Lecture 2A: Optimal Pricing I

Reading: Braeutigam Chapter in the Handbook of IO

Optimal Pricing: The Public Interest Framework

- What prices would a benevolent (often omniscient) regulator choose to set?
- Why is this an interesting or important question?
  - Information asymmetries
  - Regulatory process is political and adversarial
  - Stakeholders pursue self- (not public) interest
- PI principles are "the language of discourse"
  - Stakeholders attempt to argue that *their* interests coincide with the public interest in the matter at hand.

Ingredients of an Optimal Pricing Model

- Objective Function: What is the "Public Interest" that regulatory policy is assumed to pursue?
- Instruments: What policy variables are subject to regulatory control?
  - Prices
  - Capacity levels
  - Quality controls
- Constraints: What economic and/or political constraints limit the discretion of the regulator?
  - Break-even constraint for the firm
  - Uniform pricing constraints: rural/urban, etc.

Issues to be addressed

- Objective function: What is "total surplus?"
- What *is* marginal cost pricing?
- Why is optimal pricing aka "price discrimination in the public interest:"
  - 1st Degree: Full surplus extraction
  - 2nd Degree: Nonlinear pricing (asymmetric info, no resale)
  - 3rd Degree: Market segmentation

The Social Welfare Function: where does it come from?

- Characteristics:
  - Consumer benefits measured in dollars
  - Derivative of CB w.r.t. price = **-X**
- Achieve via
  - Utilitarian SWF
  - Redistribute income "in the background"

$$CB^u = \sum_{i\in I} \mu^i(\mathbf{p}^0;\mathbf{p},m^i)$$

$$\mu^i(\mathbf{p}^0;\mathbf{p},m^i) = e^i(\mathbf{p}^0, v^i(\mathbf{p},m^i))$$

$$\frac{\partial CB^u}{\partial p_j} = \sum_{i\in I}\frac{\partial \mu^i}{\partial p_j} = -\sum_{i\in I} x_j^i v_m^i e_u^i = -\sum_{i\in I} x_j^i = -X^j(\mathbf{p})$$

because $v_m^i e_u^i = 1$ evaluated at $\mathbf{p} = \mathbf{p}^0$

$$CB^r(\mathbf{p}) = \max_{\mathbf{t}}\{\sum_{i\in I}\alpha^i\mu^i(\mathbf{p}^0;\mathbf{p},m^i+t^i) : \sum_{i\in I} t^i = 0\}$$

$$\Rightarrow \alpha^i\mu_m^i = \alpha^k\mu_m^k = \delta > 0$$

$$\frac{\partial CB^r}{\partial p_j} = \sum_{i\in I}\alpha^i\mu_j^i = -\sum_{i\in I}\alpha^i\mu_m^i x_m^i = -\delta\sum_{i\in I} x_j^i = -\delta X^j(\mathbf{p})$$

Consumer surplus as a welfare measure

- Standard objective function in the literature is *total surplus*
  - Profits (producers' surplus).
  - Consumers' surplus (area under demand curve).
- Convenient and intuitive because it yields P=MC for unconstrained problem.
- But, what's the justification for this?
  - Consumers' surplus integral not generally path independent for market demand curves.

$$\pi(\mathbf{p}) = \mathbf{p} \cdot \mathbf{X}(\mathbf{p}) - C(\mathbf{X}(\mathbf{p}))$$

$$S(\mathbf{p}) = CS(\mathbf{p}) + \pi(\mathbf{p})$$

$$\hat{\mathbf{p}} = \arg\max_{\mathbf{p}} S(\mathbf{p}) \Rightarrow$$

$$\nabla S(\hat{\mathbf{p}}) = \nabla CS(\hat{\mathbf{p}}) + \nabla \pi(\hat{\mathbf{p}}) = 0$$

$$\nabla CS = -\mathbf{X}$$

$$\nabla \pi = \mathbf{X} + (\mathbf{p} - \nabla C) \cdot [X_j^k]$$

$$\nabla S(\hat{\mathbf{p}}) = (\hat{\mathbf{p}} - \nabla C) \cdot [X_j^k] = 0 \Rightarrow$$

$$\hat{\mathbf{p}} = \nabla C \ \text{ if } [X_j^k] \text{ is non singular}$$

Social welfare function justifications

- Actually, all that's required is that the *gradient* of the consumers' benefits function equals (the negative of) the vector of market demand functions.
- Individuals $i \in \{1, \ldots, I\}$
  - Indirect utility functions $v^i(\mathbf{p}, m_i)$
  - Ownership share σ_i
  - Total consumer income: $m_i = y_i + \sigma_i \Pi$
- Bergsonian Social Welfare Function:

$$W = W(v^1, v^2, \ldots, v^I).$$

The "neutral across individuals" property

$$S(\mathbf{p}) = W(v^1(\mathbf{p}, m_1), v^2(\mathbf{p}, m_2), \; ..., v^I(\mathbf{p}, m_I))$$

$$S_j = \sum_{i=1}^{I} \frac{\partial W}{\partial v^i} \left\{ \frac{\partial v^i}{\partial p_j} + \frac{\partial v^i}{\partial m} \sigma_i \frac{\partial \pi}{\partial p_j} \right\} = \sum_{i=1}^{I} W_i \left\{ v_j^i + v_m^i \sigma_i \pi_j \right\}$$

$$S_j = -\sum_{i=1}^{I} \delta_i x_j^i + \sum_{i=1}^{I} \delta_i \sigma_i \pi_j \quad \text{where } \delta_i \equiv W_i v_m^i$$

Suppose $\delta_{\mathrm{i}} = \delta > 0 \quad \forall i$ (What does this mean?)

$$S_j / \delta = -\sum_{i=1}^{I} x_j^i + \pi_j \sum_{i=1}^{I} \sigma_i = -X^j + \pi_j = \sum_{k=1}^{n} (p_k - C_k) X_j^k$$

Sufficient conditions for a SWF to be neutral across individuals

- Utilitarian social welfare function with quasi-linear preferences.
  - $\Rightarrow W_i = v^i_m = 1$
  - But then, we could have started with CS!
- *Any* social welfare function with optimal income redistribution
  - For any **p**, (some other branch of) government adjusts lump sum transfers t_i to maximize SWF

$$L = W(v^1(\mathbf{p}, y_1 + t_1 + \sigma_1 \pi(\mathbf{p})),\ \ldots,\ v^I(\mathbf{p}, y_I + t_I + \sigma_I \pi(\mathbf{p}))) - \gamma \sum_{i=1}^{I} t_i$$

FONCs

$$\frac{\partial L}{\partial t_i} = W_i v^i_m - \gamma = 0$$

Marginal cost pricing: the welfare benchmark

- "Simple," unconstrained models should yield p=mc
- Basic results derivable from
  - *rigorous* utility model
  - surplus maximization model

$$\Pi(\mathbf{p}) = \sum_j p_j X^j - C(\mathbf{X})$$

$$TS = CB(\mathbf{p}) + \Pi(\mathbf{p})$$

$$TS_j = \sum_k (p_k - C_k) X_j^k = (\mathbf{p} - \nabla\mathbf{C}) \cdot [X_j^l]$$

$$m^i = \overline{m}^i + s^i \Pi$$

$$W = \sum_{i \in I} \alpha^i \mu^i (\mathbf{p}^0; \mathbf{p}, m^i)$$

$$W_j = \sum_{i \in I} \alpha^i \mu_m^i [-x_j^i + s^i \Pi_j] = \delta[\Pi_j - X^j]$$

P=MC also required for Pareto Efficiency, *given* optimal income redistribution

$$L = v^1(\mathbf{p}, y_1 + t_1 + \sigma_i \pi(\mathbf{p})) + \sum_{i=2}^{I} \mu^i \left[v^i(\mathbf{p}, y_i + t_i + \sigma_i \pi(\mathbf{p})) - \bar{v}^i \right] - \delta \sum_{i=1}^{I} t_i$$

Let $\mu^1 = 1,$ then Pareto Optimality requires:

$$\frac{\partial L}{\partial p_j} = \sum_{i=1}^{I} \mu^i \left[v_j^i + v_m^i \sigma_i \pi_j \right] = \sum_{i=1}^{I} \mu^i v_m^i \left[-x_j^i + \sigma_i \pi_j \right] = 0$$

$$\frac{\partial L}{\partial t_i} = \mu^i v_m^i - \delta = 0$$

Substitution and division by $\delta > 0$ yields the FONCs:

$$\frac{\partial L}{\partial p_j}\left(\frac{1}{\delta}\right) = -\sum_{i=1}^{I} x_j^i - \pi_j \sum_{i=1}^{I} \sigma_i = \sum_{k=1}^{n} (p_k - C_k) \frac{\partial X^k}{\partial p_j} = 0$$

Optimal Pricing under Constant Returns to Scale: Peak Load Pricing

- The "Peak-Load" Problem
  - Demand varies cyclically (e.g., daily, monthly, yearly)
  - Capacity cannot be varied over the cycle
  - Output cannot be stored
- Economic efficiency issues
  - How are marginal costs defined?
  - Should prices be set equal to short-run or long-run marginal costs?
  - How to recover capacity costs?

- Basic Model:
  - Independent demands

    $D_{peak} = D_1(p) > D_2(p) = D_{off\ peak}$
  - Capacity K must be sufficient to meet demand

    $K \geq D_1(p_1) \geq D_2(p_2)$
  - Per unit capacity costs = β
  - Per unit variable costs = b
  - LRMC = b + β
  - SRMC = b
  - Total costs

    $C = \beta K + b(D_1 + D_2)$

Case I: Firm Peak

- Peak period price equals long-run marginal costs

  $p_1 = b + \beta$

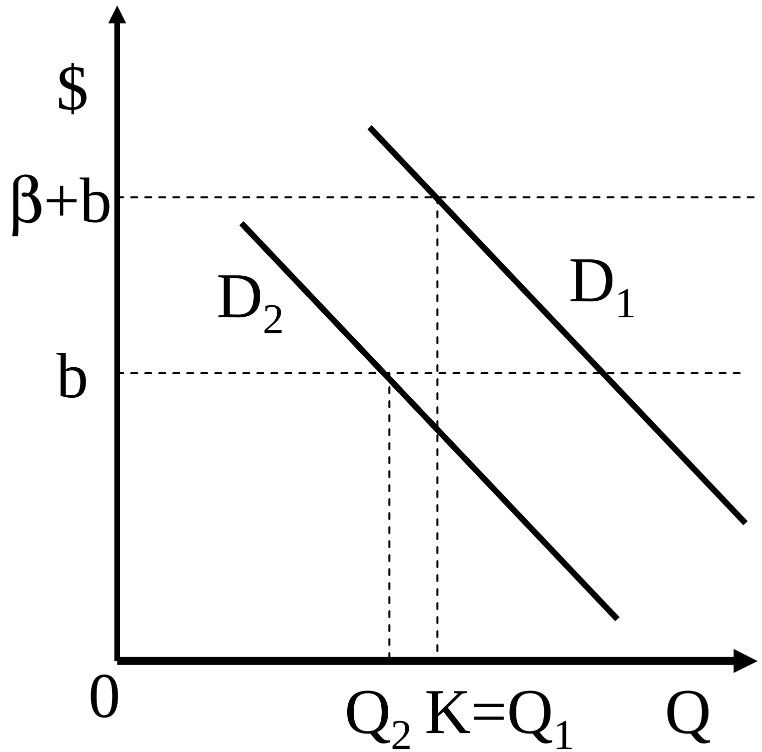
- Off-peak price equals short-run marginal cost

  $p_2 = b$

- Peak-period users "pay" for all capacity costs

  $(p_1 - b)Q_1 = \beta Q_1 = \beta K$

- Uniform pricing would result in deadweight loss



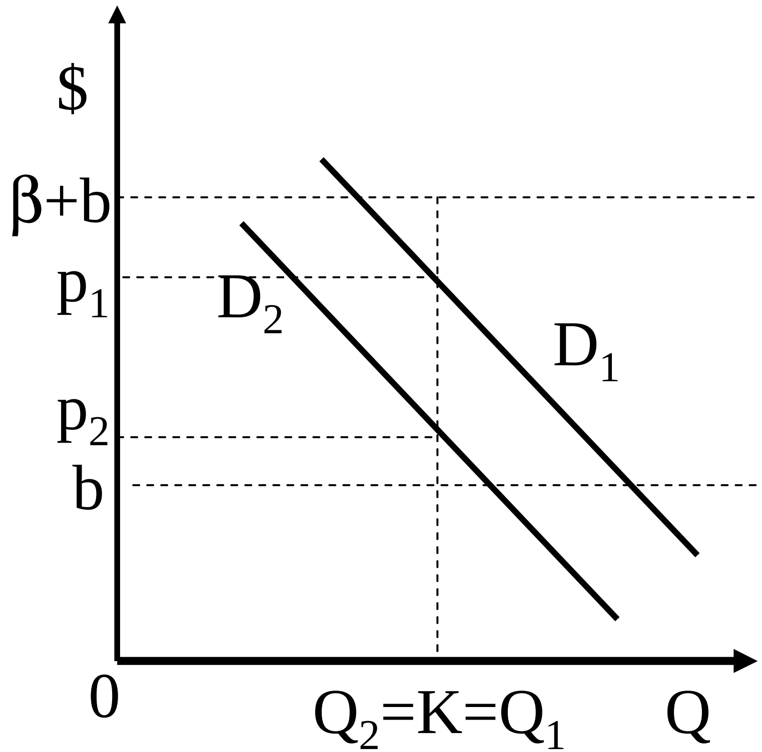
Case II: Shifting Peak

- Output in both periods fully utilizes capacity

  $D_1(p_1) = D_2(p_2) = K$
- Unequal prices:

  $\beta + b > p_1 > p_2 > b$

  $(p_1 - b) + (p_2 - b) = \beta$
- Note that
  - Both prices above SRMC
  - Both prices below LRMC
  - Both period contribute toward covering capacity costs



A Cookbook for solving peak-load pricing problems

- Identify peak and off-peak periods
- *Try* setting peak price equal to b+ β, off-peak price equal to b
  - If "peak" quantity greater than "off-peak" quantity at those prices, you're done. If not,
- Construct the *demand for capacity* schedule by adding together the *inverse* demand schedules (less variable cost) for each period:
  - $P_K(K) = (P_1(K)-b) + (P_2(K)-b)$
  - (Remember, you're now in the *shifting peak* case, so $K = Q_1 = Q_2$)
- Solve for the intersection of the demand for capacity schedule with the marginal cost of capacity: i.e., set $P_K(K) = \beta$ and solve for K
- Plug this value of K back into the inverse demand functions

  $p_1 = P_1(K)$ and $p_2 = P_2(K)$

Surplus maximization formulation of Peak Load Pricing problem

$$\pi = \sum_{t=1}^{T} \left(p_t - b\right) D^t - \beta K$$

$$L = CS(\mathbf{p}) + \pi(\mathbf{p}) + \sum_{t=1}^{T} \lambda_t (K - D^t(\mathbf{p}))$$

$$L_{p_s} = -D^s + \pi_s - \sum_{t=1}^{T} \lambda_t D_s^t = \sum_{t=1}^{T} \left(p_t - b - \lambda_t\right) D_s^t = 0$$

$$L_K = \pi_K + \sum_{t=1}^{T} \lambda_t = -\beta + \sum_{t=1}^{T} \lambda_t = 0$$

$$L_{\lambda_s} = K - D^s \geq 0; \;\; \lambda_s \geq 0; \;\; \lambda_s (K - D^s) = 0$$

Results of the basic model

- Users in peak period(s) bear all capacity costs.
- Multiple peak periods typically result in differing prices across periods.

$$\text{Let } P = \{s : D^t(\mathbf{p}^*) = K^*\}$$

$$p_t^* = b;\ \ \lambda_t^* = 0;\ \ t \notin P$$

$$p_t^* = b + \lambda_t^*;\ \ \lambda_t^* \geq 0;\ \ t \in P$$

$$\sum_{t \in P} (p_t^* - b) D^t(\mathbf{p}^*) = K^* \sum_{t \in P} \lambda_t^* = \beta K^*$$

Peak Load problem with more general technologies

- The traditional formulation of the peak load problem assumes a fixed proportions, constant returns to scale production function.
  - E.g., $q_t = f(x_t,K) = \min\{x_t/a,K\} \Rightarrow V^t = V(q_t,K) = waq_t$ for $q_t \le K$.
- Extend to general production function and multiple types of capital and variable inputs:
  - E.g., $q_t = f(\mathbf{x}^t,\mathbf{K})$; $V^t = V(q_t,\mathbf{w},\mathbf{K})$; and $V(q,\mathbf{w},\mathbf{K}) = \min_{\mathbf{x}} \{\mathbf{wx}: f(\mathbf{x},\mathbf{K}) > q\}$
- This actually makes the problem *simpler* when V is "smooth:"

$$L = CS(\mathbf{p}) + \sum_{t=1}^{T}[p_t D^t - V(D^t,\mathbf{w},\mathbf{K})] - \beta\mathbf{K}$$

$$L_{p_s} = -D^s + \pi_s = \sum_{t=1}^{T}\left(p_t - V_q(D^t,\mathbf{w},\mathbf{K})\right)D_s^t = 0$$

$$L_{K_j} = \pi_{K_j} = -\beta_j - \sum_{t=1}^{T} V_{K_j}(D^t,\mathbf{w},\mathbf{K}) \le 0;\ K_j \ge 0;\ K_j L_{K_j} = 0$$

Neoclassical Peak Load Pricing under constant returns to scale:

If the technology exhibits constant returns to scale, then

- V is linearly homogeneous in q and $\mathbf{K}$.
- V is concave in q.

From the FONCs:

- Price equals SRMC in *all* periods
- Revenues in each period exceed variable costs
  - *All* periods "pay" for capacity
- Firm breaks even.

Technological assumptions dramatically change results of standard model.

$$p_t = V_q(D^t, \mathbf{w}, \mathbf{K}) \ \ \forall t$$

$$p_t D^t = D^t V_q(D^t, \mathbf{w}, \mathbf{K}) \geq V(D^t, \mathbf{w}, \mathbf{K}) \ \ \forall t$$

$$TR = \sum_{t=1}^{T} p_t D^t = \sum_{t=1}^{T} D^t V_q(D^t, \mathbf{w}, \mathbf{K})$$

$$TC = \sum_{t=1}^{T} V(D^t, \mathbf{w}, \mathbf{K}) + \beta \mathbf{K}$$

$$= \sum_{t=1}^{T} [D^t V_q(D^t, \mathbf{w}, \mathbf{K}) + \sum_{j=1}^{n} K_j V_{K_j}(D^t, \mathbf{w}, \mathbf{K})] + \beta \mathbf{K}$$

$$= \sum_{t=1}^{T} D^t V_q(D^t, \mathbf{w}, \mathbf{K}) + \sum_{j=1}^{n} K_j \left(\sum_{t=1}^{T} V_{K_j}(D^t, \mathbf{w}, \mathbf{K}) + \beta_j \right)$$

$$= TR$$

Another example: multiple fixed proportion technologies

- Most electric utilities have several types of generating units
- Some have high capacity costs, but low operating costs; others, the opposite.
- Consider J plant types: $\beta_1 > \beta_2 > \ldots, \beta_J$; $b_1 < b_2 < \ldots, b_J$

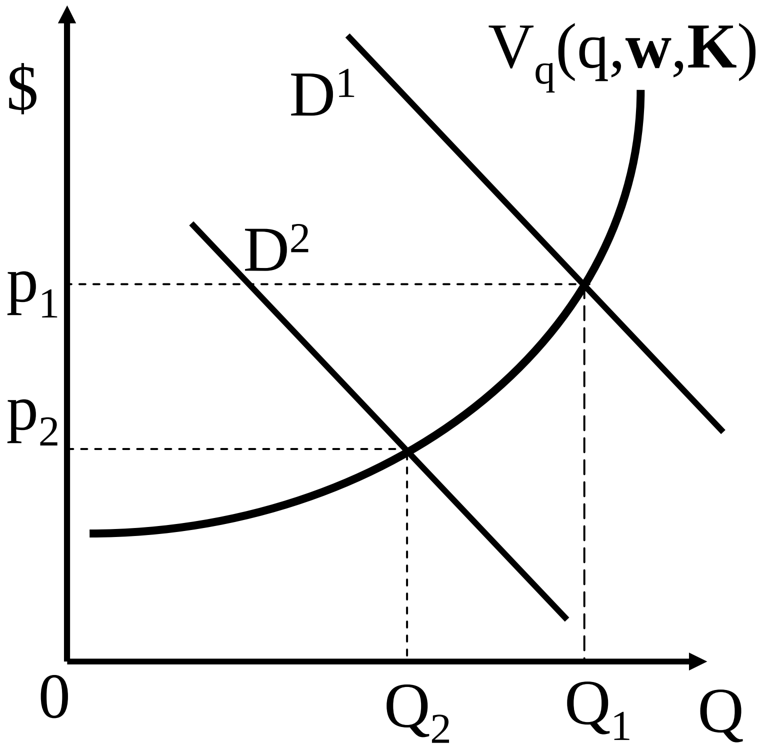
$$L = CS(\mathbf{p}) + \sum_{t=1}^{T} p_t D^t(\mathbf{p}) - \sum_{j=1}^{J} b_j \sum_{t=1}^{T} q_j^t - \sum_{j}^{J} \beta_j K_j + \sum_{t=1}^{T} \lambda_t \left(\sum_{j=1}^{J} q_j^t - D^t(\mathbf{p}) \right) + \sum_{t=1}^{T} \sum_{j=1}^{J} \gamma_j^t (K_j - q_j^t)$$

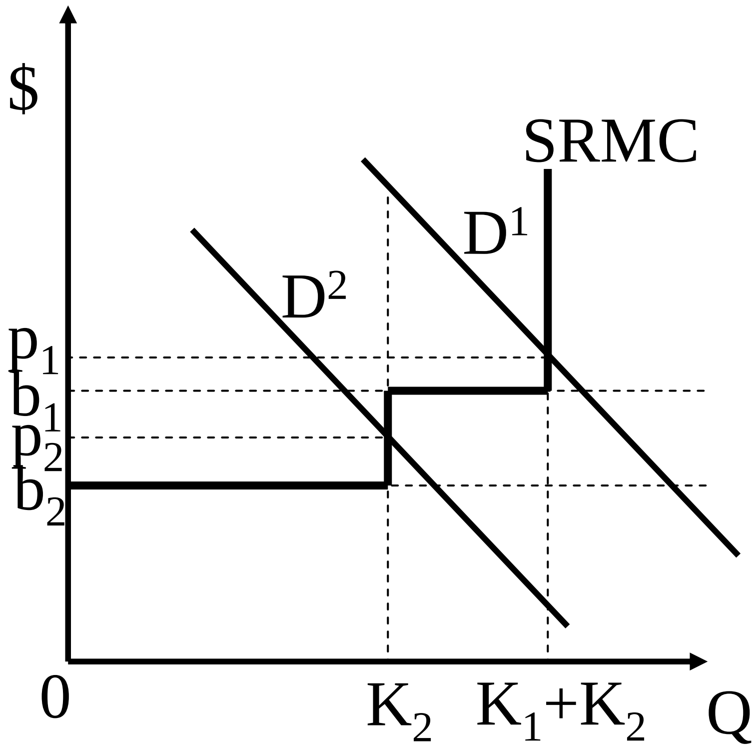
$$L_{p_s} = \sum_{t=1}^{T} (p_t - \lambda_t) D_s^t = 0$$

$$L_{q_k^s} = \lambda_s - b_k - \gamma_k^s \le 0;\ q_k^s \ge 0;\ q_k^s(\lambda_s - b_k - \gamma_k^s) = 0$$

$$L_{K_k} = \sum_{t=1}^{T} \gamma_k^t - \beta_k = 0$$

Diagrammatic analyses





Lecture 2B: Ramsey Pricing

Reading: Braeutigam Chapter in the Handbook of IO

Ramsey Pricing: The Problem

How to set prices as efficiently as possible for a multiproduct firm when:

(i) the firm must charge a uniform tariff (price) for each output

(ii) $p = \dfrac{\partial C}{\partial y_i} \quad \forall i \quad \Rightarrow \quad \pi < 0$

(iii) the firm must break even without external governmental subsidy.

Query: How would a single product firm set its uniform price to be efficient?

Let S(p) = consumers' benefit function (consumers' surplus)

π(p) = profit = py(p) - C(y(p),w)

W(p) = net economic benefit = S(p) + π(p)

First Best: What price is most efficient (without the breakeven constraint)?

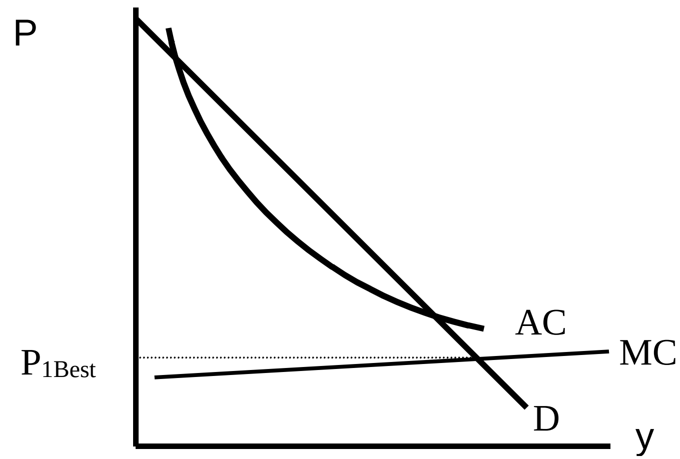
$$\max_p W(p) \Rightarrow S_p + \pi_p = -y(p) + y(p) + py_p - C_y y_p = 0 \quad .$$

$$\Rightarrow \quad p - C_y = 0$$

Second Best: What price is most efficient imposing the breakeven constraint?

$$\max_p W(p)$$

Subject to $\pi > 0 \Rightarrow p_{2Best} = ??$



Note dual relationship between two forms of the second best problem:

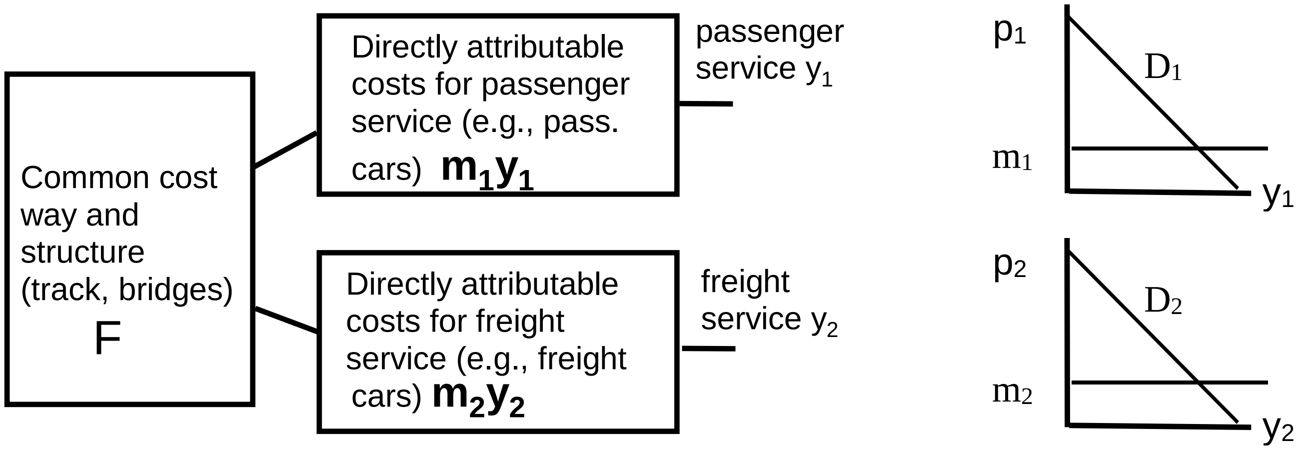
$$\max_p W(p) \qquad \min_p DWL(p)$$

Subject to $\pi > 0$. Subject to $\pi > 0$.

Problem: Efficient Pricing with Common Costs

Common costs are shared costs, incurred in a production process in which the ratios of the outputs can be varied.

Example: Railroad
Assume total cost is affine: $C = F + m_1y_1+m_2y_2$
Demands: $p_1(y_1)$ and $p_2(y_2)$



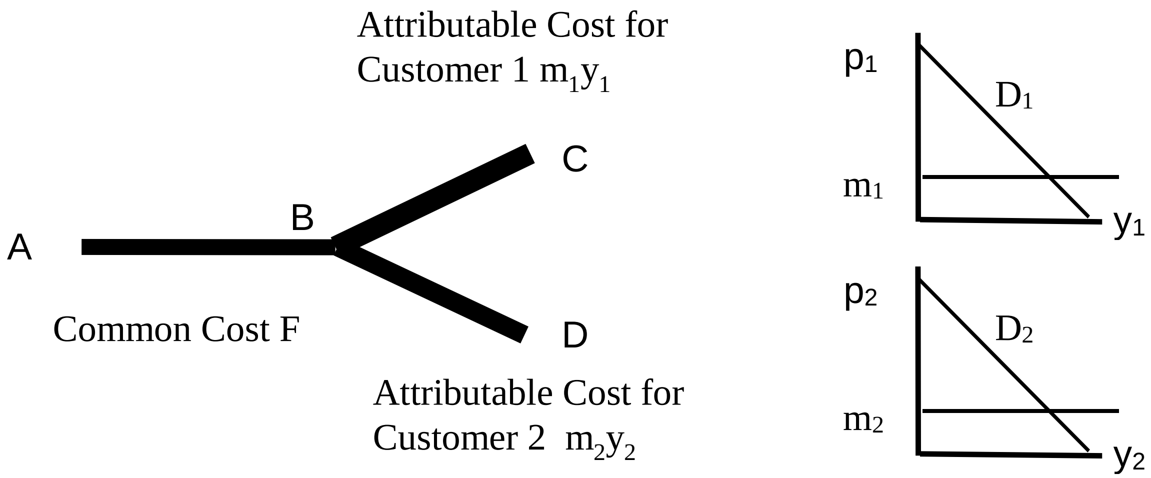
Query: How to set the most efficient uniform prices p_1 and p_2 so that enable the firm to avoid financial losses?

Problem: Efficient Pricing with Common Costs

Example: Pipeline network

Assume total cost is affine: $C = F + m_1y_1 + m_2y_2$

Demands: $p_1(y_1)$ and $p_2(y_2)$



Query: How to set the most efficient uniform prices p_1 and p_2 so that enable the firm to avoid financial losses?

Ramsey Pricing Problem: Multiproduct Firm

Consider an N product monopoly, services $i = 1, \ldots , N$

Prices: $p = (p_1 , \ldots, p_N)$.

Demands $y(p) = y_1(p), y_2(p), \ldots, y_N(p)$

$S(p)$ = consumers' benefit function (consumer surplus) .

$\pi(p) = \text{profit} = py(p) - C(y(p),w)$

$W(p) = \text{net economic benefit} = S(p) + \pi(p)$

Ramsey Optimality:

$$\max_p \; W(p) = S(p) + py(p) - C(y(p),w)$$

subject to: $\pi = py(p) - C(y(p),w) \geq 0$

Form Lagrangian

$$L = S(p) + py(p) - C(y(p),w) + \lambda[py(p) - C(y(p),w)]$$
$$= S(p) + [1 + \lambda][\, py(p) - C(y(p),w)]$$

FONC:

$$\frac{\partial L}{\partial p_i} = -y_i + [1 + \lambda][y_i + \sum_j (p_j - C_j)\frac{\partial y_j}{\partial p_i} = 0 \quad \forall i \quad .$$

(1) $$\sum_j \frac{(p_j - C_j)}{p_j}\frac{\partial y_j}{\partial p_i}\frac{p_j}{y_i} = -\frac{\lambda}{1+\lambda} \quad \forall i \quad .$$

and

(2) $$\pi = 0.$$

Ramsey Optimality: Independent Demands

$$\frac{(p_i - C_i)}{p_i}\frac{\partial y_i}{\partial p_i}\frac{p_i}{y_i} = \frac{(p_j - C_j)}{p_j}\frac{\partial y_j}{\partial p_j}\frac{p_j}{y_j} = -\frac{\lambda}{1+\lambda} \qquad \forall i,j \qquad .$$

(1) $$\frac{(p_i - C_i)}{p_i}E_{y_i,p_i} = \frac{(p_j - C_j)}{p_j}E_{y_j,p_j} = -\frac{\lambda}{1+\lambda} \qquad \forall i,j$$

and

(2) $\pi = 0.$

$\frac{(p_i - C_i)}{p_i} =$ "markup" of price over MC (Lerner index)

$\frac{(p_i - C_i)}{p_i}E_{y_i,p_i}$ = "Ramsey number" for product i

Ramsey Numbers

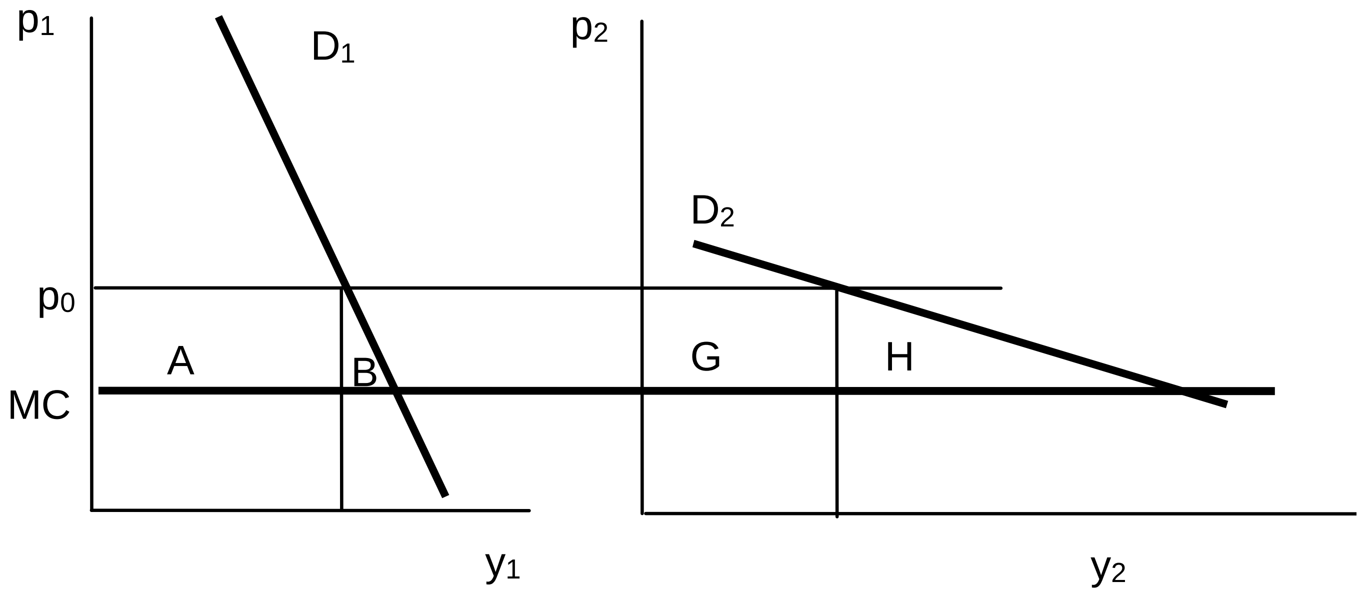
$$RN_i \equiv \frac{(p_i - C_i)}{p_i} E_{y_i, p_i} = -\frac{\lambda}{1+\lambda} \quad .$$

Range of values for RN_i ?

Implications for optimal pricing if $RN_i = 0$?

Implications for optimal pricing if $RN_i = -1$?

Inverse Elasticity Rule $\frac{(p_i - C_i)}{p_i} E_{y_i, p_i} = \frac{(p_j - C_j)}{p_j} E_{y_j, p_j}$.



Suppose $C = F + m(y_1 + y_2)$

Firm just breaks even when initial price in both markets is p_0.

What area(s) on graph measure fixed cost F?

DWL at initial price?

How does the IER suggest that prices should be changed if efficiency is to be increased (DWL decreased) while allowing the firm to break even?

Ramsey Pricing: Numerical Example

Costs:

$$C = \begin{cases} 2000 + 20y_1 + 30y_2 & \text{when } y_1 > 0, \text{ and } y_2 > 0 \quad . \\ 2000 + 20y_1 & \text{when } y_1 > 0, \text{ and } y_2 = 0 \quad . \\ 2000 + 30y_2 & \text{when } y_1 = 0, \text{ and } y_2 > 0 \quad . \end{cases}$$

Demands: $p_1 = 200 - 3y_1$ $\qquad$ $p_2 = 300 - 2y_2$.

At a Ramsey Optimum

$$(1) \quad \pi = p_1y_1 + p_2y_2 - C = 0$$

$$= (200 - 3y_1)y_1 + (300 - 2y_2)y_2 - 2000 - 20y_1 - 30y_2 = 0 \quad .$$

$$(2) \quad \frac{(p_1 - C_1)}{p_1}\frac{\partial y_1}{\partial p_1}\frac{p_1}{y_1} = \frac{(p_2 - C_2)}{p_2}\frac{\partial y_2}{\partial p_2}\frac{p_2}{y_2}$$

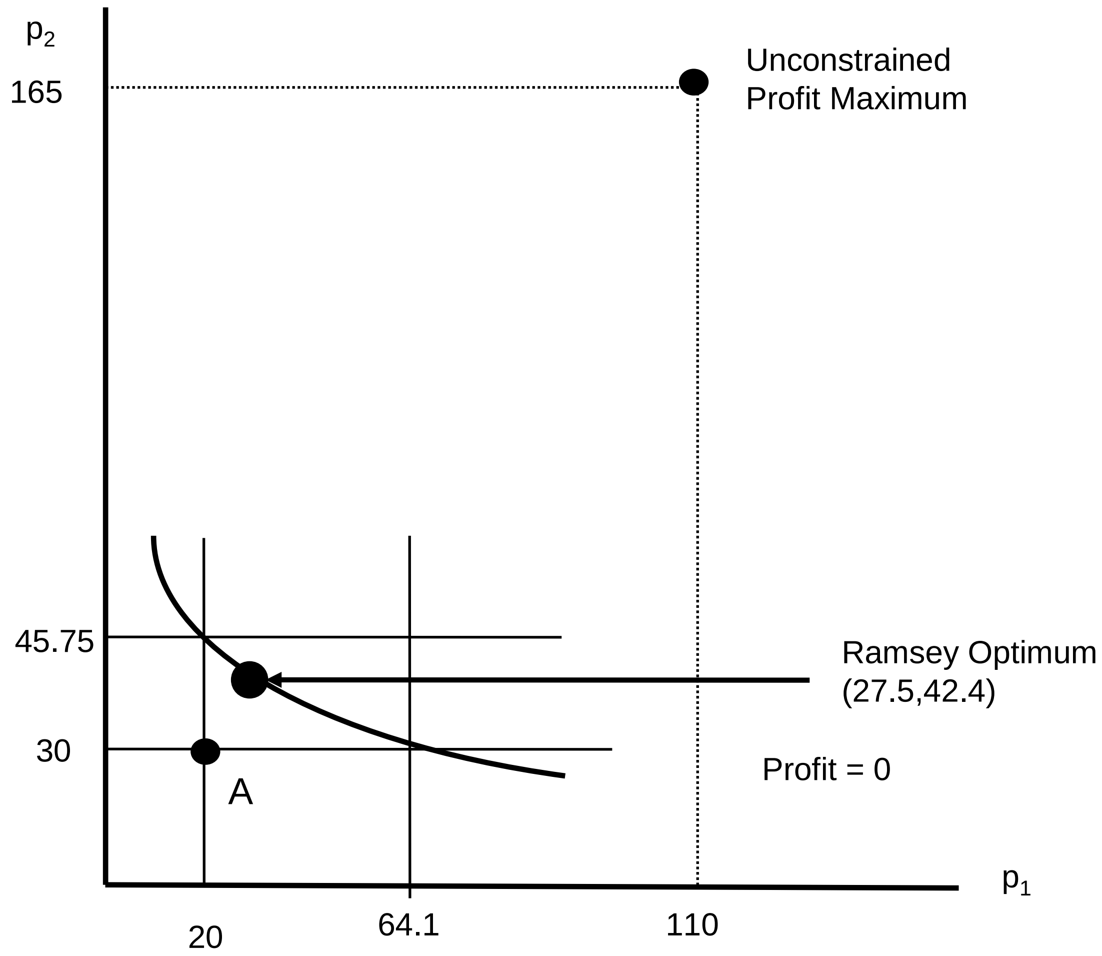
$$(200 - 3y_1 - 20)\left(-\frac{1}{3y_1}\right) = (300 - 2y_2 - 30)\left(-\frac{1}{2y_2}\right) \quad .$$

Approximate Solution:

$y_1 = 57.5$; $p_1 = 27.5$; $y_2 = 128.9$; $p_2 = 42.4$

Service	Market Revenue	Attributable Cost	Contribution	% common cost covered
y_1	p_1y_1 =27.5(57.5) = **1580**	m_1y_1 = 20(57.5) = **1150**	1580-1150 = **430**	430/2000 **21.5%**
y_2	p_2y_2 =128.9(42.4) = **5450**	m_2y_2 = 30(128.9) = **3880**	5450 - 3880 = **1570**	1570/2000 **78.5%**

Ramsey Optimum Graphically



Query: What about Ramsey optimality if the regulated firm does not have a monopoly in all of its markets?

Examples:
Railroad competes with fringe of motor carriers or water carriers.
Telephone company with a competitive fringe
Electric power generators with fringe of nonutility generators (NUGs)
Post Office, competing in some markets against UPS, Federal Express

Illustration: A RR facing a competitive fringe of truckers
RR provides N services, $\mathbf{y_r} = (y_{1r}, ..., y_{Nr})$ at prices $\mathbf{p_r} = (p_{1r}, ..., p_{Nr})$
Truckers provide same N services $\mathbf{y_t} = (y_{1t}, ..., y_{Nt})$
at prices $\mathbf{p_t} = (p_{1t}, ..., p_{Nt})$

Assume $y_{ir}(p_{ir},p_{it})$ for all i. Services are imperfect substitutes across modes of transport, but demands are independent across commodity types i.

There are supply schedules for truckers: $Q_{it}(p_{it})$ for all i.

Variations of Ramsey Optimality

I. (Totally Regulated Second Best) Regulate all prices for RR and trucks.

max W s.t. Railroad Profit $\geq$ 0
($\mathbf{p_r}$,$\mathbf{p_t}$)

- Complicated pricing rules, setting 2xN prices
  (or MxN prices, if M modes)

- Need information on cross elasticities of demand

- p_{it} > MC-$_{it}$. Will need to limit entry as well as regulate prices. (How?)

For reference later: shows that Ramsey prices may not be "sustainable" against entry.)

Another Variation

II. (Partially Regulated Second Best) Regulate all RR prices, but let the market price for trucking transport be determined by supply and demand.

$$\max_{(\mathbf{p_r})} W$$

s.t. (1) Railroad Profit ≥ 0
and (2) $Q_{it}(p_{it}) = y_{it}(p_{it}, p_{ir})$ for all i. (Markets for trucking services clear.)

- Set only N prices (for the RR)

- Don't need information on cross elasticities of demand across modes

- Get usual IER, where the elasticities of demand are the ones facing the RR firm.

Lecture 2C: Nonlinear Pricing

Reading: Braeutigam Chapter in the Handbook of IO

Notes on Nonlinear Pricing

Definition: A **nonlinear outlay schedule** is an expenditure schedule in which the average outlay is not constant as the number of units purchased varies.

Example 1: $E = e + my$

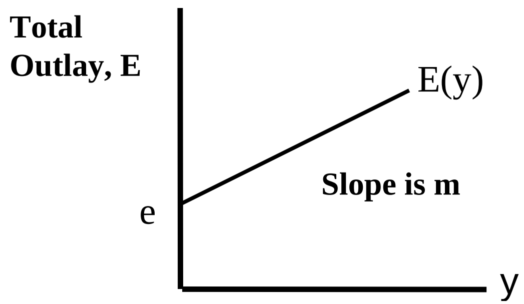
e = fixed component

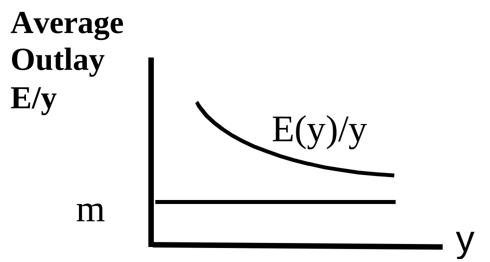
m = variable charge per unit purchased

y = number of units purchased

This affine tariff structure is often referred to as a “two-part” tariff.

Note: Although E is linear, the outlay Schedule is said to be nonlinear, because E/y is nonlinear in y

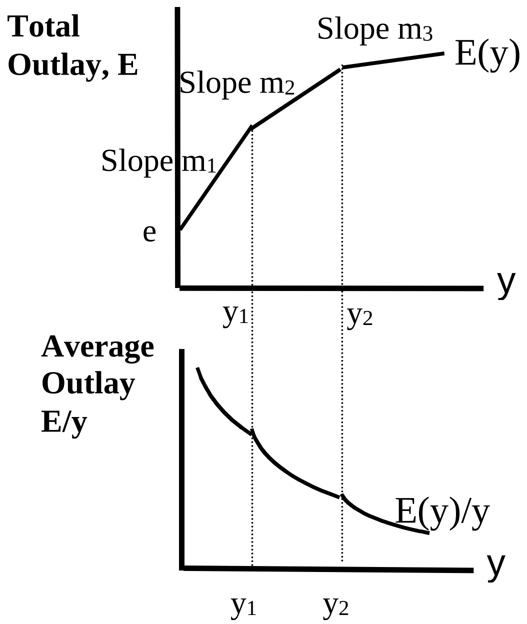




Examples of Nonlinear Pricing …

Example 2: "Four-part" tariff e, m_1, m_2, m_3

$$E = \begin{cases} e + m_1 y & \text{if } y \le y_1 \\ e + m_1 y_1 + m_2 (y - y_1) & \text{if } y_1 \le y \le y_2 \\ e + m_1 y_1 + m_2 (y_2 - y_1) + m_3 (y - y_2) & \text{if } y_2 \le y \end{cases} .$$

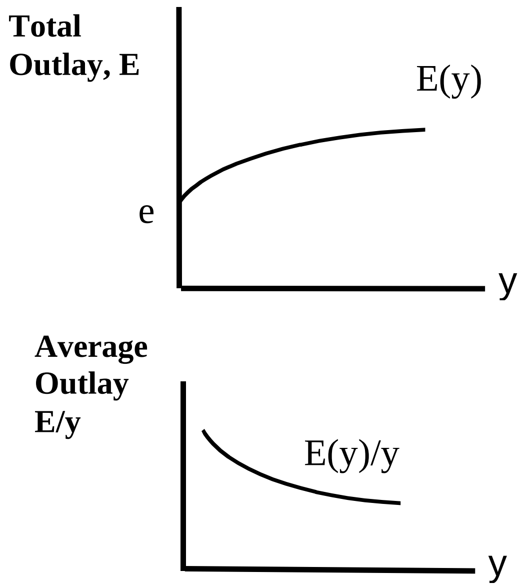


Examples of Nonlinear Pricing …

<u>Example 3</u>: Continuously varying "marginal price"

E = E(y)

Like an "infinite-part" tariff



Pareto Improving Nonlinear Outlay Schedule

Any uniform price not equal to marginal cost can be dominated by a nonlinear outlay schedule.

Willig, R., "Pareto Superior Nonlinear Outlay Schedules," *Bell Journal of Economics*, 1978

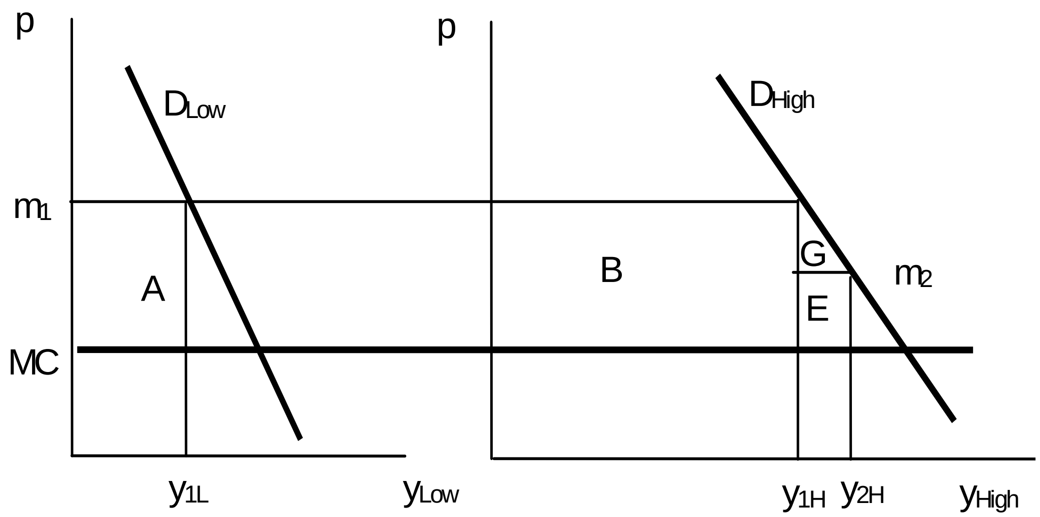
<u>Illustration</u>:
Two customers
"low" user, D_L
"high user, D_H

Affine Cost
$C = F + (MC)(y_L+y_H)$

m_1=lowest <u>uniform</u> tariff allowing the firm top break even
(area A+ area B = F)

Now introduce nonlinear tariff

$$E = \begin{cases} m_1 y_i & \text{if } y_i \le y_{1H} \\ m_1 y_{1H} + m_2(y_i - y_{1H}) & \text{if } y_{1H} \le y_i \end{cases} .$$



Effects of new tariff:
1) L unaffected, no worse off
2) H consumes more, gains consumer surplus G
3) Firm profit increases by E.
4) Firm could take part of E and lower m_1 to make everyone strictly better off.

Query: Is new tariff "first best?"

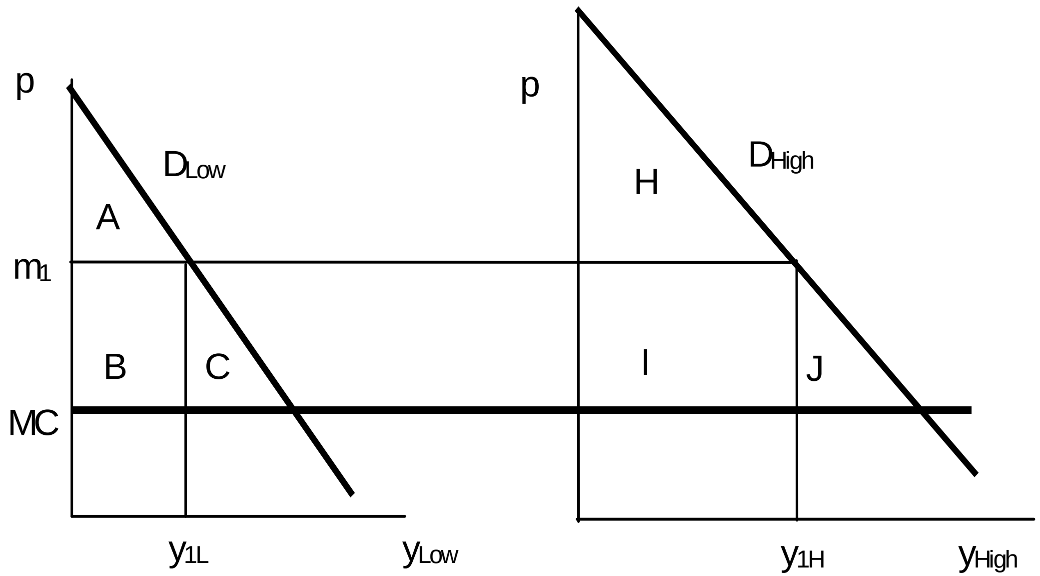
Pareto Improvements with "n-part" tariff

Illustration: Two customers

$C = F + (MC)(y_L+y_H)$

m_1 = lowest uniform tariff allowing the firm to break even (area B+ area I = F)

$CS_L = A$ $CS_H = H$ DWL = C+J



Now charge each customer the **same** two-part tariff

Fixed component e = F/2

Marginal price = b = MC

Total outlay: $E_i = F/2 + by_i$

IF A+B+C > F/2, then L purchases service (so does H because H+I+J > A+B+C. **First best ???**

Suppose A+B+C < F/2. Can we charge L fixed component $e_L < e_H$, but still have $e_L + e_H = F$? This form of price discrimination requires the firm to

(1) identify customer type and (2) prevent resale.

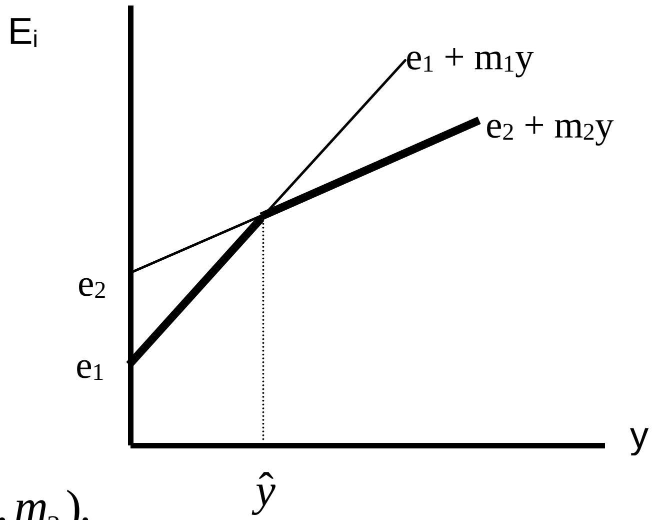
Principle: The limit on the efficiency of uniform entry fees ($e_i = e_j$ for all customers) is the elasticity of membership in the system with respect to e.

Nonlinear Pricing with Asymmetric Information

Consumer knows his own type (large, small), but the firm does not.

If price discrimination is not implementable, try offering everyone <u>the same set of tariff options</u>.

Example: $$E = \begin{cases} e_1 + m_1 y \\ e_2 + m_2 y \end{cases}$$



With $e_1 < e_2$ and $m_1 > m_2$.

Consumers will "self select" to be on the lower envelope of the options.

Mr. H, who plans on consuming $y > \hat{y}$, chooses (e_2, m_2).

Mr. L, who plans on consuming $y < \hat{y}$, chooses (e_1, m_1).

This is referred to as a "self selecting two-part tariff."

Question: Is it possible to increase total surplus by adding more two-part options?

Suppose we have J types of consumers.

have n tariff options in force, with $e_1 < e_2, \ldots < e_n$ and $m_1 > m_2 \ldots\ldots > m_n$.

•If (n+1)<J, then a Pareto improvement in welfare is possible by introducing another option.

Introduce (e_{n+1}, m_{n+1}), with $e_{n+1} > e_n$ and $m_{n+1} > m_n$ (and $m_{n+1} > b = MC$)
(This is a direct extension of Willig's result. The largest customer will now buy even more and realize higher surplus. Profits of the firm will also increase.)

•The arguments above address Pareto superior tariffs (not "optimal" tariffs).

Suggests that an <u>optimal</u> self selecting two part tariff will sell output to the largest class of users so that marginal price (m) = marginal cost (b). Otherwise an opportunity exists for a Pareto improvement.

Exception: (Ordover, J. and Panzar, J., "On the Nonexistence of Pareto Superior Outlay Schedules, *Bell Journal* (1980).
If "customers" are downstream firms in a competitive industry, then it might NOT be optimal to set m = b for the largest customer because that generates a pecuniary competitive advantage for large firms.

Optimal Nonlinear Outlay Schedules

θ index of consumer types $\qquad \theta_{\mathrm{L}} \le \theta \le \theta_U$

$p(y,\theta) =$ inverse demand for consumer of type θ

$p_y < 0$ and $p_\theta > 0$ (noncrossing assumption)

$g(\theta)$ = number of consumers of type θ (a PDF)

$$\int_{\theta_L}^{\theta_U} g(\theta)d\theta = 1$$

$G(\theta) =$ CDF on θ.

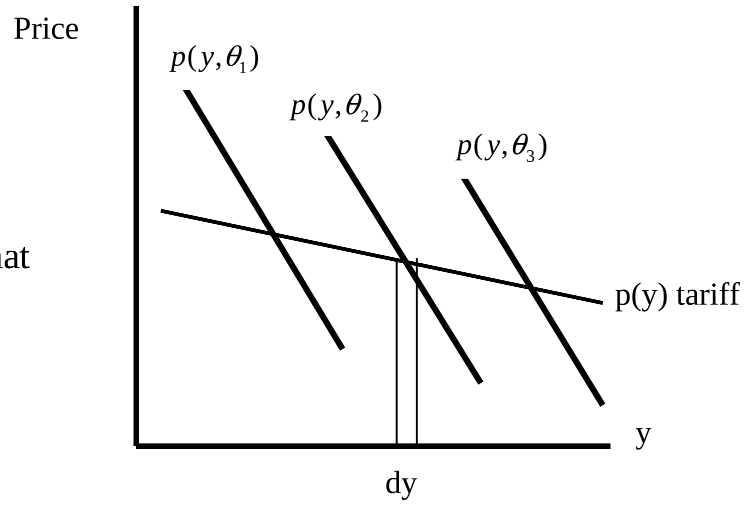
$c =$ marginal cost, a constant

$F =$ fixed cost

$p(y) =$ the tariff schedule [not to be confused with demand $p(y,\theta)$]

Optimal Nonlinear Tariffs (continued)

All consumers face tariff p(y)



For any given y, there will be a critical value of θ ($\hat{\theta}$) such that

if $\theta < \hat{\theta}$, θ will not buy the y^{th} unit

if $\theta > \hat{\theta}$, θ will buy the y^{th} unit

if $\theta = \hat{\theta}$, θ is indifferent about buying the y^{th} unit

In the graph, consider the dy^{th} market. Then $\hat{\theta} = \theta_2$.

Consumers of type θ_1 will select out of the dy^{th} market.

Consumers of type θ_3 will select into the dy^{th} market.

The self selection condition defining $\hat{\theta}$ is $p(y,\hat{\theta}) = p(y)$.

Optimal Nonlinear Tariffs (continued)

Objective: Choose $p(y)$ to max W (total surplus), subject to $\pi \geq 0$.

$$\max_{p(y)} W = \int_0^\infty \left\{ \underbrace{\int_{\hat{\theta}}^{\theta_U} \left[p(y,\theta) - p(y) \right]}_{\text{Consumer surplus in } dy^{th} \text{ market for consumers of type } \theta, \text{ "in" the market}} \underbrace{g(\theta) d\theta}_{\text{Number of consumers of type } \theta} + \underbrace{\left[1 - G(\theta) \right]}_{\text{Number of consumers in the market when } \theta = \hat{\theta}} \underbrace{\left[p(y) - c \right]}_{\text{Variable profit per unit sold}} \right\} dy - F$$

subject to: $\pi = \int_0^\infty \left[1 - G(\theta) \right] \left[p(y) - c \right] dy - F \geq 0$

Form the Langrangean $H = W + \lambda\pi$

(Don't need costate variable because there is no state equation.)

$$H = \int_0^\infty \left\{ \int_{\hat{\theta}}^{\theta_U} \left[p(y,\theta) - p(y) \right] g(\theta) d\theta + \left[1 - G(\theta) \right] \left[p(y) - c \right] \right\} dy - (1+\lambda)F$$

First Order Conditions for Optimal Tariff *p(y)*

$$\frac{\partial H}{\partial p(y)}=\int_0^{\infty}\left\{\int_{\hat\theta}^{\theta_U}[-1]g(\theta)d\theta-\overbrace{\left[p(y,\hat\theta)-p(y)\right]}^{=0}g(\hat\theta)\frac{\partial\hat\theta}{\partial p(y)}+(1+\lambda)\left[\left(-g(\hat\theta)\right)\frac{\partial\hat\theta}{\partial p(y)}[p(y)-c]+\left[1-G(\hat\theta)\right]\right]\right\}dy=0$$

The FONC is satisfied if the kernel of the integral is zero at every y:

$$\lambda\left[1-G(\hat\theta)\right]-(1+\lambda)[p(y)-c]\left[g(\hat\theta)\frac{\partial\hat\theta}{\partial p(y)}\right]=0$$

$$\Rightarrow\frac{p(y)-c}{p(y)}=\frac{\lambda}{1+\lambda}\frac{1-G(\hat\theta)}{\left[g(\hat\theta)\frac{\partial\hat\theta}{\partial p(y)}p(y)\right]}\quad(*)$$

Interpretation of the Optimal Tariff $\frac{p(y)-c}{p(y)} = \frac{\lambda}{1+\lambda} \frac{1-G(\hat{\theta})}{\left[g(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial p(y)} p(y) \right]}$ (*)

The number of customers who buy the dy^{th} unit is $1-G(\hat{\theta})$.

Thus $1-G(\hat{\theta})$ is the quantity (Q) sold in that market.

Thus $\frac{\partial Q}{\partial p(y)} = -g(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial p(y)}$.

Elasticity in the dy^{th} market: $E_{Q,p(y)} = \frac{\partial Q}{\partial p(y)} \frac{p(y)}{Q} = -g(\hat{\theta}) \frac{\partial \hat{\theta}}{\partial p(y)} \frac{p(y)}{1-G(\hat{\theta})}$

Thus, (*) can be written as

$\frac{p(y)-c}{p(y)} = \frac{\lambda}{1+\lambda} \frac{1}{E_{Q,p(y)}}$ An Inverse Elasticity Rule for the dy^{th} market !

Note: For the final unit purchased, $\theta = \theta^U$, so that G($\hat{\theta}$)=1.

Thus, in (*), $p(y) = c$. This verifies the principle of optimality suggested earlier, for the case in which all consumers are end users (rather than competitive enterprises).