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Scoring Auction by an Informed Principal

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Abstract

This paper considers a scoring auction used in procurement. In this auction, each supplier offers both price and quality, and a supplier whose offer achieves the highest score wins. The environment we consider has two features: the buyer has private information and quality is multi-dimensional. We show that a scoring auction implements the *ex ante* optimal mechanism for the buyer when the value complementarity between quality attributes is sufficiently greater than the cost substitutability. We further show how the buyer should design scoring rules.

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1 Introduction

Auction rules of public procurement have changed from single-dimensional bidding to multi-dimensional bidding. The procurement authorities have conventionally adopted price-only auctions, in which the authorities award procurement contracts based only on contractors' price-bids. A weakness of price-only auction is that the procurement authority prespecifies the quality level of product, which does not reflect technological information held by contractors. As an alternative rule, a scoring auction becomes increasingly popular. In this auction, each contractor offers both price and quality, and these offers are evaluated using a scoring rule announced by the authority. This auction allows the authority to choose the winner based not only on price-bids but also on quality-bids. In 2004, the European Union adopted a new public procurement directive, which, in effect, mandates the use of scoring auctions (Asker and Cantillon, 2008).

How effectively does a scoring auction work? How should the procurement authority design a scoring rule? They are a matter of great concern to the authorities because public projects have significant impact on society. There are, however, some difficulty in designing a scoring rule. For instance, consider the construction of a bridge. The authority should care about many attributes of the project such as building materials, a method of construction, a time for completion, and so on. Moreover, although each contractor has technological information, the authority may possess superior information about the value of each attribute. These issues render the design of scoring rule complicate.

The previous studies have confirmed the high performance of a scoring auction for the buyer. A common feature of these models is that a buyer procures a single product differentiated by its quality from one of suppliers, who have private information about production costs. In a seminal article, Che (1993) shows that a scoring auction with a

properly designed scoring rule implements the buyer's optimal mechanism (characterized by Laffont and Tirole (1987)). Branco (1997) extends this result to an environment where each supplier's production cost has a common-cost component, so that his cost is correlated with the other suppliers' costs. Asker and Cantillon (2008) consider a fully general environment where both the supplier's type and quality are multi-dimensional, and the buyer also has private information about her taste for quality. Although their main results are the characterization of equilibrium bidding behavior and the expected utility equivalence theorem, they also show that the scoring auction outperforms some other mechanisms including a price-only auction. On the other hand, they have not examined whether a scoring auction implements the optimal outcome for the buyer. The main reason is that it is extremely difficult to characterize the optimal mechanism when the supplier has multi-dimensional private information. However, Asker and Cantillon (2010) characterize the optimal mechanism in a specific environment where each supplier's type consists of two parameters (fixed cost and marginal cost) and each parameter is a binary random variable. They show that the scoring auction yields a performance close to that of the optimal mechanism, taking a numerical simulation approach. In addition to these theoretical studies, there is experimental evidence supporting the high performance of scoring auction compared to that of price-only auction (Bichler, 2000; Chen-Ritzo, 2005).

As explained above, it is an important research question whether a scoring auction, which becomes increasingly prevalent in practice, implements the optimal outcome for the buyer. This paper shows that the positive result of Che (1993) can be extended to an environment where the buyer has private information, and quality is multi-dimensional. We assume that each supplier's type is single-dimensional because Asker and Cantillon

(2010) have already obtained the negative result (i.e. a scoring auction cannot implement the optimal mechanism) in an environment where the supplier's type is two-dimensional. We proceed in two steps. First, we characterize the *ex ante* optimal mechanism for the buyer, following the approach in the informed-principal literature; see Myerson (1983), Maskin and Tirole (1990), Tan (1996), and Mylovanov and Tröger (2008). Second, we show that a scoring auction implements the *ex ante* optimal mechanism. We then characterize the optimal scoring rules, and discuss some problems of scoring rules used in practice.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the equilibrium bidding strategy. Section 4 shows that a scoring auction implements the *ex ante* optimal mechanism. Section 5 concludes. All proofs are in the Appendix.

2 The model

Consider a buyer who procures a single product from one of N suppliers. A (production) *contract* between the buyer and a supplier $i \in \{1, \dots, N\}$ is denoted by $(p_i, \mathbf{q}_i) \in \mathbb{R}_+ \times Q$, under which the supplier i must deliver a product of *quality* $\mathbf{q}_i = (q_i^1, \dots, q_i^M) \in Q \equiv \prod_{m=1}^M [0, \bar{q}^m]$ in exchange for price $p_i \in \mathbb{R}_+$; for $m \in \{1, \dots, M\}$, each q_i^m represents a non-monetary attribute.¹ The buyer's taste parameter for quality is given by $t \in [\underline{t}, \bar{t}] \subset \mathbb{R}$, and the supplier i 's cost parameter for quality by $\theta_i \in [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$. These players' types t and $\boldsymbol{\theta} = (\theta_1, \dots, \theta_N)$ are random variables which are independent across players. The cumulative distribution function of t is given by G . The cumulative distribution function

¹In this paper, bold letters denote some vectors. We say that $\mathbf{q} \geq \hat{\mathbf{q}}$ if $q^m \geq \hat{q}^m$ for all m , and $\mathbf{q} \gg \hat{\mathbf{q}}$ if $q^m > \hat{q}^m$ for all m .

of θ_i is given by F , with a density function f that is continuous and strictly positive everywhere. Each player has private information about the realized type respectively, but the prior probability distributions are common knowledge.

The buyer of type t obtains utility $v(\mathbf{q}, t) - p$ from a contract (p, \mathbf{q}) , where $v(\mathbf{q}, t)$ is her valuation for a product of quality \mathbf{q} . The supplier i of type θ_i earns profits $p - c(\mathbf{q}, \theta_i)$ from a contract (p, \mathbf{q}) , where $c(\mathbf{q}, \theta_i)$ is his production cost. We assume that v and c are three times continuously differentiable in all arguments. We also make the following assumptions.²

Assumption 1. $v_m > 0, c_m > 0$, for all m .

Assumption 2. $v_t > 0, v_{mt} > 0$, for all m .

Assumption 3. $c_\theta > 0, c_{m\theta} > 0$, for all m .

Assumption 4. c and c_m are weakly convex in θ , for all m .

Assumption 5. $\frac{F}{f}$ is increasing in θ .

Assumptions 3, 4, and 5 ensure that the function of “virtual surplus” has strictly decreasing differences in (\mathbf{q}, θ) .

There is an auction rule (mechanism) that is feasible for the buyer: a *scoring auction*. We first define a *scoring rule* as $S : \mathbb{R}_+ \times Q \rightarrow \mathbb{R} \cup \{-\infty\}$. In a scoring auction, each supplier offers both price and quality, and the function S assigns a score $S(p, \mathbf{q})$ to each price-quality pair (p, \mathbf{q}) . One can interpret this real-valued function as representing a preference relation over price-quality pairs. We assume that S is continuous in (p, \mathbf{q}) such that $S(p, \mathbf{q}) > -\infty$. The buyer can set a *reserve score*, which is normalized to zero. Then, a supplier i wins only if his score is nonnegative and the highest among suppliers.³

²Subscripts denote partial derivatives, i.e. $v_t = \partial v / \partial t$, $v_m = \partial v / \partial q^m$, $v_{mm'} = \partial^2 v / \partial q^m \partial q^{m'}$. We say that $v_m > 0$ if $v_m(\mathbf{q}, t) > 0$ for all \mathbf{q}, t . We use the same notation for other functions.

³We assume that if there is a tie, then each supplier achieving the nonnegative highest score wins with equal probability. All results hold for any other tie-breaking rule.

We consider a *first-score (sealed-bid)* format, in which the winner i is awarded a binding contract (p_i, \mathbf{q}_i) he offered in the auction; the first-score format corresponds to a first-price format in a standard price-only auction. We focus on a *quasi-linear* scoring rule S , in which the function takes a form of $S(p, \mathbf{q}) = s(\mathbf{q}) - p$. It will be shown in Section 4 that a scoring auction with properly designed quasi-linear rules implements the optimal mechanism for the buyer. Let $\mathcal{S} \equiv \{S \mid S \text{ is quasi-linear}\}$ be the set of feasible scoring rules.

The *auction game* proceeds as follows. In the first stage, all players' types $(\boldsymbol{\theta}, t)$ are realized, and the players are privately informed about their own types respectively. In the second stage, the buyer publicly announces a scoring rule $S \in \mathcal{S}$. In the third stage, each supplier i simultaneously and independently submits an offer (p_i, \mathbf{q}_i) . Then, the game ends. When a supplier i of type θ_i who offers (p_i, \mathbf{q}_i) such that $S(p_i, \mathbf{q}_i) = \max_j S(p_j, \mathbf{q}_j) \geq 0$ wins, he receives $p_i - c(\mathbf{q}_i, \theta_i)$, the other suppliers receive zero payoffs, and the buyer of type t receives $v(\mathbf{q}_i, t) - p_i$. When no supplier wins, all players receive zero payoffs.

In the following sections, we explore the (pure strategy) perfect Bayesian equilibrium of the game. The buyer's strategy is a choice of auction rule S , depending on her type t . With a slight abuse of notation, we denote a supplier i 's bidding strategy by $(p_i, \mathbf{q}_i) : [\underline{\theta}, \bar{\theta}] \times \mathcal{S} \rightarrow \mathbb{R}_+ \times Q$. A supplier i 's posterior belief about the buyer's type conditional on the announced rule S is denoted by the cumulative distribution function G_i^S on $[\underline{t}, \bar{t}]$. Because the players' types are independent, no supplier updates his belief about the other suppliers' types in equilibrium; we also assume that this is the case in any off-equilibrium path.

We finally identify the *ex post efficient outcome*: Given the realized types $(\boldsymbol{\theta}, t)$, a

supplier with the lowest type θ_i among $\boldsymbol{\theta} = (\theta_1, \dots, \theta_N)$ wins the auction, and delivers a product of the efficient quality level $\tilde{\mathbf{q}}(\theta_i, t) \in \arg \max_{\mathbf{q} \in Q} [v(\mathbf{q}, t) - c(\mathbf{q}, \theta_i)]$ to the buyer.

We assume that $v(\tilde{\mathbf{q}}(\bar{\theta}, \underline{t}), \underline{t}) - c(\tilde{\mathbf{q}}(\bar{\theta}, \underline{t}), \bar{\theta}) \geq 0$.

3 Equilibrium bidding strategy

In this section, we derive the equilibrium bidding strategy.

Note that a supplier's belief G_i^S about the buyer's type is irrelevant to his bidding behavior; all that matters is the scoring rule S announced by the buyer. This is due to the following two facts. First, the outcome is completely determined by the suppliers' bids, so that the buyer with full commitment power has no chance to affect the outcome after her announcement of the rule. Second, we consider a private-values environment, in which the suppliers' production costs are independent of the buyer's type.

The following lemma characterizes a symmetric equilibrium in the auction, where all suppliers use the same bidding strategy. We assume that no supplier uses weakly dominated strategies. Then, we apply the technique of Che (1993) to prove this lemma.

Lemma 1. *(i) For any scoring rule S , there exists a symmetric equilibrium with a cost parameter $\bar{\theta}^S$ in which the bidding strategy (p^*, \mathbf{q}^*) is determined by the following conditions: For all $\theta \in [\underline{\theta}, \bar{\theta}^S]$,*

$$\mathbf{q}^*(\theta, S) \in \arg \max_{\mathbf{q} \in Q} [s(\mathbf{q}) - c(\mathbf{q}, \theta)]$$

$$p^*(\theta, S) = c(\mathbf{q}^*(\theta, S), \theta) + \int_{\theta}^{\bar{\theta}^S} c_{\theta}(\mathbf{q}^*(z, S), z) \left(\frac{1 - F(z)}{1 - F(\theta)} \right)^{N-1} dz,$$

and for all $\theta \in (\bar{\theta}^S, \bar{\theta}]$, $(p^*(\theta, S), \mathbf{q}^*(\theta, S))$ is an arbitrary one which satisfies $S(p^*(\theta, S), \mathbf{q}^*(\theta, S)) <$

0. *(ii) In the above equilibrium, a supplier wins only if his cost parameter is the lowest among $(\theta_1, \dots, \theta_N)$ and lower than $\bar{\theta}^S$.*

Using a reserve score, the buyer can effectively exclude some suppliers who are more inefficient than a critical type $\bar{\theta}^S$. On the other hand, the equilibrium price offer may not be increasing in θ . This fact implies that a scoring auction with a reserve “price” (not reserve score) may be problematic to the buyer. We will discuss the issue after Proposition 1.

Lemma 1 implies that in equilibrium the most efficient supplier wins provided that his type is lower than $\bar{\theta}^S$. Let $\theta_{(N)} \equiv \min\{\theta_1, \dots, \theta_N\}$ be the lowest cost parameter (first-order statistic), which is also a random variable. We denote the cumulative distribution function and the probability density function of $\theta_{(N)}$ by $F_{(N)}(\theta) = 1 - (1 - F(\theta))^N$ and $f_{(N)}(\theta) = N(1 - F(\theta))^{N-1}f(\theta)$ respectively. Then, the buyer of type t 's expected utility from announcing a scoring rule S is⁴

$$\begin{aligned} U(S | t) &= F_{(N)}(\bar{\theta}^S) E_{\theta_{(N)}} [v(\mathbf{q}^*(\theta_{(N)}, S), t) - p^*(\theta_{(N)}, S) | \theta_{(N)} \leq \bar{\theta}^S] \\ &= \int_{\underline{\theta}}^{\bar{\theta}^S} \left[v(\mathbf{q}^*(\theta, S), t) - c(\mathbf{q}^*(\theta, S), \theta) - c_{\theta}(\mathbf{q}^*(\theta, S), \theta) \frac{F(\theta)}{f(\theta)} \right] f_{(N)}(\theta) d\theta; \end{aligned}$$

the second equality follows from the substitution of $p^*(\theta_{(N)}, S)$ and the interchange of the integrals. We now define the *virtual surplus* as the function $\Phi(\mathbf{q}, \theta, t) \equiv v(\mathbf{q}, t) - c(\mathbf{q}, \theta) - c_{\theta}(\mathbf{q}, \theta) \frac{F(\theta)}{f(\theta)}$. Its value $\Phi(\mathbf{q}, \theta, t)$ is the social surplus generated by trading a product of quality \mathbf{q} between the buyer of type t and a supplier of type θ , minus the sum of information rents paid to the more efficient supplier than θ . Using this virtual surplus, the buyer of type t 's expected utility can be rewritten as

$$U(S | t) = \int_{\underline{\theta}}^{\bar{\theta}^S} \Phi(\mathbf{q}^*(\theta, S), \theta, t) f_{(N)}(\theta) d\theta.$$

⁴In this paper, $E_Y[\cdot]$ represents the expectation operator with respect to the prior distributions of random variables Y .

When $\theta_i \leq \bar{\theta}^S$, a supplier of type θ_i 's expected profit in the auction is given by

$$\begin{aligned}\Pi(S | \theta_i) &= (1 - F(\theta_i))^{N-1} [p^*(\theta_i, S) - c(\mathbf{q}^*(\theta_i, S), \theta_i)] \\ &= \int_{\theta_i}^{\bar{\theta}^S} c_\theta(\mathbf{q}^*(z, S), z) (1 - F(z))^{N-1} dz.\end{aligned}$$

When $\theta_i > \bar{\theta}^S$, a supplier of type θ_i 's profit is zero because his score is negative.

4 Implementation

In this section, we consider the implementation problem. The analysis proceeds in two steps. First, we characterize the optimal mechanism for the buyer, following the approach in the informed-principal literature; see Myerson (1983), Tan (1996), and Mylovanov and Tröger (2008). Second, we examine the implementation of the optimal mechanism via a scoring auction.

In a first step, we begin by considering the following *mechanism-selection game*. This hypothetical game differs from the auction game in Section 2 only in that the buyer is allowed to use any arbitrary mechanism which satisfies individual rationality. After the players know their realized types in the first stage, the buyer announces a general mechanism in the second stage. In the third stage, each player (possibly including the buyer) simultaneously and independently reports a message from the message space specified by the mechanism; a message space must include a disagreement option which ensures zero payoff for each player.

We introduce some definitions. A *direct mechanism* is an N -tuple of measurable functions $\rho = (\rho_1, \dots, \rho_N)$ where $\rho_i = (P_i, \mathbf{Q}_i, X_i) : [\underline{\theta}, \bar{\theta}]^N \times [\underline{t}, \bar{t}] \rightarrow \mathbb{R} \times Q \times [0, 1]$. For each profile of reported types $(\boldsymbol{\theta}, t)$, a transfer schedule $P_i(\boldsymbol{\theta}, t)$ specifies the expected monetary transfer from the buyer to the supplier i , a quality schedule $\mathbf{Q}_i(\boldsymbol{\theta}, t)$ specifies

the quality level the supplier i must achieve when delivering the product, and $X_i(\boldsymbol{\theta}, t)$ specifies the trading probability between the buyer and the supplier i . A direct mechanism $(P_i^*, \mathbf{Q}_i^*, X_i^*)_{i \in \{1, \dots, N\}}$ is *ex ante optimal* (for the buyer) if it solves the following problem:

$$\begin{aligned} \max_{\rho=(P_i, \mathbf{Q}_i, X_i)_{i \in \{1, \dots, N\}}} & E_t[U^\rho(t | t)] \\ \text{s.t. } & U^\rho(t | t) \geq U^\rho(\hat{t} | t) \quad \text{for all } t, \hat{t} \in [\underline{t}, \bar{t}] \end{aligned} \quad (1)$$

$$\Pi_i^\rho(\theta_i | \theta_i) \geq \Pi_i^\rho(\hat{\theta}_i | \theta_i) \quad \text{for all } \theta_i, \hat{\theta}_i \in [\underline{\theta}, \bar{\theta}], i \in \{1, \dots, N\} \quad (2)$$

$$\Pi_i^\rho(\theta_i | \theta_i) \geq 0 \quad \text{for all } \theta_i \in [\underline{\theta}, \bar{\theta}], i \in \{1, \dots, N\} \quad (3)$$

$$\sum_{i=1}^N X_i(\boldsymbol{\theta}, t) \leq 1 \quad \text{for all } \boldsymbol{\theta} \in [\underline{\theta}, \bar{\theta}]^N, t \in [\underline{t}, \bar{t}], \quad (4)$$

where

$$\begin{aligned} U^\rho(\hat{t} | t) &= \sum_{i=1}^N E_{\boldsymbol{\theta}}[X_i(\boldsymbol{\theta}, \hat{t}) \cdot v(\mathbf{Q}_i(\boldsymbol{\theta}, \hat{t}), t) - P_i(\boldsymbol{\theta}, \hat{t})] \\ \Pi_i^\rho(\hat{\theta}_i | \theta_i) &= \int_{\underline{t}}^{\bar{t}} E_{\boldsymbol{\theta}_{-i}}[P_i(\hat{\theta}_i, \boldsymbol{\theta}_{-i}, t) - X_i(\hat{\theta}_i, \boldsymbol{\theta}_{-i}, t) \cdot c(\mathbf{Q}_i(\hat{\theta}_i, \boldsymbol{\theta}_{-i}, t), \theta_i)] dG(t). \end{aligned}$$

The first constraint is an incentive compatibility (IC) constraint for the buyer, the second one is an IC constraint for each supplier, the third one is an individual rationality (IR) constraint for each supplier, and the fourth one is a condition for the trading probability. Using the ‘‘Revelation Principle’’ and ‘‘Inscrutability Principle’’ of Myerson (1983), we can focus on this particular mechanism $(P_i^*, \mathbf{Q}_i^*, X_i^*)_{i \in \{1, \dots, N\}}$ when finding the equilibrium outcome in the mechanism-selection game that yields the highest *ex ante* utility for the buyer. Note that in a supplier’s expected profit $\Pi_i^\rho(\hat{\theta}_i | \theta_i)$, the expectation for the buyer’s type is taken with respect to the prior belief G , which implies that the announcement of a mechanism ρ conveys no information about the buyer’s type. For any separation equilibrium in which some types of the buyer announce different mechanisms, we can find a pooling equilibrium which is outcome-equivalent to the original equilibrium. This is just an argument of the Inscrutability Principle.

The next lemma characterizes the *ex ante* optimal mechanism. We define $K \equiv$

$$\max_{\mathbf{q}, m, m', \theta} c_{mm'}(\mathbf{q}, \theta)F(\theta)/f(\theta). \text{ The proof is based on Mylovanov and Tröger (2008).}$$

Lemma 2. *Suppose that $v_{mm'}(\mathbf{q}, t) - c_{mm'}(\mathbf{q}, \theta) \geq K$, for all $m \neq m'$. Then, the following direct mechanism $(P_i^*, \mathbf{Q}_i^*, X_i^*)_{i \in \{1, \dots, N\}}$ is *ex ante* optimal:*

$$X_i^*(\boldsymbol{\theta}, t) = \begin{cases} 1 & \text{if } \theta_i < \min\{\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_N, \bar{\theta}^{t*}\} \\ \frac{1}{\#\{j|\theta_j=\theta_i\}} & \text{if } \theta_i = \min\{\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_N, \bar{\theta}^{t*}\} \\ 0 & \text{if } \theta_i > \min\{\theta_1, \dots, \theta_{i-1}, \theta_{i+1}, \dots, \theta_N, \bar{\theta}^{t*}\} \end{cases}$$

$$\mathbf{Q}_i^*(\boldsymbol{\theta}, t) = \mathbf{Q}^*(\theta_i, t) \in \arg \max_{\mathbf{q} \in \mathcal{Q}} \Phi(\mathbf{q}, \theta_i, t)$$

$$P_i^*(\boldsymbol{\theta}, t) = X_i^*(\boldsymbol{\theta}, t) \left[c(\mathbf{Q}^*(\theta_i, t), \theta_i) + \int_{\theta_i}^{\bar{\theta}^{t*}} c_\theta(\mathbf{Q}^*(z, t), z) \left(\frac{1 - F(z)}{1 - F(\theta_i)} \right)^{N-1} dz \right],$$

where $\bar{\theta}^{t*} \in [\underline{\theta}, \bar{\theta}]$ is a cost parameter such that $\Phi(\mathbf{Q}^*(\theta, t), \theta, t) \geq 0$ iff $\theta \in [\underline{\theta}, \bar{\theta}^{t*}]$.

The *ex ante* optimal mechanism coincides with the mechanism that would be optimal if the buyer's type were common knowledge. This is the “irrelevance result”, which holds in many independent-private-values environments where the principal (buyer) has a quasi-linear preference (Maskin and Tirole (1990), Tan (1996)). In a more general environment, Mylovanov and Tröger (2008) provide a condition under which the irrelevance result holds.

The following lemma characterizes the optimal quality schedule $\mathbf{Q}^*(\theta, t)$ in Lemma 2.

Lemma 3. (i) $\mathbf{Q}^*(\theta, t) \geq \mathbf{Q}^*(\theta', t)$ for all $\theta < \theta'$, $t \in [\underline{t}, \bar{t}]$. (ii) $\mathbf{Q}^*(\underline{\theta}, t), \tilde{\mathbf{q}}(\underline{\theta}, t) \in \arg \max_{\mathbf{q}} [v(\mathbf{q}, t) - c(\mathbf{q}, \underline{\theta})]$ for all $t \in [\underline{t}, \bar{t}]$. (iii) Suppose that $v - c$ is supermodular in \mathbf{q} . Then, $\mathbf{Q}^*(\theta, t) \ll \tilde{\mathbf{q}}(\theta, t)$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$, $t \in [\underline{t}, \bar{t}]$.

This lemma has some important implications for the implementation possibilities. First, the part (i) states that the optimal schedule $Q^{m*}(\theta, t)$ of non-monetary attribute is nonincreasing in θ “for all” m . Without the supermodularity of the virtual surplus in

quality, $Q^{m*}(\theta, t)$ may not be nonincreasing in θ for some m . For example, assume that $M = 2$, $v(q^1, q^2, t) = q^1 q^2 + t(q^1 + q^2)$, $c(q^1, q^2, \theta) = (q^1 + q^2)^2 + \theta(q^1 + \epsilon q^2)$, $\epsilon > 0$, and $F(\theta) = (\theta - \underline{\theta})/(\bar{\theta} - \underline{\theta})$ so that the virtual surplus $\Phi(q^1, q^2, \theta, t)$ is submodular in (q^1, q^2) . Then, the optimal quality schedule can be $Q^{1*}(\theta, t) = (1/3)[t - (2 - \epsilon)(2\theta - \underline{\theta})]$ and $Q^{2*}(\theta, t) = (1/3)[t - (2\epsilon - 1)(2\theta - \underline{\theta})]$, so that Q^{2*} is increasing in θ when $\epsilon < 1/2$. Second, the optimal transfer schedule $P_i^*(\boldsymbol{\theta}, t)$ may not be increasing in θ_i . For example, assume that $M = 1$, $v(q^1, t) = tq^1$, $c(q^1, \theta) = (q^1)^2 + (\theta + 1)q^1 + \theta$, $t = 3$ and $F(\theta) = \theta$. Then, the optimal quality schedule is $Q^{1*}(\theta, t) = 1 - \theta$, and the critical type is $\bar{\theta}^{t*} = 2 - \sqrt{3}$. Assuming that $N = 2$, it follows from simple calculations that the transfer schedule $P_i^*(\theta_1, \theta_2, t = 3)$ is given by

$$P_i^*(\theta_1, \theta_2, t = 3) = X_i^*(\theta_1, \theta_2, t = 3) \frac{-\theta_i^3 + 9\theta_i^2 - 18\theta_i + 8}{3(1 - \theta_i)}.$$

In this example, provided that $X_i^*(\theta_1, \theta_2, t = 3) = 1$, the transfer is “decreasing” in $\theta_i \in [0, 2 - \sqrt{3}]$. Thus, the lower cost parameter the supplier has, the higher payment he can receive in exchange for delivering the product of the higher quality.

In a second step, we examine the implementation of the *ex ante* optimal mechanism via a scoring auction. We say that a scoring auction *implements the ex ante optimal mechanism* if in the auction game there exists an equilibrium which is outcome-equivalent to the *ex ante* optimal mechanism $(P_i^*, \mathbf{Q}_i^*, X_i^*)_{i \in \{1, \dots, N\}}$ for each realization of $(\boldsymbol{\theta}, t)$. The next proposition states that a scoring auction succeeds in the implementation.

Proposition 1. *Suppose that $v_{mm'}(\mathbf{q}, t) - c_{mm'}(\mathbf{q}, \theta) \geq K$, for all $m \neq m'$. Then: (i) There exists a quasi-linear scoring rule $S^{t*}(p, \mathbf{q}) = s^t(\mathbf{q}) - p$ for each type t with which a scoring auction implements the ex ante optimal mechanism. (ii) The scoring rule S^{t*} has the following properties: $s^t(\mathbf{q}) \neq s^{t'}(\mathbf{q})$ for all $t \neq t'$, $s^t(\mathbf{q}) \neq v(\mathbf{q}, t)$, $s^t(\mathbf{q})$ is nondecreasing in q^m for all m , and $s^t(\mathbf{q}) = -\infty$ if $q^m < Q^{m*}(\bar{\theta}^{t*}, t)$ for some m .*

This proposition is an extension of Che (1993). We discuss several implications. First, the optimal scoring rule S^{t*} for each type t differs from the other types. Although in the *ex ante* optimal mechanism the buyer is indifferent between revealing and concealing her true type at the stage of mechanism announcement, the buyer “must” reveal her type through her announcement of scoring rules. This is because the buyer has no chance to affect the outcome after the announcement of a scoring rule. The optimal scoring rule $s^t(\mathbf{q}) - p$ is, however, different from her true preference $v(\mathbf{q}, t) - p$. If $S(p, \mathbf{q}) = v(\mathbf{q}, t) - p$, then each supplier i offers the efficient quality level $\tilde{\mathbf{q}}(\theta_i, t)$, which is excessive from the buyer’s viewpoint. Second, the implementation possibilities are positively affected by the fact that the optimal schedule $Q^{m*}(\theta, t)$ is decreasing in θ for all m . When $Q^{m*}(\theta, t)$ is increasing in θ for some m as in the example after Lemma 3, a scoring auction may not be able to implement the *ex ante* optimal mechanism. Moreover, the monotonicity implies that the offer $q^m < Q^{m*}(\bar{\theta}^{t*}, t)$ is a signal that a cost parameter is more inefficient than $\bar{\theta}^{t*}$. The buyer can thus exclude some inefficient suppliers based on their quality offers. On the other hand, a quasi-linear scoring rule used in practice can be written as

$$S(p, \mathbf{q}) = \begin{cases} s(\mathbf{q}) - p & \text{if } p \leq \bar{p} \\ -\infty & \text{if } p > \bar{p}, \end{cases}$$

where $\bar{p} \in \mathbb{R}_+$ is a reserve price. With this class of scoring rules, a scoring auction cannot implement the *ex ante* optimal mechanism in general. A reserve price excludes efficient suppliers rather than inefficient suppliers when the transfer schedule $P_i^*(\boldsymbol{\theta}, t)$ is decreasing in θ_i as in the example after Lemma 3.

The scoring rule $S^{t*} = s^t(\mathbf{q}) - p$ in Proposition 1 seems complicated because the score of each attribute q^m depends on the levels of the other attributes in general (see Appendix). However, the next proposition shows that a scoring auction with quasi-

linear rules which are additively separable in (q^1, \dots, q^M) implements the *ex ante* optimal mechanism with additional conditions.

Proposition 2. *Suppose that $c_{mm'} = 0$, $v_{mm'} \geq K$, for all $m \neq m'$, and the Hessian of Φ is negative definite. Then: (i) There exists a quasi-linear scoring rule $S^{t^{**}}(p, \mathbf{q}) = \sum_{m=1}^M s^{m,t}(q^m) - p$ for each type t with which a scoring auction implements the *ex ante* optimal mechanism. (ii) The scoring rule $S^{t^{**}}$ has the following properties: $s^{m,t}(q^m) \neq s^{m,t'}(q^m)$ for all $t \neq t'$, $\sum_{m=1}^M s^{m,t}(q^m) \neq v(\mathbf{q}, t)$, $s^{m,t}(q^m)$ is nondecreasing in q^m , and $s^{m,t}(q^m) = -\infty$ if $q^m < Q^{m*}(\bar{\theta}^{t^*}, t)$.*

Appendix

Proof of Lemma 1. This is a sketch of the proof. See Che (1993).

(i) First, we show that in equilibrium a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^S]$ never submits a bid (p, \mathbf{q}) such that $\mathbf{q} \notin \arg \max_{\hat{\mathbf{q}}} [s(\hat{\mathbf{q}}) - c(\hat{\mathbf{q}}, \theta)]$. Suppose to the contrary that a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^S]$ submits such a bid (p, \mathbf{q}) . Now, consider a bid (p', \mathbf{q}') such that $\mathbf{q}' \in \arg \max_{\hat{\mathbf{q}}} [s(\hat{\mathbf{q}}) - c(\hat{\mathbf{q}}, \theta)]$ and $s(\mathbf{q}') - p' = s(\mathbf{q}) - p$. The score of (p', \mathbf{q}') is equal to that of (p, \mathbf{q}) , so that both bids yield the same winning probability given the other suppliers' strategies. As in Che (1993), we can show that $\text{Prob}[\text{win} \mid S(p, \mathbf{q})] > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}^S]$. When $\theta \in [\underline{\theta}, \bar{\theta}^S)$, the supplier's expected profit from (p, \mathbf{q}) is lower than his expected profit from (p', \mathbf{q}') because

$$\begin{aligned} & [p - c(\mathbf{q}, \theta)] \text{Prob}[\text{win} \mid S(p, \mathbf{q})] \\ & < [p - c(\mathbf{q}, \theta) + (s(\mathbf{q}') - c(\mathbf{q}', \theta) - (s(\mathbf{q}) - c(\mathbf{q}, \theta)))] \text{Prob}[\text{win} \mid S(p, \mathbf{q})] \\ & = [p' - c(\mathbf{q}', \theta)] \text{Prob}[\text{win} \mid S(p', \mathbf{q}')]; \end{aligned}$$

the inequality follows from the fact that $\mathbf{q} \notin \arg \max_{\hat{\mathbf{q}}} [s(\hat{\mathbf{q}}) - c(\hat{\mathbf{q}}, \theta)] \ni \mathbf{q}'$, and the equality

follows from the construction of (p', \mathbf{q}') . This is a contradiction because the supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^S)$ has a profitable deviation. When $\theta = \bar{\theta}^S$, we can show that the bid (p, \mathbf{q}) is weakly dominated by (p', \mathbf{q}') , by the same logic as above. This contradicts the assumption that no supplier uses weakly dominated strategies. Therefore, we can assume without loss of generality that the symmetric equilibrium bidding strategy is given by (p, \mathbf{q}^*) which satisfies $\mathbf{q}^*(\theta, S) \in \arg \max_{\mathbf{q}} [s(\mathbf{q}) - c(\mathbf{q}, \theta)]$ for all $\theta \in [\underline{\theta}, \bar{\theta}^S]$.

Second, consider the following change of variables: $k(\theta) \equiv s(\mathbf{q}^*(\theta, S)) - c(\mathbf{q}^*(\theta, S), \theta)$ and $b \equiv s(\mathbf{q}^*(\theta, S)) - p$ for all $\theta \in [\underline{\theta}, \bar{\theta}^S]$. Now, $k(\theta) = s(\mathbf{q}^*(\theta, S)) - c(\mathbf{q}^*(\theta, S), \theta) \geq s(\mathbf{q}^*(\theta', S)) - c(\mathbf{q}^*(\theta', S), \theta) > s(\mathbf{q}^*(\theta', S)) - c(\mathbf{q}^*(\theta', S), \theta') = k(\theta')$ for all $\theta < \theta'$. Hence, $k(\theta)$ is decreasing in θ , so that the inverse of k exists. Moreover, $k(\theta) = \max_{\mathbf{q}} [s(\mathbf{q}) - c(\mathbf{q}, \theta)]$ is continuous in θ by Berge's maximum theorem with the continuity of $s(\cdot)$ and $c(\cdot)$. Let $\beta : [k(\bar{\theta}^S), k(\underline{\theta})] \rightarrow \mathbb{R}_+$ denote a symmetric bidding strategy of $k(\theta)$ that is increasing in k . When the other suppliers follow this strategy β , the expected profit of the supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^S]$ who submits $(p, \mathbf{q}^*(\theta, S))$ can be rewritten as

$$\begin{aligned} & [p - c(\mathbf{q}^*(\theta, S), \theta)] \text{Prob}[\text{win} \mid S(p, \mathbf{q}^*(\theta, S))] \\ &= [k(\theta) - b] \{1 - F(k^{-1}(\beta^{-1}(b)))\}^{N-1}. \end{aligned}$$

We can then rely on the technique of first-price auction with a boundary condition $k(\bar{\theta}^S) = \beta(k(\bar{\theta}^S))$.

(ii) Because $k(\theta)$ is decreasing in $\theta \in [\underline{\theta}, \bar{\theta}^S]$ and the bidding strategy $\beta(k)$ is increasing in k , a supplier wins only if his cost parameter is the lowest among $(\theta_1, \dots, \theta_N)$ and lower than $\bar{\theta}^S$. □

Proof of Lemma 2. This is a sketch of the proof.

(a) We can show that the necessary and sufficient condition for each supplier's IC constraint (2) is given by the following two conditions: the *envelope condition* and the

monotonicity condition. We say that a direct mechanism $\rho = (P_i, \mathbf{Q}_i, X_i)_{i \in \{1, \dots, N\}}$ satisfies the envelope condition if for all i and θ_i ,

$$\Pi_i^\rho(\theta_i) = \Pi_i^\rho(\bar{\theta}) + \int_{\underline{t}}^{\bar{t}} \left[\int_{\theta_i}^{\bar{\theta}} E_{\boldsymbol{\theta}_{-i}} [X_i(z, \boldsymbol{\theta}_{-i}, t) c_\theta(\mathbf{Q}_i(z, \boldsymbol{\theta}_{-i}, t), z)] dz \right] dG(t),$$

where $\Pi_i^\rho(\theta_i) \equiv \Pi_i^\rho(\theta_i \mid \theta_i)$. We say that a direct mechanism $\rho = (P_i, \mathbf{Q}_i, X_i)_{i \in \{1, \dots, N\}}$ satisfies the monotonicity condition if for all i and $\theta_i, \hat{\theta}_i$,

$$\begin{aligned} & \int_{\underline{t}}^{\bar{t}} \left[\int_{\theta_i}^{\hat{\theta}_i} E_{\boldsymbol{\theta}_{-i}} [X_i(z, \boldsymbol{\theta}_{-i}, t) c_\theta(\mathbf{Q}_i(z, \boldsymbol{\theta}_{-i}, t), z)] dz \right] dG(t) \\ & \geq \int_{\underline{t}}^{\bar{t}} \left[\int_{\theta_i}^{\hat{\theta}_i} E_{\boldsymbol{\theta}_{-i}} [X_i(\hat{\theta}_i, \boldsymbol{\theta}_{-i}, t) c_\theta(\mathbf{Q}_i(\hat{\theta}_i, \boldsymbol{\theta}_{-i}, t), z)] dz \right] dG(t); \end{aligned}$$

if both $X_i(\theta_i, \boldsymbol{\theta}_{-i}, t)$ and $\mathbf{Q}_i(\theta_i, \boldsymbol{\theta}_{-i}, t)$ are nonincreasing in θ_i for all $\boldsymbol{\theta}_{-i}$ and t , then this condition is automatically satisfied.

(b) We solve the optimization problem for the *ex ante* optimal mechanism. The supplier's IC constraint (2) implies that $\Pi_i^\rho(\theta_i)$ is nonincreasing in θ_i because $\Pi_i^\rho(\theta_i) \geq \Pi_i^\rho(\theta'_i \mid \theta_i) \geq \Pi_i^\rho(\theta'_i)$ for all $\theta_i < \theta'_i$. Hence, the supplier's IR constraint (3) is replaced by $\Pi_i^\rho(\bar{\theta}) = 0$. Using the result (i), the IC constraint (2) and the IR constraint (3) is replaced by both the monotonicity condition and the following condition:

$$\Pi_i^\rho(\theta_i) = \int_{\underline{t}}^{\bar{t}} \left[\int_{\theta_i}^{\bar{\theta}} E_{\boldsymbol{\theta}_{-i}} [X_i(z, \boldsymbol{\theta}_{-i}, t) c_\theta(\mathbf{Q}_i(z, \boldsymbol{\theta}_{-i}, t), z)] dz \right] dG(t) \quad \text{for all } i \text{ and } \theta_i.$$

At first, we ignore the monotonicity condition and the buyer's IC constraint (1). Substituting the supplier's profit $\Pi_i^\rho(\theta_i)$, the buyer's objective function is rewritten as

$$\begin{aligned} & E_{(t, \boldsymbol{\theta})} \left[\sum_{i=1}^N [X_i(\boldsymbol{\theta}, t) [v(\mathbf{Q}_i(\boldsymbol{\theta}, t), t) - c(\mathbf{Q}_i(\boldsymbol{\theta}, t), \theta_i)] - \Pi_i^\rho(\theta_i)] \right] \\ & = E_{(t, \boldsymbol{\theta})} \left[\sum_{i=1}^N X_i(\boldsymbol{\theta}, t) \left[v(\mathbf{Q}_i(\boldsymbol{\theta}, t), t) - c(\mathbf{Q}_i(\boldsymbol{\theta}, t), \theta_i) - c_\theta(\mathbf{Q}_i(\boldsymbol{\theta}, t), \theta_i) \frac{F(\theta_i)}{f(\theta_i)} \right] \right] \\ & = E_{(t, \boldsymbol{\theta})} \left[\sum_{i=1}^N X_i(\boldsymbol{\theta}, t) \Phi(\mathbf{Q}_i(\boldsymbol{\theta}, t), \theta_i, t) \right]. \end{aligned}$$

This objective function is maximized when $\mathbf{Q}_i(\boldsymbol{\theta}, t)$ and $X_i(\boldsymbol{\theta}, t)$ are respectively given by $\mathbf{Q}^*(\theta_i, t)$ and $X_i^*(\boldsymbol{\theta}, t)$ in the lemma. This is because $\mathbf{Q}^*(\theta_i, t)$ maximizes $\Phi(\mathbf{q}, \theta_i, t)$ and the maximized value is decreasing in θ_i . The latter follows from $\Phi(\mathbf{Q}^*(\theta, t), \theta, t) \geq \Phi(\mathbf{Q}^*(\theta', t), \theta, t) > \Phi(\mathbf{Q}^*(\theta', t), \theta', t)$ for all $\theta < \theta'$; the second inequality follows from the assumption that $c_{\theta\theta} \geq 0$ and F/f is increasing in θ .

Finally, we show that the direct mechanism $\rho^* = (P_i^*, \mathbf{Q}_i^*, X_i^*)_{i \in \{1, \dots, N\}}$ satisfies the ignored constraints. It is easy to show that X_i^* is increasing in θ_i . Now, $\Phi(\mathbf{q}, \theta_i, t)$ is supermodular in \mathbf{q} by the assumption of the lemma, and has strictly increasing differences in $(\mathbf{q}, -\theta_i)$ from Assumptions 3, 4 and 5. It then follows from Topkis monotonicity theorem that $\mathbf{Q}^*(\theta_i, t) \geq \mathbf{Q}^*(\theta'_i, t)$ for all $\theta_i < \theta'_i$. Thus, this direct mechanism ρ^* satisfies the monotonicity condition. Moreover, this direct mechanism ρ^* satisfies the buyer's IC constraint (1) because

$$\begin{aligned} U^{\rho^*}(\hat{t} | t) &= \sum_{i=1}^N E_{\boldsymbol{\theta}} [X_i^*(\boldsymbol{\theta}, \hat{t})v(\mathbf{Q}^*(\theta_i, \hat{t}), t) - P_i^*(\boldsymbol{\theta}, \hat{t})] \\ &= \sum_{i=1}^N E_{\boldsymbol{\theta}} [X_i^*(\boldsymbol{\theta}, \hat{t})\Phi(\mathbf{Q}^*(\theta_i, \hat{t}), \theta_i, t)] \\ &\leq \sum_{i=1}^N E_{\boldsymbol{\theta}} [X_i^*(\boldsymbol{\theta}, t)\Phi(\mathbf{Q}^*(\theta_i, t), \theta_i, t)] = U^{\rho^*}(t | t); \end{aligned}$$

the inequality follows from the fact that $\mathbf{Q}^*(\theta_i, t) \in \arg \max_{\mathbf{q}} \Phi(\mathbf{q}, \theta_i, t)$ and the construction of $X_i^*(\boldsymbol{\theta}, t)$. Therefore, the buyer of type t cannot benefit from reporting $\hat{t} \neq t$. \square

Proof of Lemma 3. (i) As shown in the proof of Lemma 2, $\mathbf{Q}^*(\theta, t) \geq \mathbf{Q}^*(\theta', t)$ for all $\theta < \theta'$.

(ii) Lemma 2 states that $\mathbf{Q}^*(\theta, t) \in \arg \max_{\mathbf{q}} \Phi(\mathbf{q}, \theta, t)$. Now, $\Phi(\mathbf{q}, \underline{\theta}, t) = v(\mathbf{q}, t) - c(\mathbf{q}, \underline{\theta}) - c_{\theta}(\mathbf{q}, \underline{\theta})F(\underline{\theta})/f(\underline{\theta}) = v(\mathbf{q}, t) - c(\mathbf{q}, \underline{\theta})$ because $F(\underline{\theta}) = 0$. Hence, $\mathbf{Q}^*(\underline{\theta}, t), \tilde{\mathbf{q}}(\underline{\theta}, t) \in \arg \max_{\mathbf{q}} [v(\mathbf{q}, t) - c(\mathbf{q}, \underline{\theta})]$.

(iii) Define the function $v(\mathbf{q}, t) - c(\mathbf{q}, \theta) - (1 - a)c_{\theta}(\mathbf{q}, \theta)F(\theta)/f(\theta)$ where $a \in \{0, 1\}$.

This function is supermodular in \mathbf{q} because both $v - c$ and Φ are supermodular in \mathbf{q} , and has strictly increasing differences in (\mathbf{q}, a) from Assumptions 3, 4, and 5. It then follows from Topkis monotonicity theorem that $\mathbf{Q}^*(\theta, t) \leq \tilde{\mathbf{q}}(\theta, t)$. If there exists an attribute m such that $Q^{m*}(\theta, t) = \tilde{q}^m(\theta, t)$, then

$$0 = \Phi_m(\mathbf{Q}^*(\theta, t), \theta, t) \leq \Phi_m(\tilde{\mathbf{q}}(\theta, t), \theta, t) < v_m(\tilde{\mathbf{q}}(\theta, t), t) - c_m(\tilde{\mathbf{q}}(\theta, t), \theta) = 0;$$

the first inequality follows from the assumption that Φ is supermodular in \mathbf{q} , together with the fact that $\mathbf{Q}^*(\theta, t) \leq \tilde{\mathbf{q}}(\theta, t)$ and $Q^{m*}(\theta, t) = \tilde{q}^m(\theta, t)$, and the second inequality follows from the assumption that $c_{m\theta} > 0$. This is a contradiction, which implies that $\mathbf{Q}^*(\theta, t) \ll \tilde{\mathbf{q}}(\theta, t)$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$. \square

Proof of Proposition 1. For each $q^m \in [Q^{m*}(\bar{\theta}, t), Q^{m*}(\underline{\theta}, t)]$, let $\theta^{m,t}(q^m)$ be a cost parameter which satisfies $\Phi_m(q^m, \mathbf{Q}^{-m*}(\theta, t), \theta, t) = 0$; given t , $\theta^{m,t}(q^m)$ is uniquely determined by this equation because the left-hand side is monotonic in θ , i.e. $\Phi_m(q^m, \mathbf{Q}^{-m*}(\theta, t), \theta, t) > \Phi_m(q^m, \mathbf{Q}^{-m*}(\theta', t), \theta', t)$ for all $\theta < \theta'$. When $q^m > Q^{m*}(\underline{\theta}, t)$, let $\theta^{m,t}(q^m) = \underline{\theta}$. Define $s^t(\mathbf{q})$ by

$$s^t(\mathbf{q}) = \begin{cases} \sigma^t(\max\{\theta^{1,t}(q^1), \dots, \theta^{M,t}(q^M)\}) & \text{if } q^m \geq Q^{m*}(\bar{\theta}^{t*}, t) \text{ for all } m \\ -\infty & \text{if } q^m < Q^{m*}(\bar{\theta}^{t*}, t) \text{ for some } m, \end{cases}$$

where $\sigma^t(\cdot)$ is constructed as follows. It follows from Lemma 1 that a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^{t*}]$ chooses quality \mathbf{q} which maximizes $s^t(\mathbf{q}) - c(\mathbf{q}, \theta)$. Then, his problem is reduced to $\max_{\hat{\theta} \in [\underline{\theta}, \bar{\theta}^{t*}]} \sigma^t(\hat{\theta}) - c(\mathbf{Q}^*(\hat{\theta}, t), \theta)$ from the construction of $s^t(\mathbf{q})$. His production cost $c(\mathbf{Q}^*(\hat{\theta}, t), \theta)$ has strictly decreasing differences in $(\theta, \hat{\theta})$ because $c(\mathbf{Q}^*(\hat{\theta}, t), \theta) - c(\mathbf{Q}^*(\hat{\theta}', t), \theta) < c(\mathbf{Q}^*(\hat{\theta}, t), \theta') - c(\mathbf{Q}^*(\hat{\theta}', t), \theta')$ for all $\theta < \theta'$ and $\hat{\theta} < \hat{\theta}'$; the inequality follows from the part (i) of Lemma 3 and the assumption that $c_{m\theta} > 0$.

Now, consider a family of curves $\{\sigma = c(\mathbf{Q}^*(\hat{\theta}, t), \theta) + h(\theta)\}$ parametrized by $\theta \in$

$[\underline{\theta}, \bar{\theta}^{t*}]$, where $h(\theta)$ is given by

$$h(\theta) = h(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} c_{\theta}(\mathbf{Q}^*(z, t), z) dz;$$

$h(\underline{\theta})$ is an arbitrary real number which is sufficiently high. Then, let $\sigma^t(\hat{\theta})$ be the lower envelope of $\{\sigma = c(\mathbf{Q}^*(\hat{\theta}, t), \theta) + h(\theta)\}$, which is given by

$$\sigma^t(\hat{\theta}) = c(\mathbf{Q}^*(\hat{\theta}, t), \hat{\theta}) + h(\hat{\theta}).$$

Since this lower envelope is tangent to $\sigma = c(\mathbf{Q}^*(\hat{\theta}, t), \theta) + h(\theta)$ at $\hat{\theta} = \theta$, the supplier of type θ optimally chooses $\hat{\theta} = \theta$ to maximize $\sigma^t(\hat{\theta}) - c(\mathbf{Q}^*(\hat{\theta}, t), \theta)$. Hence, $\mathbf{q}^*(\theta, S^{t*}) = \mathbf{Q}^*(\theta, t)$.

It follows from Lemma 1 that in equilibrium each supplier i pays the same price as $P_i^*(\boldsymbol{\theta}, t)$ for each realization of $(\boldsymbol{\theta}, t)$.

Finally, the buyer of type t has no incentive to deviate from announcing S^{t*} because it also implements the optimal mechanism when the buyer's realized type is common knowledge. \square

Proof of Proposition 2. First, define $s^m(q^m, t)$ as follows:

$$s^m(q^m, t) = \begin{cases} \int_{Q^{m*}(\bar{\theta}^{t*}, t)}^{q^m} [v_m(q, \mathbf{Q}^{-m*}(\theta^{m,t}(q), t), t) - c_{m\theta}(q, \mathbf{Q}^{-m*}(\theta^{m,t}(q), t), \theta^{m,t}(q)) \frac{F(\theta^{m,t}(q))}{f(\theta^{m,t}(q))}] dq & \text{if } q^m \in [Q^{m*}(\bar{\theta}^{t*}, t), Q^{m*}(\underline{\theta}, t)] \\ \int_{Q^{m*}(\bar{\theta}^{t*}, t)}^{Q^{m*}(\underline{\theta}, t)} [v_m(q, \mathbf{Q}^{-m*}(\theta^{m,t}(q), t), t) - c_{m\theta}(q, \mathbf{Q}^{-m*}(\theta^{m,t}(q), t), \theta^{m,t}(q)) \frac{F(\theta^{m,t}(q))}{f(\theta^{m,t}(q))}] dq & \text{if } q^m \in (Q^{m*}(\underline{\theta}, t), \infty) \\ -\infty & \text{if } q^m \in [0, Q^{m*}(\bar{\theta}^{t*}, t)]. \end{cases}$$

Second, we show that the unique maximizer of $\sum_{m=1}^M s^m(q^m, t) - c(\mathbf{q}, \theta)$ is equal to $\mathbf{Q}^*(\theta, t)$ for all $\theta \in [\underline{\theta}, \bar{\theta}^{t*}]$. The first-order condition for q^m is given by

$$v_m(q^m, \mathbf{Q}^{-m*}(\theta^{m,t}(q^m), t), t) = c_m(q^m, \mathbf{q}^{-m}) + c_{m\theta}(q^m, \mathbf{Q}^{-m*}(\theta^{m,t}(q^m), t), \theta^{m,t}(q^m)) \frac{F(\theta^{m,t}(q^m))}{f(\theta^{m,t}(q^m))},$$

which is satisfied if $q^m = Q^{m*}(\theta, t)$ for all m . Now, we show that the Hessian of $\sum_{m=1}^M s^m(q^m, t) - c(\mathbf{q}, \theta)$ is negative definite, and thus $\sum_{m=1}^M s^m(q^m, t) - c(\mathbf{q}, \theta)$ is strictly concave in \mathbf{q} . The second-order derivative of $\sum_{m=1}^M s^m(q^m, t) - c(\mathbf{q}, \theta)$ with respect to q^m is given by

$$\begin{aligned} \Phi_{mm} &- \left[c_{m\theta\theta} \frac{F}{f} + c_{m\theta} \left(\frac{F}{f} \right)' - \sum_{m' \neq m} \left(v_{mm'} - c_{mm'\theta} \frac{F}{f} \right) \right] \frac{d\theta^{m,t}}{dq^m} \\ &= \Phi_{mm} \left[\frac{c_{m\theta} - \sum_{m' \neq m} \Phi_{mm'} (\partial Q^{m'*} / \partial \theta) + \sum_{m' \neq m} \left(v_{mm'} - c_{mm'\theta} \frac{F}{f} \right)}{c_{m\theta} + c_{m\theta\theta} \frac{F}{f} + c_{m\theta} \left(\frac{F}{f} \right)' - \sum_{m' \neq m} \Phi_{mm'} (\partial Q^{m'*} / \partial \theta)} \right]; \end{aligned}$$

the equality follows from the fact that

$$\frac{d\theta^{m,t}}{dq^m} = \frac{1}{\partial Q^{m*} / \partial \theta} = \frac{\Phi_{mm}}{c_{m\theta} + c_{m\theta\theta} \frac{F}{f} + c_{m\theta} \left(\frac{F}{f} \right)' - \sum_{m' \neq m} \Phi_{mm'} (\partial Q^{m'*} / \partial \theta)}.$$

The assumption $c_{mm'} = 0$ for all $m \neq m'$, together with the supermodularity of Φ , implies that $v_{mm'} - c_{mm'\theta} \frac{F}{f} \geq 0$. Thus, the above second-order derivative is less than zero, so that all the diagonal elements of the Hessian of $\sum_{m=1}^M s^m(q^m, t) - c(\mathbf{q}, \theta)$ are negative. The off-diagonal element is given by $-c_{mm'} = 0$ for $m \neq m'$, so that the Hessian of $\sum_{m=1}^M s^m(q^m, t) - c(\mathbf{q}, \theta)$ is negative definite. Lemma 1 implies that $\mathbf{q}^*(\theta, S^{t**}) \in \arg \max_{\mathbf{q} \in Q} [\sum_{m=1}^M s^{m,t}(q^m) - c(\mathbf{q}, \theta)]$. Hence, $\mathbf{q}^*(\theta, S^{t**}) = \mathbf{Q}^*(\theta, t)$.

It follows from Lemma 1 that in equilibrium each supplier i pays the same price as $P_i^*(\theta, t)$ for each realization of (θ, t) .

Finally, the buyer of type t has no incentive to deviate from announcing S^{t**} because it also implements the optimal mechanism when the buyer's realized type is common knowledge. □

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