Comments on "Minimal State Variable Solutions to Markov–Switching Rational Expectations Models" by R. Farmer, D. Waggoner and T. Zha

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Summary of the paper

- This paper proposes a method for solving Markov-switching rational expectations models.
- These models are important because they can deal with cyclical or structural changes in DSGE models.
- There are some papers that solve these models assuming that the agents behaved as if the current regime were to last forever even if there are some possibility of regime switching.
- This assumption makes the solution straightforward, but this is not rational.
- Rational agents will form a probability distribution over possible regimes in the future when forming expectations.
- This paper assme the rational agents in this sense.
- Then, the solution is difficult.

Summary of the paper

- This paper shows that the solution of a Markov-switching rational expectations model can be represented by a constrained Markov-switching VAR model.
- Using this result, it proposes an algorithm for solving the parameters in the constrained Markov-switching VAR model.
- This algorithm solves a larger family of models than those existing previously in the literature.
- Using this method, it conducted a calibration for the cause of the Great Moderation in the US economy.

Outline

- An alternative algorithm
- An issue around constants
- Application to the estimation of DSGE models

An alternative algorithm

A Markov switching linear rational expectations model

$$A_1(\xi_t)E_t\left(y_{t+1}\right) + A_2\left(\xi_t\right)y_t + A_3\left(\xi_t\right)y_{t-1} + B\left(\xi_t\right)\epsilon_t = 0$$

Let's assume that the solution has the form

$$y_t = G_1\left(\xi_t\right)y_{t-1} + G_2\left(\xi_t\right)\epsilon_t$$

If the Markov chain is in state j in period t and in state ℓ in period $t+1$, then

$$y_{t+1} = G_1(\ell)\left[G_1(j)y_{t-1} + G_2(j)\epsilon_t\right] + G_2(\ell)\epsilon_{t+1}$$

As the Markov process is exogenous, ξ_t and ϵ_t are independent and one can write the conditional expectation

$$E_t\left(y_{t+1}|\xi_t = j, \Omega_t\right) = \sum_{\ell=1}^{h} p_{\ell,j}G_1(\ell)\left[G_1(j)y_{t-1} + G_2(j)\epsilon_t\right]$$

An alternative algorithm

Plugging the solution in the model

The model must be true in each state j of the Markov chain, and we have the following set of nonlinear equations in $G_1(j)$ and $G_2(j)$, $j = 1, \dots, h$.

$$
\begin{aligned}
&A_1(1) \sum_{\ell=1}^{h} p_{\ell,1} G_1(\ell) \left[G_1(1) y_{t-1} + G_2(1)\epsilon_t \right] \\
&\quad + A_2(1) \left(G_1(j) y_{t-1} + G_2(j)\epsilon_t \right) + A_3(1) y_{t-1} + B(1)\epsilon_t &= 0 \\
&\vdots \\
&A_1(h) \sum_{\ell=1}^{h} p_{\ell,h} G_1(\ell) \left[G_1(h) y_{t-1} + G_2(h)\epsilon_t \right] \\
&\quad + A_2(h) \left(G_1(j) y_{t-1} + G_2(j)\epsilon_t \right) + A_3(h) y_{t-1} + B(h)\epsilon_t &= 0
\end{aligned}
$$

An alternative algorithm

The structure of the previous nonlinear system suggests the following algorithm.
Initialization: Compute $G_1^0(1),\ldots,G_1^0(h)$ for

$$A_1(j)G_1^0(j)G_1^0(j) + A_2(j)G_1^0(j) + A_3(j) = 0 \qquad j = 1,\ldots,h$$

under the assumption that each of the h Markov states last forever. This can be done by the usual methods.
Iterative step: Compute $G_1^k(1),\ldots,G_1^k(h)$ for

$$A_1(j)\left[p_{j,j}G_1^k(j)G_1^k(j) + \sum_{\ell=0,\ell\neq j}^{h} p_{\ell,j}G_1^{k-1}(\ell)G_1^k(j)\right] + A_2(j)G_1^k(j) + A_3(j) = 0 \qquad j = 1,\ldots,h$$

taking the value of $G_1^{k-1}(\ell)$, $\ell \neq j$ given by the previous iteration in the computation of the conditional expectation.

An alternative algorithm

Rewriting the previous equations as

$$A_1(j)p_{j,j}G_1^k(j)G_1^k(j) + \left[A_1(j)\sum_{\ell=0,\ell\neq j}^{h} p_{\ell,j}G_1^{k-1}(\ell) + A_2(j)\right]G_1^k(j) + A_3(j) = 0 \quad j = 1,\dots,h$$

makes it possible to use standard methods to solve for $G_1^k(j)$.
Iterate until $\left||G_1^k(j) - G_1^{k-1}(j)\right|| < tol,\ j = 1,\dots,h$
Once $G_1(1),\dots,G_1(h)$ is computed by the above procedure, $G_2(1),\dots,G_2(h)$ can be recovered directly by linear algebra.

An alternative algorithm

- If the above procedure doesn't converge, it is possible to use dampening, using in the next iteration

$$G_1^{k'}(j) = \delta G_1^k(j) + (1-\delta) G_1(j)$$

  instead of $G_1^k(j)$
- It is necessary to choose one solution in the case of indeterminacy in one regime, in order to be able to continue the algorithm.
- An optimal strategy may be to start with the present algorithm for a few iterations and refine the solution using the Newton method suggested in the paper.

An issue around constants

- There seems to be no constant in the models considered in the paper.
- Constants are important: for example, changing inflation target.
- Expanding the model by considering that one variable is always one would introduce an irrelevant unit root and is not attractive.

An issue around constants

A Markov switching linear rational expectation model with constants

$$A_1(\xi_t)E_t(y_{t+1}) + A_2(\xi_t)y_t + A_3(\xi_t)y_{t-1} + B(\xi_t)\epsilon_t + c(\xi_t) = 0$$

Let's assume that the solution has the form (this still needs to be proven)

$$y_t = g_0(\xi_t) + G_1(\xi_t)(y_{t-1} - g_0(\xi_t)) + G_2(\xi_t)\epsilon_t$$

$g_0(\xi_t)$ can be interpreted as the steady state in regime ξ_t.
If the Markov chain is in state j in period t and in state ℓ in period $t+1$, then

$$\begin{aligned} y_{t+1} =& g_0(\ell) + G_1(\ell)\left[g_0(j) - g_0(\ell) + G_1(j)(y_{t-1} - g_0(j)) + G_2(j)\epsilon_t\right] \\ &+ G_2(\ell)\epsilon_{t+1} \end{aligned}$$

An issue around constants

Conditional expectation

As the Markov process is exogenous, ξ_t and ϵ_t are independent and one can write the conditional expectation

$$E_t\left(y_{t+1}|\xi_t = j, \Omega_t\right) = \sum_{\ell=1}^{h} p_{\ell,j}\Big\{g_0(\ell) + G_1(\ell)\Big[\quad g_0(j) - g_0(\ell) + G_1(j)\left(y_{t-1} - g_0(j)\right) + G_2(j)\epsilon_t\Big]\Big\}$$

An issue around constants

Plugging the solution in the model

The model must be true in each state j of the Markov chain, and we have the following set of nonlinear equations in $G_1(j)$ and $G_2(j)$, $j = 1, \ldots, h$.

$$
\begin{aligned}
A_1(j) \sum_{\ell=1}^{h} p_{\ell,j} \Big\{ g_0(\ell) + G_1(\ell) \Big[g_0(j) - g_0(\ell) + G_1(j)\left(y_{t-1} - g_0(j)\right) \\
+ G_2(j)\epsilon_t \Big] \Big\} + A_2(j)\left(g_0(j) + G_1(j)\left(y_{t-1} - g_0(j)\right) + G_2(j)\epsilon_t\right) \\
+ A_3(1)y_{t-1} + B(1)\epsilon_t + c(j) = 0
\end{aligned}
$$

Remarks

Contrarily to the case with a single regime, the steady–state vectore, $g_0(j)$, can't be computed independently and before the multipliers $G_1(j)$.

Application to the estimation of DSGE models

- This paper applies the proposed method only to calibration.
- It is interesting to apply it to the parameter estimation in DSGE models. Is it implementable? Computaional Time?
- If implementable, there are many interesting exercises.
  - Compare the difference in the parameter estimates.
    (1) Using the proposed method.
    (2) Assuming that the agents behaved as if the current regime were to last forever.
  - Estimation of the number and the dates of structural changes (Chib (1998), Kim and Nelson (1999)).
  - Bayesian counterfactural analysis for the cause of the Great Moderation (Kim, Morley and Piger (2007)).
  - Duration dependence (Kim and Nelson (1999)).