Comments on "How much nominal rigidity is there in the US economy? Testing a New Keynesian DSGE Model using indirect inference" by V.P.M. Le, P. Minford, and M. Wickens

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The purpose of the paper

- The purpose: Test the Smets and Wouters' (2007) model (SW model hereafter) with a classical statistic method called indirect inference.
- The general idea of indirect inference is to estimate or test a structural model by comparing it with an auxiliary model. Here, the auxiliary model is taken as the (nonstructural) VAR model.
- In the literature of the Bayesian approach to the DSGE models, Schorfheide (2000) compared DSGE and VAR models. and VAR models did much better. Smets and Wouter (2007) also compared DSGE and VAR models, and their DSGE model did very well compared with the VAR models both in terms of marginal likelihood and prediction.
- Given the importance of the SW model, it is interesting and important to compare their DSGE model with VAR models with classical statistical methods.

How the authors implement their idea.

After reading their paper, I am not sure how they implemented their idea, but here is what I think they did for Table 2.

- Step 1: They applied the Bayesian method to compute the posterior means of the VAR coefficients implied by the SW model evaluated at the posterior means of the model parameters. The called these posterior means of the VAR coefficients a_T.
- Step 2: They used the SW model evaluated at the posterior means of the model parameters to compute residuals.
- Step 3: They resampled the residuals to create N bootstrap samples of the data generated from the SW model.

Step 4: They computed OLS estimates of the VAR coefficients of each of the N bootstrap sample.

Step 5: They used the empirical distribution in Step 4 to compute the average of the VAR estimates, and called them $\alpha_S(\theta)$.

Step 6: They computed test statistics to compare a_T and $\alpha_S(\theta)$ using the empirical distributions in Step 4.

Shortcomings of their method (as I understood it)

- One shortcoming is that the sampling error of a_T is ignored. I do not think that this is important because I am interested in the performance of the SW model evaluated at the posterior means of the model parameters.
- A serious shortcoming is that their bootstrap method is not proved to work. I think that it is very likely that this bootstrap method does not have an asymptotic justification.

The Bootstrap

- To start from a simple case, suppose that we have the data $X_1, X_2, ..., X_N$ with the sample size of N for an i.i.d. random variable X_i with cumulative distribution function (CDF), F_0.
- Let $Q_N(X_1, ..., X_N)$ be the statistic of our interest. Let $G_N(q, F_0) = Pr(Q_N < q)$ be the exact, finite sample CDF of Q_N. The basic idea of the bootstrap is to replace F with a known estimator, F_N.

- Depending on whether F_N is nonparametric or parametric, we have the nonparametric bootstrap or the *parametric bootstrap*.
- For the nonparametric bootstrap, we attach the probability $1/N$ to each data point X_t.
- For the parametric bootstrap, we use a parametric estimator as F_N. For example, X_t is assumed to be normally distributed with mean μ and variance σ and their consistent estimates are used.

A Monte Carlo simulation is usually used to evaluate $G_N(q, F_N)$. The procedure is as follows:

- Step 1: Generate a bootstrap sample of size N, $X_t^* : t = 1, ..., N$ by sampling the distribution corresponding to F_N randomly. For the nonparametric bootstrap, sample the data with replacement. For the parametric bootstrap, use a random number generator to randomly sample from the estimated distribution.
- Step 2: Compute $Q_N^* = Q_N(X_1^*, ..., X_N^*)$.
- Step 3: Repeat steps 1 and 2 n times to obtain n observations of Q_N^*.
- Step 4: Compute the distribution of Q_N^* by putting mass $1/n$ at each observation of Q_N^*.

- For the bootstrap to work, $G_N(\cdot, F_N)$ should be close to $G_N(\cdot, F_0)$ for large N. The concept of consistency of the bootstrap formalizes this idea.
- The consistency of the bootstrap means that the bootstrap gets the statistic's asymptotic distribution right when the sample size is large.
- The conditions for consistency of the bootstrap are given in Section 2.1 of Horowitz (2001). Roughly speaking, the conditions require that F_N is a consistent estimator for F_0 and $G_N(\cdot, F)$ is continuous for F in an appropriate sense.
- These conditions are mild, but can be violated in some time series applications especially when autoregressive roots are one or near one.

- Even for confidence interval estimation of a simple AR(1) model, Basawa, Mallik, McCormick, Reeves, and Taylor (1991, *Annals of Statistics*) show that the standard Bootstrap method does not yield appropriate confidence intervals when the autoregressive root is near one.

  Bruce Hansen (1999) proposes Grip Bootstrap methods to overcome this difficulty for AR(1). Unfortunately, this method does not easily generalizes to VAR models.

  I conclude that the Bootstrap method is NOT reliable for the application of the authors.

Their empirical results

For Table 2, the authors report that the SW model do not produce VAR variances for nominal variables (the interest rate and the nominal interest rate) that are consistent with the data.

- The idea of comparing the implied variance of a shock with the data variance is interesting.
- However, I believe that these are variances of the reduced form equations.
- Reduced form errors are considered a linear combination of structural errors.
- It seems more meaningful to compare the variances of structural shocks implied by the model and those in the data.

What may be done

- For this application, if autoregressive roots are one or close to one, then the bootstrap method is not reliable.
- So it will be better to avoid the linear trend filter. I recommend the first difference filter.
- It will be good to do Monte Carlo (with each Monte Carlo simulation including Bootstrap simulations) to see if the method is working.
- Another method to consider is the two step minimum distance method of Christiano, Eichenbaum, and Evans (2005, CEE). Indirect inference is closely related with the minimum distance method. Instead of comparing the impulse responses, one can compare all or part of the VAR estimates if we want. The two step minimum distance method has an advantage of avoiding bootstrap (This is an advantage given the problem of the bootstrap for this type of applications mentioned above).

- Comparing the variance of structural shocks implied by the model and those in the data may be interesting.
- One method is to modify the SW model to make it to be recursive as in the recursive version of the Christiano, Eichenbaum, and Vigfusson's (2006) model. Then the usual recursive identifying restrictions can be used to compare the variance of the structural shocks.
- Given that I recommended the two step minimum distance method, I would like to note that there can be serious weak identification problem with the method. See Magnusson and Mavroeidis (forthcomimg, JMCB).