$

Output, consumption, investment, capital, and the real wage can be detrended by Z_t. In terms of the detrended variables the model has a well-defined steady state. All variables that appear subsequently are expressed as log-deviations from this steady state.

The intermediate goods producers hire labor and rent capital in competitive markets and face identical real wages, w_t, and rental rates for capital, r_t^k. Cost minimization implies that all firms produce with the same capital-labor ratio

$$k_t - L_t = w_t - r_t^k \tag{12}$$

and have marginal costs

$$mc_t = (1-\alpha)w_t + \alpha r_t^k. \tag{13}$$

The intermediate goods producers sell their output to perfectly competitive final good producers, which aggregate the inputs according to a CES function. Profit maximization of the final good producers implies the following demand curve

$$\widehat{y}_t(j) - \widehat{y}_t = -\left(1 + \frac{1}{\lambda_f e^{\widetilde{\lambda}_{f,t}}}\right)(p_t(j) - p_t). \tag{14}$$

Here $\widehat{y}_t(j) - \widehat{y}_t$ and $p_t(j) - p_t$ are quantity and price for good j relative to quantity and price of the final good. The price p_t of the final good is determined from a zero-profit condition for the final good producers. We assume that the price elasticity of the intermediate goods is time-varying. Since this price elasticity affects the mark-up that intermediate goods producers can charge over marginal costs, we refer to $\widetilde{\lambda}_{f,t}$ as mark-up shock. Following Calvo (1983), we assume that in every period a fraction of the intermediate goods producers ζ_p is unable to re-optimize their prices.

A fraction ι_p of these firms adjust their prices mechanically according to lagged inflation, while the remaining fraction $1-\iota_p$ adjusts to steady state inflation π^*. All other firms choose prices to maximize the expected discounted sum of future profits, which leads to the Phillips curve:

$$\pi_t = \frac{\beta}{1+\iota_p\beta}\mathbb{E}_t[\pi_{t+1}] + \frac{\iota_p}{1+\iota_p\beta}\pi_{t-1} + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p(1+\iota_p\beta)}mc_t + \frac{1}{\zeta_p}\lambda_{f,t}, \tag{15}$$

where π_t is inflation and β is the discount rate.[5] Our assumption on the behavior of firms that are unable to re-optimize their prices implies the absence of price dispersion in the steady state. As a consequence, we obtain a log-linearized aggregate production function of the form

$$\widehat{y}_t = (1-\alpha)L_t + \alpha k_t. \tag{16}$$

Equations (13), (12), and (16) imply that the labor share lsh_t equals marginal costs in terms of log-deviations: $lsh_t = mc_t$.

Households. There is a continuum of households with identical preferences, which are separable in consumption, leisure, and real money balances. Households' preferences display (internal) habit formation in consumption, that is, period t utility is a function of $\ln(C_t - hC_{t-1})$. Households supply monopolistically differentiated labor services. These services are aggregated according to a CES function that leads to a demand elasticity $1+1/\lambda_w$ (see Equation (14)). The composite labor services are then supplied to the intermediate goods producers at real wage w_t. To introduce nominal wage rigidity, we assume that in each period a fraction ζ_w of households is unable to re-optimize their wages. A fraction ι_w of these households adjust their $t-1$ nominal wage by $\pi_{t-1}e^\gamma$, where γ represents the average growth rate of the economy, while the remaining fraction $1-\iota_p$ adjusts to steady state wage growth $\pi^* e^\gamma$. All other households re-optimize their wages. First-order conditions imply that

$$\begin{aligned}\tilde{w}_t &= \zeta_w\beta\mathbb{E}_t\Big[\tilde{w}_{t+1} + \Delta w_{t+1} + \pi_{t+1} + z_{t+1} - \iota_w\pi_{t-1}\Big] \\ &\quad + \frac{1-\zeta_w\beta}{1+\nu_l(1+\lambda_w)/\lambda_w}\left(\nu_l L_t - w_t - \xi_t + \frac{1}{1-\zeta_w\beta}\phi_t\right), \end{aligned}\tag{17}$$

[5] We used the following re-parameterization: $\lambda_{f,t} = [(1-\zeta_p\beta)(1-\zeta_p)\lambda_f/(1+\lambda_f)(1+\iota_p\beta)]\widetilde{\lambda}_{f,t}$.

where $\tilde{w}_t$ is the optimal real wage relative to the real wage for aggregate labor services, w_t, and ν_l would be the inverse Frisch labor supply elasticity in a model without wage rigidity ($\zeta_w = 0$) and differentiated labor. Moreover, ξ_t denotes the marginal marginal utility of consumption defined below and ϕ_t is a preference shock that affects the intratemporal substitution between consumption and leisure. The real wage paid by intermediate goods producers evolves according to

$$w_t = w_{t-1} - \pi_t - z_t + \iota_w \pi_{t-1} + \frac{1-\zeta_w}{\zeta_w}\tilde{w}_t. \tag{18}$$

Households are able to insure the idiosyncratic wage adjustment shocks with state contingent claims. As a consequence they all share the same marginal utility of consumption ξ_t, which is given by the expression:

$$(e^\gamma - h\beta)(e^\gamma - h)\xi_t = -(e^{2\gamma} + \beta h^2)c_t + \beta h e^\gamma \mathbb{E}_t[c_{t+1} + z_{t+1}] + h e^\gamma (c_{t-1} - z_t), \tag{19}$$

where c_t is consumption. In addition to state-contingent claims households accumulate three types of assets: one-period nominal bonds that yield the return R_t, capital $\bar{k}_t$, and real money balances.[6]

The first order condition with respect to bond holdings delivers the standard Euler equation:

$$\xi_t = \mathbb{E}_t[\xi_{t+1}] + R_t - \mathbb{E}_t[\pi_{t+1}] - \mathbb{E}_t[z_{t+1}]. \tag{20}$$

Capital accumulates according to the following law of motion:

$$\bar{k}_t = (2 - e^\gamma - \delta)\left[\bar{k}_{t-1} - z_t\right] + (e^\gamma + \delta - 1)i_t, \tag{21}$$

where i_t is investment, δ is the depreciation rate of capital. Investment in our model is subject to adjustment costs, and S'' denotes the second derivative of the investment adjustment cost function at steady state. Optimal investment satisfies the following first-order condition:

$$i_t = \frac{1}{1+\beta}\left[i_{t-1} - z_t\right] + \frac{\beta}{1+\beta}\mathbb{E}_t[i_{t+1} + z_{t+1}] + \frac{1}{(1+\beta)S'' e^{2\gamma}}(\xi_t^k - \xi_t), \tag{22}$$

[6] Since preferences for real money balances are assumed to be additively separable and monetary policy is conducted through a nominal interest rate feedback rule, money is block exogenous and we will not use the households' money demand equation in our empirical analysis.

where ξ_t^k is the value of installed capital and evolves according to:

$$\xi_t^k - \xi_t = \beta e^{-\gamma}(1-\delta)I\!\!E_t\big[\xi_{t+1}^k - \xi_{t+1}\big] + I\!\!E_t\big[(1-(1-\delta)\beta e^{-\gamma})r_{t+1}^k - (R_t - \pi_{t+1})\big]. \quad (23)$$

Capital utilization u_t in our model is variable and r_t^k in the previous equation represents the rental rate of effective capital $k_t = u_t + \bar{k}_{t-1}$. The optimal degree of utilization is determined by

$$u_t = \frac{r_*^k}{a''} r_t^k. \quad (24)$$

Here a'' is the derivative of the per-unit-of-capital cost function $a(u_t)$ evaluated at the steady state utilization rate. The aggregate resource constraint is given by:

$$\widehat{y}_t = (1+g_*)\left[\frac{c_*}{y_*}c_t + \frac{i_*}{y_*}\left(i_t + \frac{r_*^k}{e^{\gamma}-1+\delta}u_t\right)\right] + g_t. \quad (25)$$

Here c_*/y_* and i_*/y_* are the steady state consumption-output and investment-output ratios, respectively, and $g_*/(1+g_*)$ corresponds to the government share of aggregate output. The process g_t can be interpreted as exogenous government spending shock. It is assumed that fiscal policy is passive in the sense that the government uses lump-sum taxes to satisfy its period budget constraint. Finally, all stochastic processes described above are assumed to be AR(1) processes with normally distributed errors.

State-Space Representation of the DSGE Model. We use the method in Sims (2002) to solve the log-linear approximation of the DSGE model. We collect all the DSGE model parameters in the vector θ, stack the structural shocks in the vector ϵ_t, and derive a state-space representation for our vector of observables y_t, which is composed of the transition equation:

$$s_t = \mathcal{T}(\theta)s_{t-1} + \mathcal{R}(\theta)\epsilon_t, \quad (26)$$

which summarizes the evolution of the states s_t, and of the measurement equations:

$$y_t = \mathcal{Z}(\theta)s_t + \mathcal{D}(\theta), \quad (27)$$

which map the states onto the vector of observables y_t, where $\mathcal{D}(\theta)$ represents the vector of steady state values for these observables. Specifically, for our standard set of macro time series the set of measurement equations is:

$$
\begin{array}{lcl}
\text{Real output growth (\%, annualized)} & & \\
400(\ln RGDP_t - \ln RGDP_{t-1}) & = & 400(\widehat{y}_t - \widehat{y}_{t-1} + z_t) \\
\text{Hours (\%)} & & \\
100 \ln L_t & = & 100(L_t + \ln L^{adj}) \\
\text{Labor Share (\%)} & & \\
100 \ln LSH_t & = & 100(L_t + w_t - \widehat{y}_t + \ln lsh_*) \\
\text{Inflation (\%,annualized)} & & \\
400(\ln P_t - \ln P_{t-1}) & = & 400(\pi_t + \ln \pi_*) \\
\text{Interest Rates (\%,annualized)} & & \\
400 \ln R_t & = & 400(R_t + \ln R_*),
\end{array}
\tag{28}
$$

where $RGDP_t$, L_t, LSH_t, P_t, and R_t represent real per capita GDP, total per capita hours, the labor share, the price level, and the Federal Funds Rate, respectively. The quantities lsh_*, π_*, and R_* are the steady states of the labor share, the inflation rate, and the nominal interest rate, respectively, and the parameter L^{adj} captures the units of measured hours (it can be viewed as a re-parameterization of the steady state associated with the time-varying preference parameter ϕ_t that appears in the households' utility function).

Whenever we include observed k-quarter ahead inflation expectations $\pi_t^{O,t+k}$ to our set of time series, the set of equations (28) is augmented to include:

$$
\begin{aligned}
\pi_t^{O,t+k} &= 400(\mathbb{E}_t^{dsge}[\pi_{t+k}] + \ln \pi_*) \\
&= 400(\mathcal{Z}(\theta)_{\pi,.}\mathcal{T}(\theta)^k s_t + \ln \pi_*),
\end{aligned}
\tag{29}
$$

where $\mathbb{E}_t^{dsge}[.]$ are the inflation expectations obtained from the DSGE model. The second line shows how to compute these expectations using the transition equation (26), where $\mathcal{Z}(\theta)_{\pi,.}$ is the row of $\mathcal{Z}(\theta)$ corresponding to inflation. In our application $k = 4$.[7]

[7]Equations (29) embodies the assumption that observed expectations are rational, which arguably clashes with some of the evidence mentioned in the introduction. See section 5.5 for a discussion of this issue.

4 Measurement and Issues with Modeling Inflation Expectations

Several issues arise in using inflation expectations as observables in the estimation of DSGE models. First, there are several measures of inflation expectations available, for different inflation measures, and at different horizons. Our measurement choice of inflation expectations for the benchmark specification coincides with that of Erceg and Levin (2003): we use four-quarter ahead expectations for the GDP deflator obtained from the Survey of Professional Forecasters. We check for the robustness of the results using different sources of expectations (Blue Chip versus SPF), and different inflation measures (CPI versus GDP deflator). An alternative source of inflation expectations is the Michigan Survey of households, which are available at the one and ten years horizons. However in that Survey households are asked about inflation in general, as opposed to any specific measure, and that makes it hard to have a measurement of expectations that is consistent with the chosen measure of inflation.

In terms of forecast horizons, we choose the longest forecast horizon for which data are available since the 1980s, namely four quarters, since arguably longer forecast horizons are more informative on agents' views about the policymakers' inflation target. Erceg and Levin (2003) also choose this horizon.[8] Measures of inflation expectations for forecasting horizon longer than 4 quarters ahead are available but with limitations in terms of sample length and frequency. SPF provides average CPI inflation forecasts for the following 10 years but the sample starts in 1990Q4. Bluechip and the Philadelphia Fed's Livingston survey also provide 10-years CPI inflation forecast staring 1979Q4 but the forecasts are taken only twice a year. In one of our specifications we use the SPF 10-years CPI inflation forecast together with the four-quarter ahead expectations.[9]

[8] Shorter horizons forecasts are available but are less informative.

[9] Concerning the 5-years horizon, Bluechip includes forecasts which are also taken twice a year, while SPF produces quarterly forecast starting only in 2005Q3. SPF also provides quarterly 5 and 10 years forecast for PCE inflation but those start in 2007Q1. Finally, SPF produces 2 year

Another serious issue is that forecasters (whether SPF or Blue Chip) have only the latest vintage of data available, while the econometrician often uses the final vintage. This is a potentially important issue, especially for revision in the inflation measure itself, which will heavily condition the forecasts. This is not the only paper that uses inflation expectations together with revised data for macroeconomic variables (Canova and Gambetti (2007), Leduc et al. (2007), and CLARK AND DAVIG, who use structural VARs, are recent examples). Addressing the issue of data revisions when using observed expectations as observables represents a major challenge, which we do not undertake in this paper. We do however show the robustness of the results when we use CPI as a measure of inflation, as opposed to the more heavily revised GDP deflator. Non-seasonally CPI is never revised. Seasonally adjusted CPI adjusted has revisions, but these are fairly small compared to those for the GDP deflator, as shown by Figure 1. Of course, measured expectations are also function of measures of economic activity. Hence the issue of data revisions is by no means fully addressed by using CPI inflation.

A third issue is the one of information synchronization. SPF forecasters provide their forecasts in the middle of the quarter, and hence have partial information about the state of the economy in the current quarter. We deal with this issue by checking the robustness of the results to different assumptions regarding the timing of the agents' information set. The benchmark results are obtained assuming that observed expectations are formed using current quarter information (which is also the assumption used in Canova and Gambetti (2007)). The alternative assumption, which we call "Lagged Information" specification, is that the forecasters are only endowed with information up to the previous quarter. Last, forecasts are heterogeneous, and our model cannot account for such heterogeneity (sticky information models can produce heterogeneous expectations, see Mankiw et al. (2003)). This is a very interesting and important avenue of research, which we do not pursue in this paper. [10]

forecasts for CPI (core and total) and PCE (core and total) inflation but they are available since 2007Q1 (CPI is available since 2005Q3).

[10] QUOTE recent papers.

The standard set of macro data used in the estimation includes the following variables: Output growth (log differences, quarter-to-quarter, in %); hours worked (log, in %); labor share (log, in %); inflation (annualized, in %, we use either GDP deflator and CPI, depending on the the corresponding inflation expectation measure); nominal interest rate (annualized, in %). See Appendix A for details. In our benchmark specification we use 97 quarters of data spanning the Volcker-Greenspan period: 1984Q2 to 2008Q2.

5 Comparing Perfect and Imperfect Information Models of Time-Varying Inflation Target

5.1 Prior Choice and Prior Predictive Checks

Table 1 shows the priors for the parameters of the policy rule (4) and the associated law of motion for the inflation target π_t^* (5), which are the key parameters for the exercise conducted here. Priors for the responses to inflation (ψ_1) and the measure of economic activity (ψ_2) – output growth in the baseline specification – in the policy rule, persistence (ρ_r), and steady state inflation target (π^*) are as chosen as follows. In particular, The prior on π_* is centered using pre-sample information on inflation, as in DelNegro and Schorfheide (2008). The prior on ψ_1 and ψ_2 are centered at 2 and .2 respectively, and imply a fairly strong response to inflation and a much moderate response to output. Priors on variance of i.i.d. policy shocks σ_r is centered at .15. In general the priors on the standard deviations of the shocks are chosen so that overall variance of endogenous variables is roughly close to that observed in the pre-sample 1959Q3-1984Q1, informally following the approach in DelNegro and Schorfheide (2008). Key priors are those on persistence and standard deviation of the innovation to π_t^* process, as they determine, together with the prior on σ_r, the agents' Kalman gain in the Imperfect Information model. We follow Erceg and Levin (2003) and make the process followed by π_t^* very persistent: The prior for ρ_{π^*} is centered at .95 and the 90% bands range from about .91 to .99.

In the Benchmark prior the prior on σ_{π^*}, centered at .05, is independent from all other parameters, and is fairly loose.[11] An alternative prior ("Signal-to-Noise Ratio Prior") places a prior directly on the Signal-to-Noise ratio (and hence induces dependence between σ_{π^*} and σ_r) and is centered at the value that delivers a Kalman gain of approximately .13, the value calibrated by Erceg and Levin (2003).

Priors on nominal rigidities parameters are shown in the top panel of Table 2). To check robustness to the degree of nominal rigidities in the economy we consider two priors, as in DelNegro and Schorfheide (2008): "Low Rigidities" (loosely calibrated at Bils and Klenow (2004) values of average duration less than 2 quarters), and "High Rigidities" (duration about 4 quarters).

Priors on remaining parameters are shown in the bottom panel of Table 2). The priors on "Endogenous Propagation and Steady State" are all chosen as in DelNegro and Schorfheide (2008). Specifically, the prior for the habit persistence parameter h is centered at 0.7, which is the value used by Boldrin et al. (2001). The prior for a' implies that in response to a 1% increase in the return to capital, utilization rates rise by 0.1 to 0.3%. These numbers are considerably smaller than the one used by Christiano et al. (2005). The 90% interval for the prior distribution on ν_l implies that the Frisch labor supply elasticity lies between 0.3 and 1.3, reflecting the micro-level estimates at the lower end, and the estimates of Kimball and Shapiro (2003) and Chang and Kim (2006) at the upper end. We use a pre-sample of observations from 1959Q3-1984Q1 to choose the prior means for the parameters that determine steady states.

The priors on standard deviations and autocorrelations are chosen so that overall variance and autocorrelations of endogenous variables is roughly close to that observed in the pre-sample 1959Q3-1984Q1 (see Table 3). Table 3 also shows that although we use the same prior for both the models under consideration – the Imperfect and Perfect Information models – the prior predictive statistics are fairly similar across models.

[11]In this and all other tables the standard deviations σ_{π^*} and σ_r are not annualized.

5.2 Model Comparison Results

Table 4 shows the log marginal likelihood for three models: Imperfect Information, Perfect Information, and the model with constant inflation target (Fixed-π^*). For all models we use the Benchmark prior. The Dataset with Expectations uses the SPF 4-quarters ahead median forecast for the GDP deflator. For these results we assume that the expectations are generated using current quarter information. In the remainder of the paper we condition on two lags of the variables included in Y^0 when computing both marginal likelihoods and posteriors unless we indicate otherwise.

Table 4 shows that for the dataset without expectations (column *(1)*) all three models perform about the same, with the Fixed-π^* model performing slightly worse. The difference in $lnp(Y^0_{1,T}|\mathcal{M}_i)$ for the Imperfect and Perfect Information models is .69, which implies a posterior odd of roughly 2 in favor of the Imperfect Information model. The difference in $ln\ p(Y^0_{1,T}|\mathcal{M}_i)$ for the Fixed-π^* is larger, about 5. Although this difference implies that the posterior odds are heavily against the Fixed-π^* model, DelNegro and Schorfheide (2008) show that for marginal likelihoods for DSGE models are quite sensitive to the choice of priors, so that a difference of 5 can in principle be overturned by choosing a slightly different prior.

When SPF inflation expectations are included among the observables, the Perfect Information model with time-varying π^* performs significantly better than both the Fixed-π^* and, most importantly, the Imperfect Information model. The difference in the log marginal likelihoods $ln\ p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i)$ between the Perfect and Imperfect Information models is about 25 in favor of the latter. The data disfavors the Fixed-π^* even more strongly. Since the marginal likelihoods $ln\ p(Y^0_{1,T}|\mathcal{M}_i)$ are similar across models, these differences translate into differences in $ln\ p(Y^1_{1,T}|Y^0_{1,T}, \mathcal{M}_i)$. They imply that the Perfect Information model fits observed inflation expectations much better than either the Imperfect Information or the Fixed-π^* model. The fact that the differences are large indicates that the extra observable included in $Y^1_{1,T}$ contains quite a lot of information as to which model describes it best. In the remainder of the section we will provide additional evidence that the Imperfect

Information model has a much harder time at explaining observed inflation expectations than the Perfect Information one. Next, we will provide the intuition as to why this is the case.

Table 5 shows the median in-sample forecast errors for the Imperfect and Perfect Information models computed using the Kalman filter. In the top panel we compute the RMSEs using for each model the respective parameter values that maximize the posterior for the dataset without expectations (that is, the value of θ that maximizes $p(\theta|Y^0_{1,T}, \mathcal{M}_i)$). Columns (1) and (2) show the errors for the two models computed without providing the econometrician with the information about observed inflation expectations. Specifically, for each variable x_t we show the average value of $(x_t - I\!E[x_t|Y^{0,t-1}, \mathcal{M}_i])^2$. The next column shows the ratio of the RMSEs for the two models. These figures are all in the neighborhood of one, indicating that the forecasting performance of the models is roughly equal. In fact, the log likelihood $ln\ p(Y^0_{1,T}|\mathcal{M}_i, \theta)$ is quite similar for the two models. Interestingly, the ratio of RMSEs is about one also for observed inflation expectations, which are not part of the econometrician's information set (the numbers for inflation expectations are in parenthesis to emphasize that the corresponding forecast errors are computed without including this variable in the information set).

For the same set of parameters the forecast performance of the two models for the variables in $Y^0_{1,T}$ worsens considerably when inflation expectations are included into the econometrician's information set, and this is particularly the case for the Imperfect Information model. This is apparent from columns (3) and (4), which show $(x_t - I\!E[x_t|Y^{0,t-1}, Y^{1,t-1}, \mathcal{M}_i])^2$ for the two models. The last two columns of Table 5 show the ratio of the RMSEs with and without including inflation expectations among the observables for the Imperfect and Perfect Information models, respectively. All these figures are larger than one for both models for all the variables included in $Y^0_{1,T}$ (of course, for inflation expectations the RMSEs decrease). The worsening of in-sample forecasting performance is particularly large for the Imperfect Information model, where the increase in RMSEs range from 7% to 46%. As a consequence, when inflation expectations are included in the set of observables the Perfect Information model performs better than the Imperfect Information one: The ratios between the figures in column (3) and (4) are all larger than one (and the log likelihood $ln\ p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i, \theta)$ is much larger for the Perfect Information model).

The values of θ that maximizes $p(\theta|Y^0_{1,T}, \mathcal{M}_i)$ for the two models are of particular interest because it is the mode of the prior in formula (2). Nonetheless, such value may overemphasize the effect of including inflation expectations among the observables, since it maximizes the model's fit (adjusting for the prior) when this variable is excluded. Therefore the bottom panel of Table 5 repeats the exercise using the value of θ that maximizes $p(\theta|Y^0_{1,T}, Y^1_{1,T}, \mathcal{M}_i))$ for each of the two models, respectively. For these parameters it is still the case that the ratios of the RMSEs in column (3) and (4) are all larger than one, except for inflation expectations were the in-sample forecasting performance is the same. Moreover, for the Imperfect Information model it is also still the case that forecasting performance worsens for all variables when inflation expectations are included into the econometrician's information set. For the Perfect Information model this is the case only for some of the variables, notably for inflation.

Including inflation expectations among the observables worsens the fit of Imperfect Information model relative to that of the Perfect Information, consistently with the marginal likelihood results in Table 4. In order to understand this result we ask what kind of inflation expectations the two models generate whenever actual inflation expectations are not among the observables. Figure 3 plots the projections for the 4-quarter ahead inflation forecasts generated by the Imperfect (black solid) and Perfect (gray solid) Information models. This exercise is performed using the value of θ that maximizes $p(\theta|Y^0_{1,T}, \mathcal{M}_i)$ for the two models – the mode of the prior in formula (2).

To the extent that the inflation expectations generated by the model are roughly in line with the observed data, including measured expectations among the observables is unlikely to change the estimates of the states, and hence the forecasts of the other variables. However, if there is a large discrepancy between a model's forecasts of inflation expectations and what we observe in the data, we expect both the estimates of the states and the forecasts of the other variables to change substantially following the addition of measured expectations to econometrician's information set. Figure 3 also plots the actual inflation expectation data – namely, the SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted) – along with the projections. It is clear that the inflation forecasts generated by the both models are at odds with the data. They are too low in the early part of the sample, and too high in the later part. Interestingly, the inflation expectations generated by the two models are very similar.

The top panel of Figure 4 plots the mean estimate of the latent variable $\pi^*_{t|t}$ for the Imperfect Information model for the dataset without (black line) and with (gray line) inflation expectations. Similarly, the middle panel shows the mean estimate of the latent variable π^*_t for the Perfect Information model for the dataset without (black line) and with (gray line) inflation expectations. Since in the Imperfect Information model agents do not observe the actual π^*_t, these two latent variables are conceptually equivalent in that in each model they drive the agents' beliefs about the inflation target.[12] In both panels these figures are computed using the value of θ that maximizes $p(\theta|Y^0_{1,T}, \mathcal{M}_i)$. Both panels also show observed inflation expectations (dashed-and-dotted line).

The time series for $\pi^*_{t|t}$ and π^*_t look very similar across the two models when the econometrician does not have information about inflation expectations (black lines in top and middle panels). Not surprisingly, for both models the movement in these time series mirrors that of the model-generated inflation expectations in Figure 3. When inflation expectations are included among the observables, the path for π^*_t in the Perfect Information model moves closer to that of observed inflation expectations. Very loosely speaking, the filtering procedure realizes that the model is failing to match the new observable, and adjusts the latent state π^*_t accordingly. For the Imperfect Information model the path for $\pi^*_{t|t}$ barely move, and only at the

[12] In the Imperfect Information model all the econometrician can infer from the data is the agents' belief about π^*_t.

very beginning. The law of motion of the agents' perception of the inflation target $\pi^*_{t|t}$ is given by:

$$\pi^*_{t|t} = (1 - K)\rho_{\pi^*}\pi^*_{t-1|t-1} + K\tilde{\pi}_t, \tag{30}$$

which obtains rearranging equation (10). As we iterate this law of motion forward starting from the initial condition $\pi^*_{0|0}$, we realize that the econometricians only degree of freedom lies in the choice of this initial condition. After that, the path for $\pi^*_{t|t}$ is pinned down by that of the interest feedback rule residual $\tilde{\pi}_t$, defined in equation (6). In the baseline model where the interest rate responds to inflation and output growth this residual is pinned down by the data, for given parameters (the bottom panel of Figure 4 plots $\tilde{\pi}_t$, and shows that its fluctuations are consistent with the evolution of $\pi^*_{t|t}$). Hence the filtering procedure cannot adjust $\pi^*_{t|t}$ to match inflation expectations, and needs to rely on large, and likely persistent, shocks to fill the gap between $\pi^*_{t|t}$ and observed expectations. These large shocks negatively affect the fit for the other observables. In conclusion, the Imperfect Information model imposes tighter cross-equation restrictions than the Perfect Information model, in the sense that it cannot rely on adjusting the latent variable π^*_t to fit the data.

5.3 Robustness to the Choice of Priors, Datasets, Timing Conventions, Initial Conditions, Policy Rules, and Choice of Shocks

This section investigates the robustness of the model comparison results to the choice of priors, datasets, timing conventions, and policy rules. **Robustness to Priors**: Lines (1) and (2) of Table 6 report the model comparison results under the "'High Nominal Rigidities" prior and "Signal-to-Noise Ratio" prior described in section 5.1, respectively. We find that the "High Nominal Rigidities" prior favors the Perfect Information relative to the Imperfect Information model, in that the difference in $ln\ p(Y^1_{1,T}|Y^0_{1,T}|\mathcal{M}_i)$ is larger in favor of the Perfect Information model (we use the "Low Nominal Rigidities" prior precisely because it gives the Imperfect Information model the best shot). Using the "Signal-to-Noise Ratio" prior makes little difference.

Robustness to Data Sets and Timing Assumptions: Lines (3) through (6) show the log marginal likelihoods for the two models under different timing assumptions ("Lagged Information" specification), source for inflation expectations ("Blue Chip" versus SPF), and inflation measure (CPI versus GDP deflator). Under the "Lagged Information" specification the forecasters in the SPF Survey are only endowed with information up to the previous quarter. Results are robust to both timing assumptions and measurement choices. The gap in $ln\ p(Y^1_{1,T}|Y^0_{1,T}|\mathcal{M}_i)$ between the Perfect and Imperfect Information models varies among the different specifications, but is always larger than 20. The gap widens substantially whenever we use CPI (which is less subject to revisions) as opposed to the GDP Deflator.

Robustness to Conditioning Assumptions: As mentioned above, in our benchmark specification we condition on two lags of the variables included in Y^0 when computing marginal likelihoods, so that effectively we compute $ln\ p(Y^0_{1,T}|\mathcal{M}_i, Y^0_{-1,0})$ and $ln\ p(Y^0_{1,T}, Y^1_{1,T}|\mathcal{M}_i, Y^0_{-1,0})$ (for simplicity of notation we mostly omit the conditioning on $Y^0_{-1,0}$). Given that during the first part of our sample both inflation and inflation expectations are trending down, conditioning may play a non trivial role (see Sims ...). For this reason, line (7) reports the marginal likelihoods without conditioning on any variables, while line (8) reports the results when conditioning also on the first two lags of inflation expectations. While initial conditions matter in terms of marginal likelihood computations, from the perspective of model comparison the results do not change.

Robustness to Policy Rule Specification: Lines (9) through (11) report the model comparison results under different specifications of the policy rule, where the policy makers target output growth as opposed to the output gap ("Output Growth"), a four-quarter moving average of inflation as opposed to current inflation ("4Q Inflation"), or where the the law of motion for the inflation target follows the rule suggested by Gurkaynak et al. (2005) ("GSS"). Under this rule the marginal likelihood gap between the Imperfect and Perfect Information models stays roughly constant or increases. Under the rule proposed by Gurkaynak et al. (2005) the gap narrows, but it is still larger than 17.[13] The last two rows of Table 6 show the marginal likelihoods for the models where we allow for measurement error in expectations. We discuss this case in detail in section 5.5.

Robustness to Choice of Shocks (Discount Rate): The Imperfect Information models in principle has seven shocks, as discussed in section 3. Due to the fact that two of the shocks (the i.i.d. policy shock and the shocks to the target π^*_t) are not separately observed by either the agents in the model or the econometrician, but commingle into the policy rule innovation $\tilde{\pi}_t$, effectively this model has six independent disturbances. Whenever observed inflation expectations are added to the data set, this model has therefore as many shocks as observables. DelNegro and Schorfheide (forthcoming) show that introducing additional shocks into a model is tantamount to relaxing the cross-equation restrictions: the additional shocks can improve the model's fit by capturing dynamics that the existing set of shocks was not able to capture. One may wonder to what extent the worse fit for the imperfect information model relative to the perfect information is partly due to the set of shocks originally chosen. If so, introducing another shock may improve the imperfect information's model ability to explain inflation expectations. We therefore introduce a shock that is commonly used in DSGE models, namely a shock to the rate at which the representative agent discounts the future, which we refer to as b_t. Like most other shocks, b_t is also assumed to follow an AR(1) process. In terms of the log-linearized conditions this shock enters equation (19), which becomes:

$$\begin{aligned}(e^{\gamma} - h\beta)(e^{\gamma} - h)\xi_t &= -(e^{2\gamma} + \beta h^2)(c_t - b_t) \\ &+ \beta h e^{\gamma} I\!\!E_t[c_{t+1} + z_{t+1}] + h e^{\gamma}(c_{t-1} - z_t) - \beta h e^{-\gamma}(e^{2\gamma} + \beta h^2) I\!\!E_t[b_{t+1}],\end{aligned} \tag{31}$$

and equation (17), which becomes:

$$\begin{aligned}\tilde{w}_t \;&=\; \zeta_w \beta I\!\!E_t\left[\tilde{w}_{t+1} + \Delta w_{t+1} + \pi_{t+1} + z_{t+1} - \iota_w \pi_{t-1}\right] + \\ &\frac{1-\zeta_w\beta}{1+\nu_l(1+\lambda_w)/\lambda_w}\left(\nu_l L_t - w_t + \frac{e^{\gamma}(e^{\gamma}-h)}{(e^{2\gamma}+\beta h^2)} b_t - \xi_t + \frac{1}{1-\zeta_w\beta}\phi_t\right).\end{aligned} \tag{32}$$

[13] In the estimation of the GSS model we used the value of $\chi = .02$ in expression (9), which is the value used by Gurkaynak et al. (2005).

Line (12) of Table 6 reports the marginal likelihoods under these different model specification. For the standard set of macro variables the addition of discount rate shocks barely improves the marginal likelihood, especially for the Imperfect Information model. For the data set with observed inflation expectations the improvement over the benchmark specification is substantial. This is the case for both models, however, so the relative ranking is unaffected.

5.4 Posterior Estimates and Variance Decomposition

Table 7 shows the posterior mean and standard deviation (in parenthesis) of the parameters. The differences in parameter estimates between the posterior without ($p(\theta|Y^0_{1,T}, \mathcal{M}_i)$) and with inflation expectations ($p(\theta|Y^0_{1,T}, Y^1_{1,T}, \mathcal{M}_i)$) are not particularly noticeable for the Imperfect Information model. The ratio of σ_{π^*} to σ_r decreases from .13 to .11 between columns 1 ($p(\theta|Y^0_{1,T}, \mathcal{M}_i)$) and 2 ($p(\theta|Y^0_{1,T}, Y^1_{1,T}, \mathcal{M}_i)$) , and the estimates of ρ_{π_*} and $\rho_r$decrease as well. The importance of nominal rigidities decreases, consistently with the results in line (1) of Table 6. The importance of investment adjustment increases by about 60%, which implies that investment specific shocks become much more powerful when inflation expectations are used in the estimation. The persistence of shocks all increase, except for productivity shocks, and the increase is particularly noticeable for preference shocks to leisure ϕ_t (recall that hours is the variable for which the RMSE in Table 5 worsens the most when inflation expectations become part of the econometrician's information set). The shocks standard deviations generally rise, and particularly that of government spending shocks g_t.

Changes in parameters for the Perfect Information model are even less dramatic. The curvature of the dis-utility from working nu_l decreases between columns 1 ($p(\theta|Y^0_{1,T}, \mathcal{M}_i)$) and 2 ($p(\theta|Y^0_{1,T}, Y^1_{1,T}, \mathcal{M}_i)$), thereby making hours more elastic, and the persistence of ϕ_t shock decreases (with a more elastic labor supply the reliance on ϕ_t shocks to explain movements in hours decreases). Movements in the inflation target become larger and more persistent (both σ_{π_*} and ρ_{π_*} increase).

Table 8 shows the (unconditional) variance decomposition computed using the posterior distribution for the Imperfect and Perfect Information models obtained using the dataset that includes observed inflation expectations. The time -varying inflation target π_t^* is the main driver of inflation expectations in the Perfect Information model, while it explains very little under Imperfect Information, consistently with the intuition discussed in section 5.2.

5.5 Allowing for Measurement Error/Irrationality in Observed Inflation Expectations

Taken at face value, the measurement equation (29):

$$\pi_t^{O,t+k} = 400(\mathbb{E}_t^{dsge}[\pi_{t+k}] + \ln \pi_*)$$

may seem to be at odds with part of the body of literature that studies the "rationality" of observed inflation expectations, and which we briefly discussed in the introduction. Don't we know that observed inflation expectations are "irrational"? If we accept this preposition, how can we assume that they are generated by a rational expectation model, as implied by expression (29)? First, the irrationality of observed expectations is not an accepted fact in this literature. For instance, Andolfatto et al. (2008) show that in small samples a rational expectation model with imperfect information can often generate rejections of the null hypothesis. But even if we accept that from a statistical perspective rationality of observed expectations is often rejected, it is not clear from the existing literature that deviations from rationality are large enough from an economic standpoint that equation (29) is not a reasonable first pass assumption. Recall that the issue studied in this paper – explaining the joint dynamics of observed inflation expectations – is different from that addressed by that literature – whether one can reject the rationality of expectations. At the same time it is legitimate to ask whether our results are robust to violations (29), whether these are due to "irrationality" of private forecasters or to issues of data revisions and data synchronization. This is the question addressed in this section.

First, we allow for measurement error in equation (29):

$$\pi_t^{O,t+k} = 400(\mathbb{E}_t^{dsge}[\pi_{t+k}] + \ln \pi_*) + \chi_t, \tag{33}$$

where the error χ_t is assumed to be either i.i.d. ("i.i.d. Meas. Error" case) or to follow and AR(1) process ("AR(1) Meas. Error" case). In both cases χ_t evolves independently from all other shocks in the model. Rows (1) and (2) of Table 9 show the marginal likelihoods for the models where we allow for measurement error in expectations.[14] The Perfect Information model is still superior to the specification with Imperfect Information when the measurement error is i.i.d.. The difference in $ln\ p(Y^1_{1,T}|Y^0_{1,T}|\mathcal{M}_i)$ is about 16, which is smaller than in Table 4 but still substantial. The fit of the two models are essentially the same under AR(1) measurement error.

We conjecture that the autoregressive measurement error largely "takes care" of the misspecification in the Imperfect (and to some extent also in the Perfect) Information model, so we revert to the original result that when the dataset does not include inflation expectations the fit of the two models is about the same. We substantiate this conjecture using the variance decomposition for observed inflation expectations – both unconditional and 10-quarters ahead – shown in Table 10. We find that i.i.d. measurement error is not all that important for both the Imperfect and Perfect Information models. Its contribution is small for the unconditional variance, and between 30 and 45% at the 10-quarters ahead horizon. The AR(1) measurement error is the most important source of variation for observed expectations in both models, however. Measurement error explains about 60 and 40-45 percent of the variance for the Imperfect and Perfect Information models, respectively. These results may be easily explained just by appealing to irrationality of forecasters, or to data revisions. While issues of data revisions and data synchronization are likely to introduce a mismatch between measured and model-generated inflation expectations, our prior would be that this mismatch is relatively short-

[14]Note that the marginal likelihood for the data set without expectations $ln\ p(Y^0_{1,T}|\mathcal{M}_i)$ is the same as in the benchmark case: Whenever observed expectations are not part of the observables, the measurement error parameters do not enter the likelihood. Since the priors on these parameters are proper (they integrate to one), the marginal likelihood is the same.

lived. The results for the AR(1) measurement error show otherwise: For both the Perfect and Imperfect Information model the mean estimate of the AR(1) coefficient for measurement error is about .87.

We conclude that introducing classical measurement error, especially if AR(1), tends to mask the difference among models as it largely takes care of model misspecification by explaining a large fractions of fluctuations in the variable of interest. For this reason, we choose to put more structure on the discrepancy between model implied and observed expectations. First, even under specification (33) the measurement error is assumed to have mean zero: forecasts are unbiased. Some literature (QUOTE) argues to the contrary that a bias exist. Row (3) of Table 9 investigates this hypothesis by introducing a constant in (29) (the prior for the constant has mean zero and standard deviation .75%). The marginal likelihood results indicate that the evidence in favor of a bias is very weak. The marginal likelihood is actually worse for the imperfect information model, and only slightly better for the perfect information one. The 90% posterior bands for the bias parameter are on both sides of zero.

Next, we assume that the discrepancy between model implied and observed expectations depends on observables. This is a natural assumption for two reasons. First, the "irrationality" of observed expectations literature shows that inflation forecast errors depend on current information. Second, there is evidence that data revisions are dependent from the state of the economy (ARUOBA?). We therefore use the alternative measurement equation:

$$\pi_t^{O,t+k} = 400(I\!E_t^{dsge}[\pi_{t+k}] + \ln \pi_*) + \gamma' x_t, \tag{34}$$

where γ is a $\kappa \times 1$ parameter vector, and x_t consists of a $\kappa \times 1$ vector of time t observables. We always allow for the possibility of a bias in this exercises, so the first element of x_t is 1. Note that to the extent that there is a mapping between x_t and the model's states s_t – or even more simply, if the elements of x_t are already part of the measurement equation, as is the case here – equation can be recast in terms of the canonical form

$$y_t = \mathcal{Z}(\theta)s_t + \mathcal{D}(\theta),$$

so the likelihood can be computed via usual methods. Row (4) of Table 9 investigates whether the discrepancy depends on current inflation, and row (5) in addition allows for dependence on current output and labor share (the prior for the constant has mean zero and standard deviation .5%). None of these models significantly improves over the benchmark specification. The only parameters for which 90% posterior bands are not on both sides of zero is the response to output growth, but economically this coefficient is small. In summary, we find that the Imperfect Information model fits observed inflation expectations worse than the Perfect Information one regardless of whether we allow for a discrepancy between model implied and observed expectations. The only exception is the AR(1) measurement error. In this case the two model have roughly the same fit, but that is because the measurement error explains about half of the fluctuations in observed inflation expectations.

5.6 Using Data from the Great Disinflation

According to Erceg and Levin (2003) the Great Disinflation of the early eighties is the poster child for the Imperfect Information model: The Central Bank raised rates in order to bring down inflation, but agents initially have trouble telling whether it represented a shift in π^*_t or a temporary interest rate shock. As a consequence, inflation expectations decline very gradually. One would therefore think that our conclusions about the ability of the Imperfect and Perfect Information model could be reversed using data from that period.

The trouble with the Great Disinflation period, and particularly with its early phase, is that the rule adopted by the monetary authorities may have been different from that employed since the mid-eighties. At the same time, estimating the models over the 1980-1984 period only would imply using a very short time series. For the sake of notation, call T_0, T_1, and T_2 the quarters corresponding to 1980Q1 (beginning of time series considered in Erceg and Levin), 1984Q2 (beginning of our benchmark estimation period) to 2008Q2 (end of our sample). The first row of Table 11 performs the standard model comparison exercise over the benchmark estimation period T_1 to T_2, computing the usual quantities $\ln p(Y^0_{T_1,T_2}|\mathcal{M}_i)$ and $\ln p(Y^0_{T_1,T_2}, Y^1_{T_1,T_2}|\mathcal{M}_i)$. The only difference between these numbers and those in Table 4 is that we do not condition on any pre-sample observations for reasons that will soon become apparent (these numbers correspond to those in row (7) of Table 6). The second row of Table 11 performs the model comparison exercise over the period T_0 to T_2, and computes the quantities $\ln p(Y^0_{T_0,T_2}|\mathcal{M}_i)$ and $\ln p(Y^0_{T_0,T_2}, Y^1_{T_0,T_2}|\mathcal{M}_i)$. We find that our findings are once again fairly robust: For the standard set of observables $Y^0_{T_0,T_2}$ the two models perform very similarly, but once observed expectations are added to the set of observables the Perfect Information model fares better.

We now ask a slightly different question. Suppose we have estimated the model over the post-84 period and made an assessment about the relative fit of the two models, how does the additional information from the Great Disinflation period update our view of the two models? The objects of interest are now

$$\ln p(Y^0_{T_0,T_1}, Y^1_{T_0,T_1}|Y^0_{T_1,T_2}, Y^1_{T_1,T_2}, \mathcal{M}_i) = \ln p(Y^0_{T_0,T_2}, Y^1_{T_0,T_2}|\mathcal{M}_i) - \ln p(Y^0_{T_1,T_2}, Y^1_{T_1,T_2}|\mathcal{M}_i),$$

where the equality shows that these objects can computed as the difference between quantities we already have computed (not conditioning on any pre-sample observations makes this convenient). Note that this exercise can be seen as a training sample *in reverse*. Usually in training sample exercises we move forward: We form a prior over the T_0 to T_1 sample and then estimate the model using data between T_1 and T_2. Here we go backward: We form a prior over the post-84 period and then compare the models over the Great Disinflation. The result is that during the Great Disinflation period the Imperfect Information model fares better, consistently with Erceg and Levin (2003). The third row of Table 11 shows that the differences in log marginal likelihoods is about 10 in favor of the Imperfect Information model.

6 Conclusions

The paper shows that data on inflation expectations are quite informative as to which model – Perfect and Imperfect Information – fits the data best, unlike standard macro time series. The paper finds that the model comparison exercise always favors what we would consider to be the most "implausible" of the two models, namely Perfect Information. The paper documents that this conclusion is robust to a number of auxiliary assumptions, and provides a fairly straightforward intuition behind this result: The Imperfect Information poses tighter restrictions on the econometrician than the Perfect Information model. Under Perfect Information the econometrician can use the latent variable π_t^* to fit the data. Under Imperfect Information the path of (the agents' perception of) π_t^* is tied down by the realizations of the residual in the policy rule. Under this latter model the econometrician has, in a sense, less free parameters. Of course, having less free parameters is not an issue to the extent that the restrictions agree with the data. In the present situation it appears that they do not, however.

These results are informative, in that arguably they help updating economists' views on the Imperfect Information model's ability to describe the data, At the same time these findings leave the researcher with two bad options: a "plausible" model that does not fit the data versus an "implausible" one that fits better.[15] A natural solution is to improve the "plausible" model. We leave this task for future research.

[15]The extent to which the Perfect Information model can be considered implausible is of course open to debate.

References

Adam, Klaus and Mario Padula, "Inflation Dynamics and Subjective Expectations in the United States," CSEF Working Papers 78, Centre for Studies in Economics and Finance (CSEF), University of Naples, Italy March 2002.

Andolfatto, David, Scott Hendry, and Kevin Moran, "Are inflation expectations rational?," *Journal of Monetary Economics*, March 2008, *55* (2), 406–422.

Aruoba, S. Boragan and Frank Schorfheide, "Sticky Prices Versus Monetary Frictions: An Estimation of Policy Trade-offs," NBER Working Papers 14870, National Bureau of Economic Research, Inc April 2009.

Bernanke, Ben, Jean Boivin, and Piotr S. Eliasz, "Measuring the Effects of Monetary Policy: A Factor-augmented Vector Autoregressive (FAVAR) Approach," *The Quarterly Journal of Economics*, January 2005, *120* (1), 387–422.

Bils, Mark and Peter Klenow, "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 2004, *112*, 947–985.

Boivin, Jean and Marc Giannoni, "DSGE Models in a Data-Rich Environment," NBER Technical Working Papers 0332, National Bureau of Economic Research, Inc December 2006.

Boldrin, Michele, Lawrence Christiano, and Jonas Fisher, "Habit Persistence, Asset Returns, and the Business Cycle," *American Economic Review*, 2001, *91*, 149–166.

Calvo, Guillermo, "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 1983, *12*, 383–398.

Canova, Fabio and Luca Gambetti, "Do expectations matter? The Great Moderation revisited," Economics Working Papers 1084, Department of Economics and Business, Universitat Pompeu Fabra November 2007.

Chang, Youngsun and Sun-Bin Kim, "Individual to Aggregate Labor Supply: A Quantitative Analysis based on Heterogeneous Agent Macroeconomy," *International Economic Review*, 2006, *47*, 1–27.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 2005, *113*, 1–45.

DelNegro, Marco and Frank Schorfheide, "Forming priors for DSGE models (and how it affects the assessment of nominal rigidities)," *Journal of Monetary Economics*, 2008, *55* (7), 1191 – 1208.

_ **and** _ , "Monetary Policy with Potentially Misspecified Models," *American Economic Review*, forthcoming.

_ **,** _ **, Frank Smets, and Raphael Wouters**, "On the Fit of New Keynesian Models," *Journal of Business and Economic Statistics*, 2007, *25* (2), 123 – 162.

Erceg, Christopher J. and Andrew T. Levin, "Imperfect Credibility and Inflation Persistence," *Journal of Monetary Economics*, 2003, *50*, 915–944.

Giannone, Domenico, Francesca Monti, and Lucrezia Reichlin, "Incorporating Conjunctural Analysis in Structural Models," *in "The Science and Practice of Monetary Policy Today", Volker Wieland editor, Springer*, 2008.

Gurkaynak, Refet S., Brian Sack, and Eric Swanson, "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implications for Macroeconomic Models," *American Economic Revie*, 2005, *95* (1), 425–36.

Keen, Benjamin D., "The Signal Extraction Problem Revisited: A Note on its Impact on a Model of Monetary Policy," *Macroeconomic Dynamics*, 2009, *forthcoming.*

Kimball, Miles and Matthew Shapiro, "Labor Supply: Are the Income and Substitution Effects Both Large or Both Small?," *Manuscript, University of Michigan*, 2003.

Leduc, Sylvain, Keith Sill, and Tom Stark, "Self-fulfilling expectations and the inflation of the 1970s: Evidence from the Livingston Survey," *Journal of Monetary Economics*, March 2007, *54* (2), 433–459.

Lloyd, B. Thomas Jr., "Survey Measures of Expected U.S. Inflation," *Journal of Economic Perspectives*, Fall 1999, *13* (4), 125–144.

Mankiw, N. Gregory, Ricardo Reis, and Justin Wolfers, "Disagreement about Inflation Expectations," in "NBER Macroeconomics Annual 2003, Volume 18" 2003.

Melecky, Martin, Diego Rodriguez Palenzuela, and Ulf Soderstrom, "Inflation Target Transparency and the Macroeconomy," *Mimeo, Bocconi University*, 2008.

Milani, Fabio, "Expectations, Learning, and Macroeconomic Persistance," *Journal of Monetary Economics*, 2007, *54* (7), 2065 – 2082.

Nunes, Ricardo, "Inflation Dynamics: The Role of Expectations," 2009. Manuscript.

Ormeno, Arturo, "Inflation Expectations, Learning, and DSGE Model Estimation," *mimeo UPF*, 2009.

Rich, Robert W., "Testing the Rationality of Inflation Forecasts from Survey Data: Another Look at the SRC Expected Price Change Data," *The Review of Economics and Statistics*, 1989, *71* (4), 682–686.

Roberts, John M., "Is inflation sticky?," *Journal of Monetary Economics*, July 1997, *39* (2), 173–196.

Rudebusch, Glenn and Eric Swanson, "The bond premium in a DSGE model with long-run real and nominal risks," Working Paper Series 2008-31, Federal Reserve Bank of San Francisco 2008.

Schorfheide, Frank, "Learning and Monetary Policy Shifts," *Review of Economic Dynamics*, April 2005, *8* (2), 392–419.

Sims, Christopher A., "Solving Rational Expectations Models," *Computational Economics*, 2002, *20*, 1–20.

Smets, Frank and Raf Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 2003, *1* (5), 1123 – 1175.

_ **and** _ , "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 2007, *97* (3), 586 – 606.

A Data

The data set is obtained from Haver Analytics (Haver mnemonics are in italics). We compile observations for the variables that appear in the measurement equation (28). Real output is obtained by dividing the nominal series (*GDP*) by population 16 years and older (*LN16N*), and deflating using the chained-price GDP deflator (*JGDP*). We compute quarter-to-quarter output growth as log difference of real GDP per capita and multiply the growth rates by 100 to convert them into percentages. Our measure of hours worked is computed by taking total hours worked reported in the National Income and Product Accounts (NIPA), which is at annual frequency, and interpolating it using growth rates computed from hours of all persons in the non-farm business sector (*LXNFH*). We divide hours worked by *LN16N* to convert them into per capita terms. We then take the log of the series multiplied by 100 so that all figures can be interpreted as percentage changes in hours worked. The labor share is computed by dividing total compensation of employees (*YCOMP*) obtained from the NIPA by nominal GDP. We then take the log of the labor share multiplied by 100. Inflation rates are defined as log differences of the GDP deflator and converted into annualized percentages. The nominal rate corresponds to the effective Federal Funds Rate (*FFED*), also in percent. As an alternative measure of the nominal rate we use the three months Tbill (*FTBS3*),

We use Survey of Professional Forecasters (SPF) quarterly measures of expected inflation. We consider both expectations for GDP deflator[16] and for CPI inflation. In particular, we use the median four -quarters-ahead forecast of inflation in annualized terms. Concerning the information available to the forecasters, the survey is sent out at the end of the first month of each quarter and responses deadlines occur in the middle month of each quarter. Therefore, respondents have knowledge about the BEA advance report of the National Income and Product Accounts. We also compute the revisions in GDP deflator and CPI occurred since 1982 using the real time dataset available from the Federal Reserve Bank of Philadelphia.

[16]In more detail, the forecast are for the GDP price index, seasonally adjusted (base year varies). Prior to 1996, the forecast variable was the GDP implicit deflator. Prior to 1992, the GNP deflator.

(http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/)

As an alternative measure of inflation expectations, we use Bluechip monthly forecasts of CPI inflation. We choose forecast horizons of 3 and 4 quarters ahead. In order to compare Bluechip and SPF quarterly forecast of CPI inflation, we use the Bluechip forecasts available in the middle month of each quarter. This roughly corresponds to the time period when SPF participants provide their forecasts.

Table 1: Priors on Policy Parameters

Parameter	Domain	Density	Para (1)	Para (2)	5%	95%
ψ_1	$\mathbb{R}^+$	Gamma	2.00	0.25	1.592	2.408
ψ_2	$\mathbb{R}^+$	Gamma	0.20	0.10	0.049	0.349
ρ_r	[0,1)	Beta	0.50	0.200	0.170	0.827
π^*	$\mathbb{R}$	Normal	4.3	2.5	0.520	8.17
σ_r	$\mathbb{R}^+$	InvGamma	0.150	4.00	0.080	0.298
ρ_{π^*}	[0,1)	Beta	0.950	0.025	0.913	0.989
Benchmark Prior						
σ_{π^*}	$\mathbb{R}^+$	InvGamma	0.050	8.000	0.032	0.078
Signal-to-Noise Ratio Prior						
$\sigma_{NR} = \frac{\sigma_P}{\sigma_T}$	$\mathbb{R}^+$	Gamma	0.180	0.150	0.001	0.380

Notes: Para (1) and Para (2) correspond to means and standard deviations for the Beta, Gamma, and Normal distributions and to s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The last two columns report the 5^{th} and 95^{th} quintile of the prior distribution.

Table 2: Priors on Non-Policy Parameters

Parameter	Domain	Density	Para (1)	Para (2)	5%	95%
Priors on Nominal Rigidities Parameters						
Low Rigidities (Benchmark)						
ζ_p	[0,1)	Beta	0.450	0.100	0.285	0.614
ζ_w	[0,1)	Beta	0.450	0.100	0.285	0.614
High Rigidities						
ζ_p	[0,1)	Beta	0.750	0.100	0.590	0.913
ζ_w	[0,1)	Beta	0.750	0.100	0.590	0.913
Priors on "Endogenous Propagation and Steady State" Parameters						
α	[0,1)	Beta	0.330	0.020	0.297	0.362
s''	$\mathbb{R}^+$	Gamma	4	1.500	1.614	6.303
h	[0,1)	Beta	0.700	0.050	0.619	0.782
a'	$\mathbb{R}^+$	Gamma	0.200	0.100	0.049	0.349
ν_l	$\mathbb{R}^+$	Gamma	2	0.75	0.787	3.137
r^*	$\mathbb{R}^+$	Gamma	1.5	1	0.106	2.883
γ	$\mathbb{R}^+$	Gamma	1.650	1	0.204	3.073
g^*	$\mathbb{R}^+$	Gamma	0.300	0.100	0.143	0.459
ι_p	[0,1)	Beta	0.5	0.280	0.043	0.922
ι_w	[0,1)	Beta	0.5	0.280	0.049	0.932
Priors on ρs and σs						
ρ_z	[0,1)	Beta	0.400	0.250	0.000	0.764
ρ_ϕ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_{λ_f}	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_μ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_g	[0,1)	Beta	0.750	0.150	0.530	0.982
σ_z	$\mathbb{R}^+$	InvGamma	0.200	4.000	0.107	0.395
σ_ϕ	$\mathbb{R}^+$	InvGamma	2.500	4.000	1.326	4.930
σ_{λ_f}	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.161	0.596
σ_μ	$\mathbb{R}^+$	InvGamma	0.500	4.000	0.264	0.99
σ_g	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.159	0.594

Notes: Para (1) and Para (2) correspond to means and standard deviations for the Beta, Gamma, and Normal distributions and to s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu,s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The last two columns report the 5^{th} and 95^{th} quintile of the prior distribution.

Table 3: Prior Implications for Moments of the Endogenous Variables

Variables	St. Dev.			Autocorr.		
	Imperfect Information	Perfect Information	*Data*	Imperfect Information	Perfect Information	*Data*
$OutputGrowth$	3.48	3.47	*4.33*	0.39	0.39	*0.28*
$LaborSupply$	2.98	2.98	*3.20*	0.93	0.93	*0.96*
$LaborShare$	1.39	1.39	*2.24*	0.86	0.86	*0.95*
$Inflation$	3.13	3.15	*2.77*	0.71	0.72	*0.88*
$InterestRate$	4.34	4.38	*4.30*	0.85	0.85	*0.87*
$Exp.\ Inflation$	1.37	1.40		0.86	0.85	

Notes: II: imperfect information; PI: perfect information. The pre-sample statistics (column *Data*) are in italics. These statistics are computed over the sample 1959Q3-1984Q1. Inflation expectations are not available during most of the pre-sample. The in-sample standard deviation and first-order autocorrelation of inflation expectations are 1.21, and 0.86, respectively.

Table 4: Model Comparison

	$\ln p(Y^0_{1,T})$	$\ln p(Y^0_{1,T}, Y^1_{1,T})$	$\ln p(Y^1_{1,T} \vert Y^0_{1,T})$
	Dataset without Expectations	Dataset with Expectations	
	(1)	(2)	(2) - (1)
Imperfect Information	-703.62	-811.04	-107.42
Perfect Information	-704.31	-786.35	-82.04
Fixed π^*	-709.29	-821.84	-112.55

Notes: The Table shows the log marginal likelihood for three models: Imperfect Information, Perfect Information, and the model with constant inflation target (Fixed-π^*). For all models we use the Benchmark prior. The Dataset with Expectations uses the SPF 4-quarters ahead median forecast for the GDP deflator. We assume that the expectations are generated using current quarter information.

Table 5: In-sample RMSEs

	Dataset without Expectations			Dataset with Expectations			Increase/Decrease in RMSE	
	Imperfect Info.	Perfect Info.		Imperfect Info.	Perfect Info.			
	(1)	(2)	(1)/(2)	(3)	(4)	(3)/(4)	(1)/(3)	(2)/(4)
Posterior mode estimates for the dataset without expectations								
Output Growth	2.221	2.258	*0.984*	3.056	2.374	*1.287*	1.376	1.052
Labor Supply	0.573	0.563	*1.017*	0.838	0.660	*1.271*	1.463	1.171
Labor Share	0.537	0.540	*0.994*	0.575	0.559	*1.029*	1.071	1.036
Inflation	0.869	0.895	*0.971*	1.014	0.996	*1.018*	1.167	1.114
Interest Rate	1.526	1.543	*0.989*	1.758	1.673	*1.051*	1.152	1.084
Exp. Inflation	(0.987)	(.959)	(*1.029*)	0.512	0.487	*1.051*	0.518	0.507
Likelihood	-656.7	-660.3		-1088.4	-791.7			
Posterior mode estimates for the dataset with expectations								
Output Growth	2.271	2.166	*1.048*	2.370	2.119	*1.119*	1.044	0.978
Labor Supply	0.580	0.556	*1.043*	0.654	0.554	*1.182*	1.129	0.996
Labor Share	0.544	0.530	*1.025*	0.563	0.539	*1.044*	1.035	1.016
Inflation	1.010	0.922	*1.096*	1.030	0.977	*1.054*	1.020	1.060
Interest Rate	1.583	1.487	*1.064*	1.635	1.499	*1.090*	1.033	1.008
Exp. Inflation	(0.774)	(0.703)	(*1.101*)	0.477	0.479	*0.996*	0.616	0.682
Likelihood	-685.2	-668.7		-760.9	-732.4			

Notes: The table shows the in-sample Root Mean Square Errors (RMSEs) for the Imperfect and Perfect Information models computed using the Kalman filter. The top panel shows the RMSEs using for each model the respective posterior mode for the dataset without expectations. The bottom panel shows the RMSEs using for each model the respective posterior mode for the dataset with expectations.

Table 6: Robustness of Model Comparison Results

Imperfect Information			Perfect Information		
$\ln p(Y^0_{1,T})$	$\ln p(Y^0_{1,T}, Y^1_{1,T})$	$\ln p(Y^1_{1,T}\|Y^0_{1,T})$	$\ln p(Y^0_{1,T})$	$\ln p(Y^0_{1,T}, Y^1_{1,T})$	$\ln p(Y^1_{1,T}\|Y^0_{1,T})$
Dataset without Expectations	Dataset with Expectations		Dataset without Expectations	Dataset with Expectations	
(1)	(2)	(2) - (1)	(3)	(4)	(4) - (3)
Robustness to Priors					
(1) High Nominal Rigidities Prior					
-701.65	-820.84	-119.19	-705.39	-789.26	-83.87
(2) Signal-to-Noise Ratio Prior					
-703.86	-811.97	-108.11	-709.66	-786.59	-76.93
Robustness to Data Sets and Timing Assumptions					
(3) Lagged Information					
-703.62	-800.74	-97.12	-704.31	-780.53	-76.22
(4) Blue Chip Expectations					
-703.62	-761.68	-58.06	-704.31	-742.11	-37.80
(5) CPI and SPF Expectations					
-761.28	-844.98	-83.70	-763.72	-771.38	-7.66
(6) CPI and Blue Chip Expectations					
-761.28	-865.04	-103.76	-763.72	-779.31	-15.59
Robustness to Conditioning Assumptions					
(7) No Conditioning					
-711.641	-816.67	-105.03	-711.67	-789.84	-78.17
(8) Conditioning on Initial Level of Inflation Expectations					
	-810.49			-784.83	

Notes: The table shows the log marginal likelihood for the Imperfect Information and Perfect Information models under different choices of priors, datasets, timing conventions, policy rules, and set of shocks. Lines (1) and (2) report the results under the "High Nominal Rigidities" prior and "Signal-to-Noise Ratio" prior, respectively. Lines (3) to (6) show the log marginal likelihood for the two models under different timing assumptions ("Lagged Information" specification), measures of inflation and measures of inflation expectations ("Blue Chip Expectations", "CPI and SPF Expectations", "CPI and Blue Chip Expectations"). Lines (7) and (8) report the results under different conditioning assumptions. Lines (9)-(11) report the results under different specifications of the policy rule, where the policy makers target output growth as opposed to the output gap ("Output Growth"), a four-quarter moving average of inflation as opposed to current inflation ("4Q Inflation"), or where the the law of motion for the inflation target follows the rule suggested by Gurkaynak et al. (2005) ("GSS"). Line (12) reports the results after augmenting the model with an additional shock (discount rate shock).

Table 6: Robustness of Model Comparison Results – Continued

Imperfect Information			Perfect Information		
$\ln p(Y^0_{1,T})$	$\ln p(Y^0_{1,T}, Y^1_{1,T})$	$\ln p(Y^1_{1,T} \mid Y^0_{1,T})$	$\ln p(Y^0_{1,T})$	$\ln p(Y^0_{1,T}, Y^1_{1,T})$	$\ln p(Y^1_{1,T} \mid Y^0_{1,T})$
Dataset without Expectations	Dataset with Expectations		Dataset without Expectations	Dataset with Expectations	
(1)	(2)	(2) - (1)	(3)	(4)	(4) - (3)
Robustness to Policy Rule Specification					
(9) Output Level					
-715.46	-816.23	-100.77	-709.17	-791.74	-82.57
(10) 4Q Inflation					
-703.74	-820.96	-117.22	-698.88	-790.42	-91.5
(11) GSS					
-707.79	-805.64	-97.85	-709.45	-789.99	-80.54
Other Robustness Checks					
(12) Additional Shocks (Discount Rate)					
-701.09	-792.10	-90.01	-703.58	-773.04	-69.46

Table 7: Posterior Estimates for Selected Parameters

Parameters	**Imperfect Information** Dataset without Expectations (1)	**Imperfect Information** Dataset with Expectations (2)	**Perfect Information** Dataset without Expectations (3)	**Perfect Information** Dataset with Expectations (4)
Policy Parameters				
ψ_1	2.442 (0.225)	1.915 (0.123)	2.497 (0.247)	2.324 (0.191)
ψ_2	0.282 (0.112)	0.255 (0.106)	0.232 (0.093)	0.264 (0.110)
ρ_r	0.407 (0.077)	0.375 (0.065)	0.454 (0.067)	0.592 (0.043)
$\rho_{\pi^\star}$	0.945 (0.025)	0.907 (0.021)	0.943 (0.025)	0.974 (0.011)
σ_r	0.404 (0.037)	0.422 (0.033)	0.389 (0.036)	0.435 (0.035)
σ_{π^*}	0.054 (0.010)	0.048 (0.009)	0.058 (0.012)	0.066 (0.009)
Nominal Rigidities Parameters				
ζ_p	0.579 (0.061)	0.530 (0.057)	0.558 (0.051)	0.580 (0.061)
ι_p	0.285 (0.182)	0.494 (0.202)	0.346 (0.181)	0.317 (0.167)
ζ_w	0.249 (0.069)	0.186 (0.031)	0.238 (0.061)	0.353 (0.098)
ι_w	0.400 (0.251)	0.540 (0.257)	0.375 (0.253)	0.370 (0.236)
Other "Endogenous Propagation and Steady State" Parameters				
α	0.340 (0.003)	0.340 (0.004)	0.340 (0.003)	0.341 (0.003)
s''	2.831 (0.880)	4.529 (1.152)	3.002 (0.902)	3.543 (1.205)
h	0.649 (0.047)	0.636 (0.053)	0.658 (0.049)	0.640 (0.046)
a'	0.291 (0.112)	0.212 (0.097)	0.275 (0.102)	0.274 (0.095)
ν_l	2.153 (0.534)	2.690 (0.649)	2.271 (0.588)	1.327 (0.510)
r^*	1.000 (0.423)	1.424 (0.541)	1.019 (0.452)	1.259 (0.471)
π^*	2.470 (0.996)	3.068 (0.574)	2.106 (0.759)	3.662 (1.134)
γ	1.629 (0.333)	1.511 (0.330)	1.646 (0.362)	1.454 (0.314)
g^*	0.272 (0.090)	0.304 (0.100)	0.287 (0.092)	0.306 (0.107)
ρs and σs				
ρ_z	0.203 (0.094)	0.200 (0.095)	0.247 (0.090)	0.177 (0.098)
ρ_ϕ	0.837 (0.071)	0.980 (0.013)	0.850 (0.062)	0.569 (0.218)
ρ_{λ_f}	0.823 (0.073)	0.838 (0.059)	0.840 (0.058)	0.803 (0.071)
ρ_μ	0.885 (0.050)	0.910 (0.025)	0.897 (0.044)	0.894 (0.051)
ρ_g	0.810 (0.116)	0.824 (0.056)	0.798 (0.140)	0.982 (0.016)
σ_z	0.699 (0.055)	0.693 (0.052)	0.709 (0.055)	0.689 (0.047)
σ_ϕ	3.008 (0.516)	3.327 (0.589)	3.055 (0.638)	2.656 (0.660)
σ_{λ_f}	0.146 (0.031)	0.175 (0.031)	0.156 (0.026)	0.149 (0.023)
σ_μ	0.468 (0.115)	0.410 (0.083)	0.464 (0.111)	0.398 (0.099)
σ_g	0.291 (0.050)	0.426 (0.047)	0.267 (0.050)	0.410 (0.050)

Notes: The table reports the posterior mean and standard deviation (in parenthesis) of the parameters for the Imperfect and Perfect Information models obtained from both the datasets with and without inflation expectations.

Table 8: Variance Decomposition

Variables	Tech	ϕ	μ	g	λ_f	π^*	Money
Imperfect Information							
Output Growth	0.25	0.35	0.11	0.22	0.05	0.00	0.01
Labor Supply	0.00	0.94	0.05	0.01	0.01	0.00	0.00
Labor Share	0.05	0.03	0.00	0.02	0.88	0.00	0.01
Inflation	0.13	0.15	0.44	0.07	0.08	0.03	0.07
Interest Rate	0.08	0.09	0.60	0.08	0.05	0.00	0.00
Exp. Inflation	0.01	0.01	0.90	0.00	0.00	0.05	0.00
Perfect Information							
Output Growth	0.29	0.12	0.17	0.20	0.10	0.00	0.04
Labor Supply	0.03	0.09	0.31	0.3	0.06	0.00	0.01
Labor Share	0.06	0.07	0.00	0.01	0.83	0.00	0.02
Inflation	0.06	0.08	0.11	0.01	0.08	0.59	0.05
Interest Rate	0.05	0.08	0.35	0.01	0.06	0.28	0.14
Exp. Inflation	0.01	0.00	0.14	0.00	0.00	0.84	0.00

Notes: The Table shows the (unconditional) variance decomposition computed using the posterior distribution for the Imperfect and Perfect Information models obtained using the dataset that includes observed inflation expectations.

Table 9: Allowing for Measurement Error/Irrationality in Observed Inflation Expectations

Imperfect Information			Perfect Information		
$\ln p(Y^0_{1,T})$	$\ln p(Y^0_{1,T}, Y^1_{1,T})$	$\ln p(Y^1_{1,T} \mid Y^0_{1,T})$	$\ln p(Y^0_{1,T})$	$\ln p(Y^0_{1,T}, Y^1_{1,T})$	$\ln p(Y^1_{1,T} \mid Y^0_{1,T})$
Dataset without Expectations	Dataset with Expectations		Dataset without Expectations	Dataset with Expectations	
(1)	(2)	(2) - (1)	(3)	(4)	(4) - (3)
(0) Benchmark specification					
-703.62	-811.04	-107.42	-704.31	-786.35	- 82.04
Measurement Error					
(1) i.i.d. Measurement Error					
-703.62	-796.31	-92.69	-704.31	-780.89	-76.58
(2) AR(1) Measurement Error					
-703.62	-775.31	-71.69	-704.31	-775.21	-70.90
(3) Bias					
-703.62	-811.48	-107.86	-704.31	-784.82	-80.51
(4) Bias + Response to Current Inflation					
-703.62	-811.08	-107.46	-704.31	-788.69	-84.38
(5) Bias + Response to Current Inflation, Labor Share, and Output Growth					
-703.62	-814.29	-110.67	-704.31	-786.16	-81.85

Notes: The table shows the log marginal likelihood for the Imperfect Information and Perfect Information models when allowing for discrepancies between observed and model generated expectations. Line (0) shows the results from the benchmark specification for ease of comparison. Lines (1) and (2) report the log marginal likelihood for the two models measurement errors are added ("i.i.d. Measurement Error", and "AR(1) Measurement Error").

Table 10: Variance Decomposition for Observed Inflation Expectations: Models with Measurement Errors

Variables	Tech	ϕ	μ	g	λ_f	π^*	meas.	Money
Unconditional								
Imperfect Information								
i.i.d. Meas. Error	0.02	0.02	0.63	0.00	0.01	0.16	0.14	0.01
AR(1) Meas. Error	0.01	0.01	0.26	0.00	0.01	0.12	0.57	0.00
Perfect Information								
i.i.d. Meas. Error	0.01	0.01	0.21	0.00	0.01	0.67	0.07	0.00
AR(1) Meas. Error	0.01	0.00	0.25	0.00	0.00	0.27	0.41	0.00
10 Quarters Ahead								
Imperfect Information								
i.i.d. Meas. Error	0.01	0.01	0.39	0.00	0.01	0.12	0.44	0.01
AR(1) Meas. Error	0.01	0.00	0.25	0.00	0.01	0.08	0.63	0.01
Perfect Information								
i.i.d. Meas. Error	0.01	0.02	0.23	0.00	0.02	0.41	0.28	0.00
AR(1) Meas. Error	0.01	0.00	0.26	0.00	0.01	0.24	0.46	0.00

Notes: The Table shows the posterior means of the variance decomposition for observed inflation expectations – both unconditional and 10 quarters ahead – for the Imperfect Information and Perfect Information models with both i.i.d. and AR(1) measurement error. The posteriors are obtained using the dataset that includes observed inflation expectations.

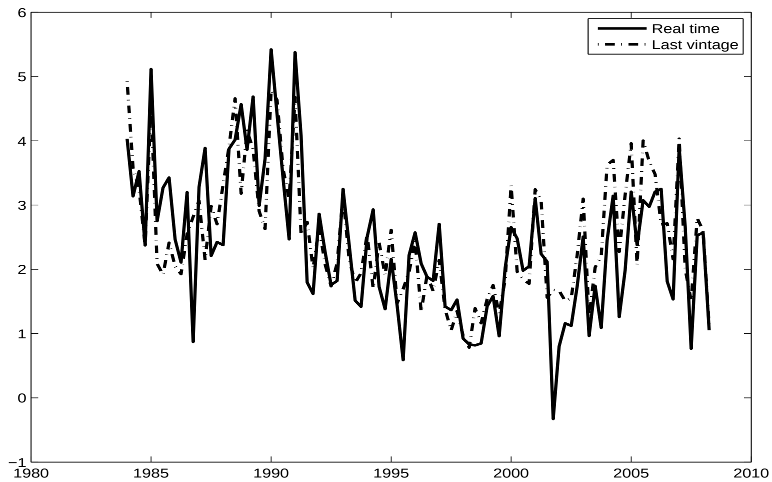
Table 11: Using Data from the Great Disinflation

Imperfect Information			Perfect Information		
$\ln p(Y^0_{T_i,T_j})$ Dataset without Expectations	$\ln p(Y^0_{T_i,T_j}, Y^1_{T_i,T_j})$ Dataset with Expectations	$\ln p(Y^1_{T_i,T_j} \mid Y^0_{T_i,T_j})$	$\ln p(Y^0_{T_i,T_j})$ Dataset without Expectations	$\ln p(Y^0_{T_i,T_j}, Y^1_{T_i,T_j})$ Dataset with Expectations	$\ln p(Y^1_{T_i,T_j} \mid Y^0_{T_i,T_j})$
(1)	(2)	(2) - (1)	(3)	(4)	(4) - (3)
(1) Post-Disinflation Data Set (1984-2008, standard)					
-711.641	-816.67	-105.03	-711.67	-789.84	-78.17
(2) Post-1980 Data Set (1980-2008)					
-918.02	-1057.45	-139.43	-921.38	-1041.51	-120.13
(3): (2)-(1), Updating over the Great Disinflation Period					
	-240.78			-251.67	

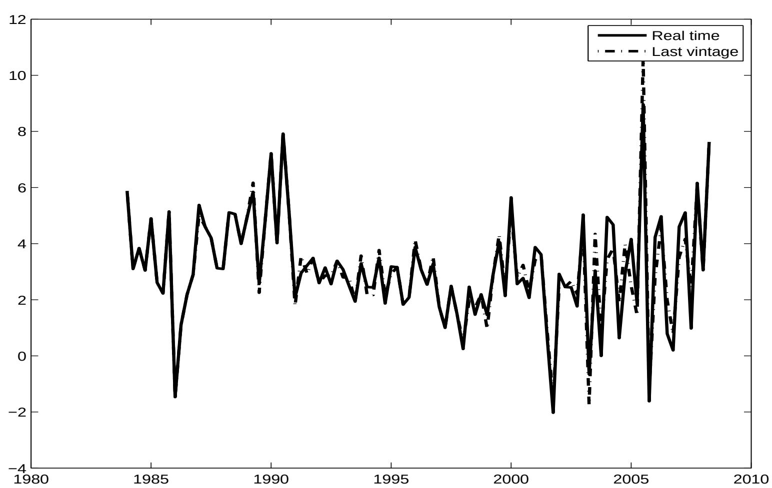
Notes: The table shows the log marginal likelihood for the Imperfect Information and Perfect Information models under different data sets.

Figure 1: Revisions in Inflation Data: Real Time vs Last Vintage

GDP Deflator

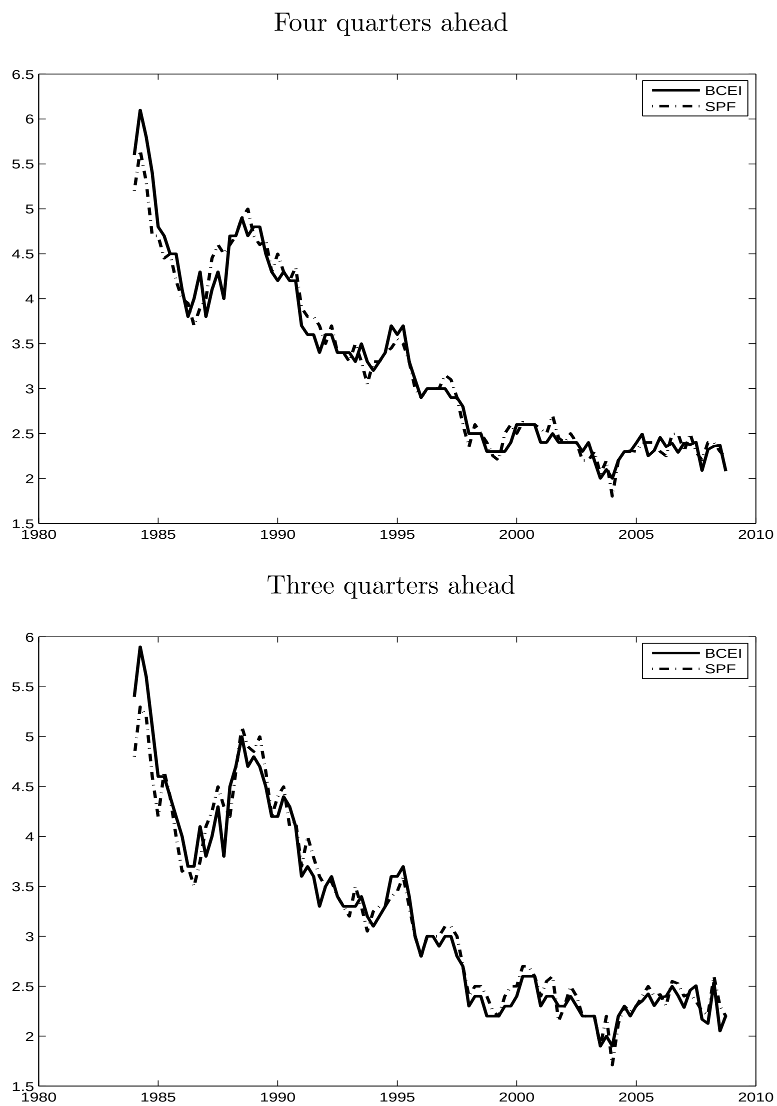


CPI



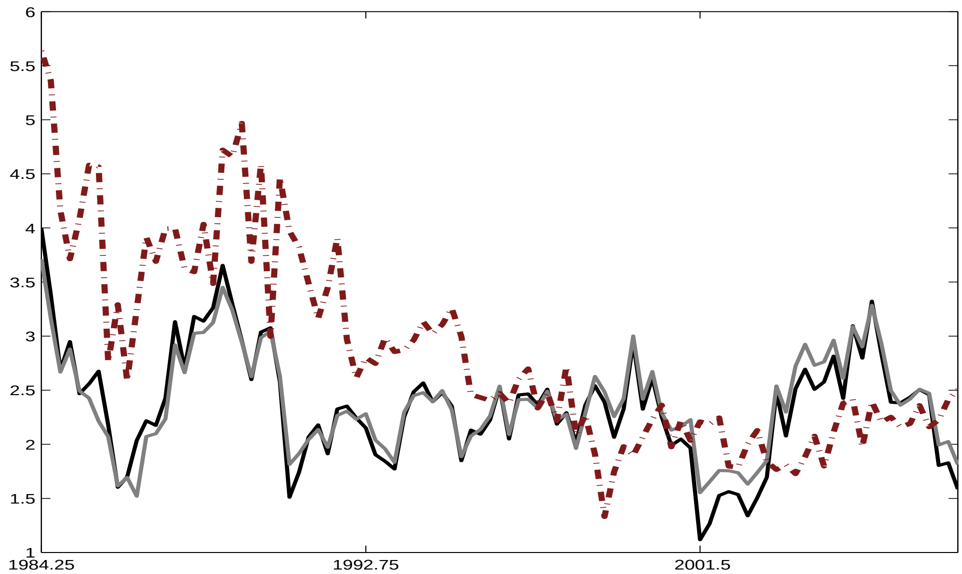
Notes: The figure plots data revisions for two measures of inflation: GDP deflator and CPI. The solid line shows the real time measure (that is, first vintage available) while the dashed-dotted line shows the most recent vintage.

Figure 2: Inflation Expectations: SPF vs Blue Chip



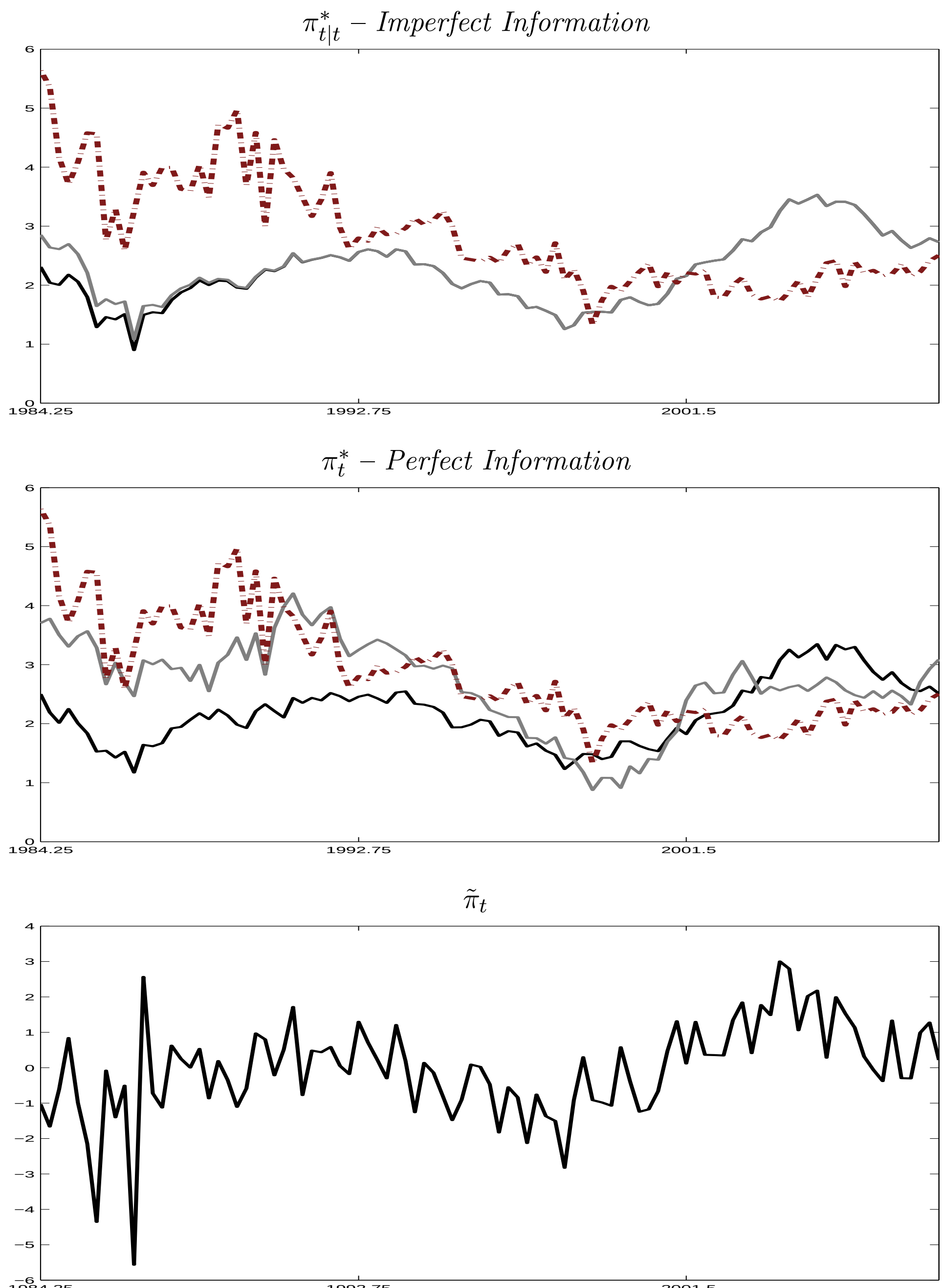
Notes: The figure plots inflation expectations from Blue Chip (solid line) and SPF (dashed line). The top panel shows quarterly (annualized) inflation expectations four quarters ahead, while the bottom panel shows expectations three-quarters ahead.

Figure 3: Inflation Expectations: Data vs Model Prediction



Notes: The figure plots SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted), together with the projections for the 4-quarter ahead inflation forecasts generated by the Imperfect (black solid) and the Perfect (gray solid) information models. The projections are computed using for each model the respective posterior mode for the dataset without expectations.

Figure 4: π_t^*



Notes: The top panel of the figure plots the mean estimate of the latent variable $\pi^*_{t|t}$ for the Imperfect Information model for the dataset without (black line) and with (gray line) inflation expectations. The middle panel shows the mean estimate of the latent variable π^*_t for the Perfect Information model for the dataset without (black line) and with (gray line) inflation expectations. The bottom line shows the interest feedback rule residual $\tilde{p}i_t$.