A Dark Side of Patent Pools with Compulsory Independent Licensing [*]

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1 Introduction

This paper examines the roles of patent pools with compulsory independent licensing. It is now well known that patent pools have positive roles for economic welfare. If patent pools include substitute patents, however, they may decrease the total welfare, since they become a mechanism for promoting the collusive behaviors of patent holders. A seminal work by Lerner and Tirole (2004) have shown that requiring independent licensing or compulsory independent licensing is a useful tool to select only desirable patent pools. They have shown that by requiring independent licensing, only welfare improving patent pools are stable, and welfare decreasing patent pools, in which substitutable patents are included, become unstable.

Requiring independent licensing may have another positive role. By supplying independent licensing, those users who demand only a part of the pooled technology get benefit. For example, Commission Notice Guidelines on the application of Article 81 of the EC Treaty to technology

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transfer agreements (2004/C 101/02) states that "in cases where the pooled technologies have different applications some of which do not require

use of all of the pooled technologies, the pool offers the technologies only as a single package or whether it offers separate packages for distinct applications. In the latter case it is avoided that technologies which are not essential to a particular product or process are tied to essential technologies". (222 (c)).

In this paper, however, we are going to show that this intuition is not correct. The above argument is implicitly assuming that patent pools ignore the users who demand a part of the pooled technologies. If the pools price by consider such users, they argument becomes quite different. Without independent licensing, the price of the pooled technology becomes low to promote the independent users. On the other hand, if the independent licensing is required, the price for the patent pool can be higher, since the profit from the independent market is derived by the independent licensing. In other words, the compulsory independent licensing gives a tool for price discrimination for the patent holders. It is well known that such price discrimination is welfare decreasing under some conditions. Hence, independent licensing has a negative impact for economic welfare.

2 The Model

There are two patent (or license) holders, denoted by A and B respectively, and many potential consumers for their patents. Consumers are classified to the following three types: (i) those who demand only patent A, (ii) those who demand only patent B, or (iii) those who treat patent A and B are perfect complements and demand both A and B. We will call the market of type (i) and (ii) the "single market" and that of type (iii) the "bundled market". The demand function of the single market is given by

$$D(p_i) = \begin{cases} 1 - p_i & \text{if} \quad p_i \in [0, 1] \\ 0 & \text{if} \quad p_i > 1 \end{cases}$$

for $i = A, B$ where $p_i \geq 0$ is the price of patent i.[1)] We assume, for simplicity, that the cost for supplying the patent i is normalized to be zero. Hence the profit from the single market of patent i is written as $\Pi(p_i) = p_i D(p_i)$. On the other hand, the demand of the bundled market is given by

$$\overline{D}(P) = \begin{cases} a - bP & \text{if} \quad P \in [0, a/b] \\ 0 & \text{if} \quad P > a/b \end{cases}$$

where $P \geq 0$ is the price to obtain both patent A and B. We assume $a > 0$, and $b > 0$. Each of the patent holders maximizes the sum of the profits from these markets. The profit from the bundled market depends on the licensing process. Here we consider the following three paterns of licensing strategies.

Competitive Licensing (C) The patent holders do not form the patent pool and they simultaneously and noncooperatively choose their own patent price p_i. Given p_A and p_B, consumers in the bundled market face the sales price $p_A + p_B$ to get both of the patents. Thus the demand function of the bundled market is given by $\overline{D}(p_A + p_B)$. The patent holder i gets the profit $\Pi(p_i)$ from her own single market and $\overline{\Pi}_i^C(p_i, p_j) := p_i \overline{D}(p_i + p_j)$ $(j \neq i)$ from the bundled market. Patent holder i maximizes the sum of them, $\Pi(p_i) + \overline{\Pi}_i^C(p_i, p_j)$ given p_j for $j \neq i$.

Patent Pool (P) The patent holders form the patent pool and jointly choose the price for the bundled patents to maximize their joint profit. Specifically, if the patent holders choose P as the price for the bundled patents, consumers in all the markets, including each of the single markets, face P as the sales price. We assume that the profit gained from the pool is equally divided into the patent holders (as Lerner and Tirole (2004)). Thus the profit from the single market is $\Pi(P)$ and that of the bundled market is $\overline{\Pi}^P(P)/2 := P\overline{D}(P)/2$, and then the joint profit is given as $\Pi(P) + \overline{\Pi}^P(P)/2$.

[1)] Because we assume symmetry on the single markets, the demand function itself does not depend on i.

Patent Pool with Independent Licensing (I) It is a two stage game. First, the patent holders form the patent pool and choose the price of bundled patents P. Second, each of them simultaneously and noncooperatively chooses her patent price p_i.Since the profit function for each patent holder is a little complicated in this case, we examine the profit function from each market carefully.

In the single market, consumers who only demand the patent i has two options, to purchase the independent patent i or to purchase the bundled patent. Hence, as long as p_i is lower than the price for the bundled patent, P, the each patent holder gets $\Pi(p_i)$. On the other hand, if $p_i \geq P$, the patent pool supplies the bundled patent, and each patent holder gets the profit $\Pi(P)/2$. Hence, the profit of the patent holder i from a single markets is[2)]

$$\begin{cases} \Pi(p_i) & \text{if} \quad p_i < P \\ \dfrac{1}{2}\Pi(P) & \text{if} \quad p_i \geq P. \end{cases}$$

Similarly, in the bundled market, consumers decide from whom they purchase the patents, by comparing P and $p_A + p_B$. The profit of patent holder i from the bundled market is then

$$\begin{cases} \overline{\Pi}_i^C(p_i, p_j) & \text{if} \quad p_i + p_j < P \\ \dfrac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad p_i + p_j \geq P. \end{cases}$$

In the second stage, given the pool price P, the patent holder i chooses p_i so as to maximize the

[2)] Assume that if indifferent between purchasing patents from the individual and the pool, a consumer goes to the pool. In what follows, we adopt this manner.

sum of them;

$$
\Pi_i^I(p_i, p_j | P) := \begin{cases} \Pi(p_i) + \overline{\Pi}_i^C(p_i, p_j) & \text{if} \quad p_i < P - p_j \\ \Pi(p_i) + \dfrac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad 0 < P - p_j \leq p_i < P \\ \Pi(p_i) + \dfrac{1}{2}\Pi(P) + \dfrac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad P - p_j \leq 0 \leq p_i < P \\ \dfrac{1}{2}\Pi(P) + \dfrac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad p_i \geq P > p_j \\ \Pi(P) + \dfrac{1}{2}\overline{\Pi}^P(P) & \text{if} \quad \min\{p_A, p_B\} \geq P. \end{cases}
$$

Let $(p_A^*(P), p_B^*(P))$ be a Nash equilibrium price in the second stage given P. In the first stage, the patent holders decide P to achieve the Pareto optimum profits. Formally, we say that price of the pool P^* is supported by a subgame perfect equilibrium if there does not exist P such that

$$
\begin{aligned}
\Pi_A^I(p_A^*(P), p_B^*(P)|P) &\geq \Pi_A^I(p_A^*(P^*), p_B^*(P^*)|P^*) \\
\Pi_B^I(p_A^*(P), p_B^*(P)|P) &\geq \Pi_B^I(p_A^*(P^*), p_B^*(P^*)|P^*)
\end{aligned}
$$

where at least either of the inequalities strictly holds.

Recall that the monopoly price in the single market is $1/2$ and that in the bundle market is $a/2b$. Since it is natural to assume that the market size of the bundle market is larger than that of the single market, we assume that $1/2 < a/2b$. In other words, $b < a$. Moreover, we consider the situation in which the single market is sufficiently large and the patent pool does not exclude or ignore the single market. Thus it is natural to assume $a/2b < 1$ or $a < 2b$. In the following analysis, we assume;

Assumption 1 *$b < a < 2b$.*

3 Equilibrium

3.1 Competitive Licensing

We first consider the equilibrium under the competitive licensing. For patent holder i, given the opponent price p_j, the profit is $\Pi(p_i) + \overline{\Pi}_i^C(p_i, p_j)$. In particular, for $p_i \in [0, \min\{1, a/b - p_j\}]$ the profit function becomes

$$\max_{p_i} p_i(1 - p_i) + p_i(a - b(p_i + p_j)).$$

The equilibrium pricing strategy is derived by the first-order condition, $1 + a - bp_j - 2(1 + b)p_i = 0$. Rigorously, we have to care about the case where $p_i > \min\{1, a/b - p_j\}$, but the proof of the following proposition shows that the equilibrium solution can be derived directly from the first-order condition.

Proposition 1 *The equilibrium of licensing strategy (C) is* $p_1 = p_2 = p^C := (1 + a)/(2 + 3b)$.

Proof *See the appendix.* ■

Since each patent holder consider both the single market and the bundle market, the equilibrium price is different from the monopoly price 1/2. We can easily show $p^C < 1/2$, that is the single market price becomes lower by the existence of the bundle market. On the other hand, the equilibrium price for the bundle market $2p^C$ is higher than the monopoly price $a/2b$.

3.2 Patent Pool

In the case of patent pool, the patent holders jointly maximize their aggregate profit $2\Pi(P)+\overline{\Pi}^P(P)$ by choosing P, the price for the bundled patent. In particular, for $P \in [0, 1]$, the aggregate profit is $2(1 - P)P + (a - bP)P$. It can be verified that its first order condition satisfies the optimal price while we have to carefully check the case where $P > 1$.

Proposition 2 *When the licensing strategy is (P), the patent holders choose the price as* $P^P := (2 + a)/2(2 + b)$.

Proof *See the appendix.* ■

We obtain that $P^P < a/2b$, which means that existence of the single markets pushes the price down from the monopoly price of the bundled market. If the patent holders maximize the profit only in the bundled market, they choose the monopoly price in the bundled market $a/2b$. Here, however, the patent holders account for not only the bundled market but also each of the single markets, and reduce the price to P^P.

Moreover, we can see tht $P^P < 2p^C$, that is the equilibrium price for the bundled market under the patent pool is lower than that under the competitive licensing. In other words, the total surplus from the bundled market becomes higher by formulating the patent pool. The reason is simple. Those two patents are complements for the bundled market, and thus we can aviod the double marginalization problem by formulating the patent pool. This logic is wellknown in the literature.

3.3 Patent Pool with Independent Licensing

In this game, first the patent holders form the pool and choose the price of bundled patents P and given P, each of the patent holders noncooperatively and simultaneously choose her own patent price p_i.

Recall that the monopoly prices are $1/2$ in the single markets and $a/2b$ in the bundled market. Then, given the pool's price $P = a/2b$, if each of the patent holders chooses $1/2$ as her own patent price, consumers in the bundled market purchase the patents from the patent pool and those in the single market purchase the patent from the patent holder individually. It means that the patent holders can keep their monopoly profit in all of the markets. We can show that such a pair of the prices is supported by a subgame perfect equilibrium.[3)]

Proposition 3 *In licensing strategy (I), the price pair of $p_A = p_B = p^I := 1/2$ and $P = P^I := a/2b$ are supported by a subgame perfect equilibrium. Furthermore, this is a unique symmetric*

[3)] We do not exclude the possibility of the asymmetric equilibrium. However, we believe that the fact that the monopoly prices are chosen in all the market and then the joint profits are maximized is enough for the focal point.

equilibrium.

Proof *See the appendix.* ■

The intuitive reason of this result is quite simple. In the case of patent pool (without independent licensing), the price P must be lower than the monopoly price $a/2b$ to attract the consumers of the single market. In the case of the patent pool with independent licensing, each patent holder can attract the users of the single market by the independent licensing. Hence the price for the bundled market can be equal to the monopoly price, $a/2b$.

4 Welfare Comparison

Now that we have derived the equilibrium price in each licensing strategy, we will demonstrate the welfare comparison among them.

From the equilibrium price, the supply quantity can be immediately computed as follows;

$$\begin{aligned} q^C &:= D\left(p^C\right) = \frac{1+3b-a}{2+3b}, & Q^C &:= \overline{D}\left(2p^C\right) = \frac{ab+2a-2b}{2+3b}, \\ q^P &:= D\left(P^P\right) = \frac{2-a+2b}{2(2+b)}, & Q^P &:= \overline{D}\left(P^P\right) = \frac{4a-2b+ab}{2(2+b)}, \\ q^I &:= D\left(p^I\right) = \frac{1}{2}, & Q^I &:= \overline{D}\left(P^I\right) = \frac{a}{2}. \end{aligned}$$

Let w and W (with the corresponding superscript) be the social welfare of (one of) the single markets and that of the bundled market, which can be computed as follows[4];

$$w^\ell := \int_0^{q^\ell} D^{-1}(q)dq = q^\ell - \frac{q^{\ell 2}}{2}, W^\ell := \int_0^{Q^\ell} \overline{D}^{-1}(Q)dQ = \frac{1}{b}\left(aQ^\ell - \frac{Q^{\ell 2}}{2}\right),$$

for $\ell = C, P, I$. Then the aggregate social welfare is the sum of the welfare of the single markets and of bundle markets, that is, $W^\ell + 2w^\ell$. By comparing this value among $\ell = C, P, I$, we obtain the order of the welfare.

[4] $D(\cdot)^{-1}$ and $\overline{D}(\cdot)^{-1}$ are the inverse functions with domain $[0,1]$ and $[0,a]$, respectively.

Proposition 4 $W^P + 2w^P > W^I + 2w^I > W^C + 2w^C$.

This result shows that the total welfare can be improved by only allowing the patent pool. This result is natural since the price for the bundled market can be lower by formulating the patent pool. On the other hand, by requiring the indipendent licensing, the total welfare must be decreased.

A Proofs

A.1 Proof of Proposition 1

Suppose that the equilibrium satisfies $p_A + p_B \geq a/b$. Then on the equilibrium there are no demand on the bundled market and patent holder i's profit would be $p_i(1-p_i)$ if $p_i \in [0, a]$ and 0 if $p_i > 1$. If $p_i \neq 1/2$, its profit can be improved by choosing $p_i = 1/2$. Thus in this case p_i must be 1 / 2 for $i = A, B$. However, it implies that $p_A + p_B = 1$ which is strictly less than a/b by Assummption 1. Thus it cannot be the case that $p_A + p_B \geq a/b$ on equilibria.

Then an equilibrium must satisfy $p_A + p_B < a/b$ and patent holder i's profit would be

$$\begin{cases} (1-p_i)p_i + (a - b(p_i + p_j))p_i & \text{if} \quad p_i \in [0, 1] \\ (a - b(p_i + p_j))p_i & \text{if} \quad p_i \in [1, a/b - p_j] \\ 0 & \text{if} \quad p_i \geq a/b - p_j. \end{cases}$$

Now we will investigate the optimal choice of p_i given p_j. For $p_i \in [1, a/b - p_j]$, the derivative of $(a - b(p_i + p_j))p_i$ is $a - 2bp_i - bp_j$ which satisfies, by the restriction of p_i,

$$a - 2bp_i - bp_j \leq a - 2b \cdot 1 - b \cdot 0 = a - 2b < 0.$$

Then for $p_i \in [1, a/b - p_j]$, the profit is decreasing in p_i which means that

$$\max_{p_i \in [1, a/b - p_j]} (a - b(p_i + p_j))p_i = (a - b(1 + p_j)).$$

We will now show that it is less than $\max_{p_i \in [0,1]}[(1-p_i)p_i + (a - b(p_i + p_j))p_i]$ which implies that patent holder i chooses $p_i \in [0,1]$ rather than $p_i \in [1, a/b - p_j]$. Let $R(p_j) := (1 + a - bp_j)/2(1+b)$ which satisfies the first order condition with respective to p_i when $p_i = R(p_j)$. Note that since $R(p_j)$ is decreasing in p_j,

$$R(p_j) \leq R(0) = \frac{1+a}{2(1+b)} \leq \frac{1+2b}{2(1+b)} < 1$$

which means that if $p_i = R(p_j)$, the profit would be optimized for the case where $p_i \in [0,1]$. It follows that

$$(1 - R(p_j))R(p_j) + (a - b(R(p_j) + p_j))R(p_j) = \max_{p_i \in [1, a/b - p_j]} (a - b(p_i + p_j))p_i > (a - b(1 + p_j)).$$

Finally note that the profit when $p_i = R(p_j)$ is positive which implies that it is better to choose $p_i = R(p_j)$ than any p_i larger than $a/b - p_j$. Therefore the best response of patent holder i is $p_i = R(p_j)$.

Then, by symmetry between patent holder A and B, the equilibrium must satisfy $p_A = R(p_B)$ and $p_B = R(p_A)$. those are linear equations and the solution is simply given by $p_A = p_B = (1+a)/(2+3b)$. Note that $p_A + p_B = 2(1+a)/(2+3b)$ and

$$\frac{a}{b} - \frac{2(1+a)}{2+3b} = \frac{2(a-b) + ab}{(2+3b)b} > 0$$

which is consistent with the hypothesis that $p_A + p_B < a/b$.

A.2 Proof of Proposition 2

The objective function can be described as following

$$\begin{cases} 2(1-P)P + (a-bP)P & \text{if} \quad P \in [0,1] \\ (a-bP)P & \text{if} \quad P \in [1, a/b] \\ 0 & \text{if} \quad P \in [a/b, \infty). \end{cases}$$

First consider

$$\max_P 2(1-P)P + (a-bP)P \quad \text{s.t.} \quad P \in [0,1].$$

The first orede condition gives us the solution $P^P = (2+a)/2(2+b) \in (0,1)$ Notice that to choose P^P yields positive profits which implies that to choose $P \in [a/b, \infty)$ is never optimal. Furthermore since arg $\max_P (a-bP)P = a/2b$, we obtain

$$\begin{aligned} 2(1-P^P)P^P + (a-bP^P)P^P &\geq 2\left(1-\frac{a}{2b}\right)\frac{a}{2b} + \left(a - b\left(\frac{a}{2b}\right)\right)\frac{a}{2b} \\ &> \left(a - b\left(\frac{a}{2b}\right)\right)\frac{a}{2b} \\ &= \max_P (a-bP)P \end{aligned}$$

which means that to choose $P \in [1, a/b]$ is never optimal neither. Therefore the solution is P^P.

A.3 Proof of Proposition 3

Fix the pool's price $P = a/2b$. Then given the opponent price $p_j = 1/2$, the profit of patent holder i is

$$\Pi_i^I\left(p_i, \frac{1}{2} \,\Big|\, \frac{a}{2b}\right) := \begin{cases} (1-p_i)p_i + \left(a - b\left(p_i + \frac{1}{2}\right)\right)p_i & \text{if} \quad p_i < \frac{a}{2b} - \frac{1}{2} \\ (1-p_i)p_i + \frac{a^2}{8b} & \text{if} \quad \frac{a}{2b} - \frac{1}{2} \leq p_i < \frac{a}{2b} \\ \frac{1}{2}\left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{a^2}{8b} & \text{if} \quad p_i \geq \frac{a}{2b}. \end{cases}$$

First we show that to choose $p_i \in [a/2b, \infty)$ is never optimal. To see it, notice that $a/2b - 1/2 < 1/2 < a/2b$. Then if $p_i = 1/2$, the profit is

$$\begin{aligned} \frac{1}{4} + \frac{a^2}{8b} &= \max_{p_i} [(1-p_i)p_i] + \frac{a^2}{8b} \\ &\geq \left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{a^2}{8b} \\ &> \frac{1}{2}\left(1 - \frac{a}{2b}\right)\frac{a}{2b} + \frac{a^2}{8b} \end{aligned}$$

the right hand side of which is the profit when $p_i \geq [a/2b, \infty)$.

Thus in order to show that $p_i = 1/2$ is the best response, it suffices to show that $1/4 + a^2/8b \geq (1-p_i)p_i + (a - b(p_i + 1/2))p_i$ for all $p_i \in [0, a/2b - 1/2)$. Let $H(p_i) := (1-p_i)p_i + (a - b(p_i + 1/2))p_i$. It is easy to see that $H(p_i)$ is increasing in p_i for any $p_i \in [0, a/2b - 1/2]$, because

$$\begin{aligned} \frac{\partial H(p_i)}{\partial p_i} &= 1 + a - \frac{b}{2} - 2(1+b)p_i \\ &\geq 1 + a - \frac{b}{2} - 2(1+b)\left(\frac{a}{2b} - \frac{1}{2}\right) \\ &= 2 - \frac{a}{b} + \frac{b}{2} > 0 \end{aligned}$$

and it follws that $H(a/2b - 1/2) > H(p_i)$ for any $p_i \in [0, a/2b - 1/2)$. Thus if $1/4 + a^2/8b > H(a/2b - 1/2)$, the proof is completed. It is actually true because

$$\begin{aligned} \frac{1}{4} + \frac{a^2}{8b} - H\left(\frac{a}{2b} - \frac{1}{2}\right) &= \frac{1}{4} + \frac{a^2}{8b} - \left(1 - \frac{a}{2b} + \frac{1}{2}\right)\left(\frac{a}{2b} - \frac{1}{2}\right) - \left(a - b\left(\frac{a}{2b} - \frac{1}{2} + \frac{1}{2}\right)\right)\left(\frac{a}{2b} - \frac{1}{2}\right) \\ &= \frac{1}{4} + \frac{a^2}{8b} - \frac{1}{4}\left(3 - \frac{a}{b}\right)\left(\frac{a}{b} - 1\right) - \frac{a}{4}\left(\frac{a}{b} - 1\right) \\ &= \frac{1}{4} + \frac{a^2}{8b} - \frac{1}{4}\left[-\left(\frac{a}{b} - 2\right)^2 + 1\right] - \frac{a^2}{4b} + \frac{a}{4} \\ &= \frac{1}{4}\left(\frac{a}{b} - 2\right)^2 + \frac{a}{4}\left(1 - \frac{a}{2b}\right) > 0. \end{aligned}$$

Now we have shown that given $P = a/2b$, $p_A = p_B = 1/2$ is the equilibrium behavior in the second stage. Since both of the prices are monopoly ones in each of the markets, the attained profit must be Pareto optimal. Then this pair of the prices is supported by a subgame perfect equilibrium.

Finally, in the class of symmetric prices, any other pair of the price attains the payoff less than the pair of p^I and P^I. No other pair of the price is chosen on equilibrium.

A.4 Proof of Proposition 4

The proof consists of two claims.

Step 1: $W^P + 2w^P > W^I + 2w^I$. It is easy to verify that $Q^P > Q^I$ and $q^P < q^I$ which imply that

$$\begin{aligned}
W^P - W^I &= \frac{1}{b}\left[a(Q^P - Q^I) - \left(\frac{{Q^P}^2}{2} - \frac{{Q^I}^2}{2}\right)\right] \\
&= \frac{1}{b}(Q^P - Q^I)\left[a - \frac{1}{2}(Q^P + Q^I)\right] \\
&> \frac{1}{b}(Q^P - Q^I)\left[a - \frac{1}{2}(Q^P + Q^P)\right] \\
&= \frac{1}{b}(Q^P - Q^I)(a - Q^P) \\
&= P^P(Q^P - Q^I)
\end{aligned}$$

and

$$\begin{aligned}
w^P - w^I &= \left[q^P - q^I - \frac{{q^P}^2}{2} + \frac{{q^I}^2}{2}\right] \\
&= (q^P - q^I)\left[1 - \frac{1}{2}(q^P + q^I)\right] \\
&= -(q^I - q^P)\left[1 - \frac{1}{2}(q^P + q^I)\right] \\
&> -(q^I - q^P)\left[1 - \frac{1}{2}(q^P + q^P)\right] \\
&= -(q^I - q^P)(1 - q^P) \\
&= -P^P(q^I - q^P).
\end{aligned}$$

Then

$$\begin{aligned} W^P + 2w^P - W^I - 2w^I &> P^P(Q^P - Q^I) - 2P^P(q^I - q^P) \\ &= P^P(q^I - q^P)\left[\frac{Q^P - Q^I}{q^I - q^P} - 2\right]. \end{aligned} \tag{1}$$

Finally,

$$\frac{Q^P - Q^I}{q^I - q^P} = \frac{\dfrac{4a - 2b + ab}{2(2+b)} - \dfrac{a}{2}}{\dfrac{1}{2} - \dfrac{2 - a + 2b}{2(2+b)}} = 2$$

which implies that equation (1) is 0.

Step 2: $W^I + 2w^I > W^C + 2w^C$ Recall that since there is no production cost on the supply side, the welfare unambiguously goes up when the equilibrium quantity is increasing. Now it is easy to check that $Q^I > Q^C$. Then if $q^I \geq q^C$, the result is immediately established. Notice that $q^I \geq q^C$ if and only if $3b \leq 2a$. Then suppose that $3b > 2a$ and then $q^I < q^C$.

Since $Q^I > Q^C$,

$$\begin{aligned} W^I - W^C &= \frac{1}{b}\left[a(Q^I - Q^C) - \left(\frac{{Q^I}^2}{2} - \frac{{Q^C}^2}{2}\right)\right] \\ &= \frac{1}{b}(Q^I - Q^C)\left[a - \frac{1}{2}(Q^I + Q^C)\right] \\ &> \frac{1}{b}(Q^I - Q^C)\left[a - \frac{1}{2}(Q^I + Q^I)\right] \\ &= \frac{1}{b}(Q^I - Q^C)(a - Q^I) \\ &= P^I(Q^I - Q^C) \\ &= \frac{a}{2b}(Q^I - Q^C) \end{aligned}$$

and since $q^I < q^C$,

$$\begin{aligned} w^I - w^C &= \left[q^I - q^C - \frac{q^{I2}}{2} + \frac{q^{C2}}{2} \right] \\ &= (q^I - q^C) \left[1 - \frac{1}{2}(q^I + q^C) \right] \\ &= -(q^C - q^I) \left[1 - \frac{1}{2}(q^I + q^C) \right] \\ &> -(q^C - q^I) \left[1 - \frac{1}{2}(q^I + q^I) \right] \\ &= -(q^C - q^I)(1 - q^I) \\ &= -p^I(q^C - q^I) \\ &= -\frac{1}{2}(q^C - q^I). \end{aligned}$$

Then

$$\begin{aligned} W^I + 2w^I - W^C - 2w^C &> \frac{a}{2b}(Q^I - Q^C) - (q^C - q^I) \\ &= \frac{a}{2b}(q^C - q^I) \left[\frac{Q^I - Q^C}{q^C - q^I} - \frac{2b}{a} \right]. \end{aligned} \tag{2}$$

Finally,

$$\begin{aligned} \frac{Q^I - Q^C}{q^C - q^I} - \frac{2b}{a} &= \frac{\frac{a}{2} - \frac{ab + 2a - 2b}{2 + 3b}}{\frac{1 + 3b - a}{2 + 3b} - \frac{1}{2}} - \frac{2b}{a} \\ &= \frac{ab - 2a + 4b}{3b - 2a} - \frac{2b}{a} \\ &= \frac{a^2 b + 2(3b - a)(a - b)}{a(3b - 2a)} > 0 \end{aligned}$$

which implies that equation (2) is greater than 0.

References

Lerner, J. and J. Tirole (2004): "Efficient Patent Pools," *American Economic Review*, 94, 691-711.