The Silk Road:

Tax competition among nation states

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Abstract

All international trade involves the shipment of commodities from one nation to another. Many commodities, before reaching their final destinations, are transshipped through several nations, each having independent authorities to tax commodities in transit. When trade is repeated over time, such “middle” nations may lose monopoly power over commodities in transit.

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1. Introduction

The ancient civilizations of China and the Mediterranean were connected by the legendary Silk Road, the nearly 5,000-mile-long trade route that meandered through many areas of influence, in which the independent nation states exacted tolls from traveling merchants. Analyzing a game-theoretic model of such trade routes, Karni and Chakrabarti (1997) have found that the nation states collectively suffer from the double marginalization problem so that unification of them into one empire, such as achieved under the Mongols, leads to more trade and tax revenues. Gardner, Gaston and Masson (2002) have reached similar conclusions in their theoretical analysis of cargo shipments down the medieval Rhine when local barons constructed castles along the river to exact tolls for the right of passage.

As these examples vividly illustrate, in market economies trade between producers and final users of the products involve successive layers of intermediaries. Manufactures and farm produce are sold to wholesalers and then to retailers before reaching their final consumers. Oil and natural gas can reach overseas consumers through pipelines across foreign countries, which demand user fees. Other examples include telephone calls involving local and long-distance or international telephone service providers, and vacation trips consisting of hub-to-hub flights by one carrier and local flights by another. In each of these cases, the double marginalization problem can arise when more than one intermediary have monopoly power over the commodities in transit. As is well known, however, the problem can be mitigated if two or more in a chain of monopolies are integrated to coordinate prices. Furthermore, vertical integration is known to be socially desirable as the price is lowered and output increased.
As Feinberg and Kamien (2001) have argued, however, the double marginalization problem arises from the implicit assumption that prices and tolls are pre-committed to before transactions take place. This indeed is the assumption made in the works of Karni and Chakrabarti (1997) and Gardner et al. (2002). If trade instead occurs sequentially, the lack of commitment leads to the disappearance of the double marginalization problem, but as shown by Feinberg and Kamien (2001) the economy instead suffers from the holdup problem. To demonstrate this result, suppose that a merchant must deliver his ware to the market over the road that goes through two domains. Assuming that the merchandise loses its value unless delivered to the market, suppose that the baron of the second domain, when the merchant has arrived there, demands the entire market value of the merchandise in exchange for the right of passage through his domain. Then, the merchant is indifferent between acceding to this demand and discontinuing the journey, for either way he will lose the value of his merchandise. Therefore the baron of the second domain can capture (almost) the entire value of the merchandise with a slightly lower tax to induce the merchant to complete his journey.

However, a forward-looking merchant anticipates the taxing strategy of the second baron. Since he loses all his value once having reached the second domain, the merchant will not embark on a journey unless he can cross the first domain tax-free. The conclusion: when trade occurs the baron of the second domain captures the entire value of the merchandise. Since only the second baron exercises monopoly power, there is no double marginalization. Furthermore, social welfare is greater than when the two barons commit to the taxes before the merchant embarks on a journey.
In this paper I argue that the hold-up problem in the Feinberg-Kamien analysis is a consequence of the assumption that the game is played only once. Although one can think of real-world examples in which one-shot games are appropriate, many transactions are rarely one-shot affairs. Therefore, in this paper I analyze a model of infinitely repeated transactions. I show that trade thrives despite trade costs, and that the equilibrium price is lower than when the barons contract on prices in a one-shot game, but higher than when all the domains are integrated into a single monopoly. The intuition underlying these results is as follows. In a repeated-game setting, the baron of the domain which merchants last cross can always act myopically, exacting the value of the commodity as in the one-shot game. However, if it is assumed that no merchant will embark on a journey in the future once he suffers financial losses in the past, then the last baron may want to exact a lower tax to guarantee the continuation of trade. If this long-run strategy yields greater revenues than the one-time gain from acting myopically, then trade can continue occur. The objective of this paper is to investigate the nature of such an equilibrium outcome.

When there are more than two monopolies in a succession, we show that the entire value of the merchandise is shared between the first and the last baron while the barons of all the “middle” domains fail to capture any rent despite their monopoly positions over commodities in transit. The following is the reason. What makes the baron of the last domain act myopically or patiently is the sum of taxes the merchants paid before getting there. If this sum is high, the last baron has to lower his tax for the continuation of trade in the future, which makes it more attractive to act myopically. I show that there is a threshold level of sum of taxes collected by all the barons before the
last. If the actual sum of taxes the merchant paid exceeds this threshold the last baron acts myopically, thereby ending future trade. Thus, it is in the best interest of all the other barons to keep this sum below the threshold to ensure future trade. But then the baron of the first domain exercises a first-mover advantage, setting his tax equal to that threshold, leaving no room for maneuver for the barons of the middle domains. Thus, the first and the last monopolists exact the entire value of the merchandise.

The remainder of the paper is organized in four sections. The next section revisits the Feinberg-Kamien model. Section 3 extends the model to cases of repeated interplays. Section 4 extends the model further to a case of multiple road segments. Section 5 concludes.

2. The one-shot game

Suppose that merchants produce a homogeneous product in one country and sell it in another country. Merchants must deliver the product to the market. Choose units so that each merchant delivers one unit of merchandise to the market. Let x denote the quantity of the product delivered to the market. Treat x as a real number to ease exposition. Market (inverse) demand is given by $p(x)$, a differentiable function with respect to $x > 0$, with first and second derivatives denoted by $p'(x) < 0$ and $p''(x) \leq 0$.

To reach the market merchants must go through two nation states and pay taxes there. For example, the first nation may be their home country, which imposes an export tax while the second nation may be where the market is, imposing an import tax. Consider the following four-stage game.

Stage 1: The first state announces a tax rate $t_1 \geq 0$.  
Stage 2: Observing \( t_1 \), all the merchants simultaneously decide whether to cross the first state or not.

Stage 3: After the merchants have crossed the first state, the second posts a tax rate \( t_2 \geq 0 \).

Stage 4: Observing \( t_2 \), the merchants decide whether to cross the second state.

Nation state \( i \) incurs costs \( c_i (i = 1, 2) \) per merchant crossing its territory \((c_i \geq 0)\). Each state’s payoff is the net tax revenue it collects from the merchants (the difference between the total taxes collected and the total cost incurred). A merchant’s payoff is the difference between the market price of the commodity (zero in case of non-delivery) and the sum of taxes he pays to get to the market. There are no other markets, and all the taxes are sunk after they are paid. We normalize each merchant’s default payoff to zero, and adopt the tie-breaking rule that a merchant sets out on a journey as long as he expects a non-negative payoff. Finally, we assume that \( p(0) > c_1 + c_2 \), so delivery of the commodity to the market is socially desirable.

We solve the game backwards. Let \( x_1 \) be the number of merchants who has paid \( t_1 \) and crossed the first state. Now they observe \( t_2 \), and each merchant reasons as follows. If he crosses the second state and expects \( x_2 (\leq x_1) \) other merchants to do the same, his payoff will be

\[
p(x_2) - t_2 - t_1.
\]

On the other hand, if he does not, his payoff will be \(-t_1\), since the first tax payment is sunk. Therefore, a merchant goes forward if and only if

\[
p(x_2) - t_2 - t_1 \geq -t_1
\]
or

\((1) \quad p(x_2) - t_2 \geq 0.\)

Moving backward to the third stage of the game, state 2 chooses \(t_2\) to maximize the net tax revenue \((t_2 - c_2)x_2\) subject to the constraint \((1)\) and \(x_2 \leq x_1\). Ignoring the second inequality for the moment, maximization of the Lagrangian

\[(t_2 - c_2)x_2 + \lambda[p(x_2) - t_2]\]

yields the optimality conditions

\[t_2 - c_2 + \lambda p'(x_2) = 0; \quad \text{and} \quad x_2 - \lambda = 0,\]

where \(\lambda\) is the Lagrangian multiplier. The two equations above combine to yield

\[(2) \quad p(x_2) + p'(x_2)x_2 - c_2 = 0.\]

The first two terms constitute marginal revenue. Thus, \((2)\) is the standard first-order condition for a monopolist facing demand \(p(x)\) and constant marginal cost \(c_2\). Given the assumption on \(p(x)\), \((2)\) has a unique solution, denoted by \(x^m_2\). The (maximum) monopoly rent to state 2 equals

\[\pi^m_2 \equiv [p(x^m_2) - c_2]x^m_2.\]

Substituting into the constraint equation from \((1)\), we obtain state 2’s optimal tax rate:

\[t^m_2 = p(x^m_2).\]

Now return to the constraint \(x_2 \leq x_1\) that we have so far ignored. Suppose that \(x^m_2 < x_1\). Then, if all \(x_1\) merchants paid the tax \(t^m_2 = p(x^m_2)\) and crossed the second state, the payoff to each merchant will be \(p(x_1) - p(x^m_2) - t_1\) which is less than \(-t_1\). Therefore, not
all merchants will cross the second state to reach the market. In equilibrium, the merchants have rational expectations so that exactly \( x_2^m \) merchants cross the second state while \( x_1 - x_2^m \) do not. However, each merchant has the same payoff of \( -t_1 \).

Suppose alternatively that \( x_1 \leq x_2^m \). Then, the left-hand side of (2) is strictly positive at \( x_1 < x_2^m \). Therefore, the optimal tax is \( t_2 = p(x_1) \), implying all the merchants \( x_1 \) cross the second segment, but again they all earn \( -t_1 \). To sum, state 2’s optimal strategy is \( t_2 = p(x_1) \) if \( x_1 \leq x_2^m \) and \( t_2 = p(x_2^m) \) if \( x_1 > x_2^m \).

We now move back to the second stage of the game. Given state 2’s optimal strategy, the merchants, once having crossed the first state, expect to earn the negative profit, \( -t_1 \), whether they would cross the second or not. Therefore, the merchants will embark on journeys only if \( t_1 \leq 0 \). But state 1 is willing to charge \( t_1 = 0 \) only if \( c_1 = 0 \).

Alternatively, suppose that that \( c_1 = 0 \) but that each merchant incurs some trade cost \( c > 0 \), which can also be his opportunity cost. Since state 2 exacts the entire sales revenue from the merchants that have crossed the first state, the merchants cannot recover their trade costs. Thus, the merchants never travel. Thus, the presence of trade cost borne by the merchants or by the first nation state, no matter how small, results in the absence of trade in the one-shot game, even though trade is socially desirable. Thus,

**Proposition 1:** In the one-shot game with two nation states, trade takes place if and only if the first nation state incurs no trade cost \( (c_1 = 0) \) and the merchants’ opportunity costs
are zero. When trade occurs, the second state captures the entire value of the merchandise.

3. Repeated interactions

In this section we reconsider the above problem in a repeated-game setting. There are an infinite number of periods. Each merchant makes one delivery per period. While no trade is still a possible equilibrium outcome under the conditions of the previous section, there is an alternative equilibrium outcome in which trade occurs. In this section we examine the properties of such an equilibrium.

Assume that each merchant adopts the following strategy. In the first period he decides whether to embark on a journey. In any subsequent period, he sets out on a new journey if and only if he has never suffered a loss in the past. The merchants who stayed home in the first period never travel in later periods.

I next specify the equilibrium strategies of two nation states that result in trade. I look for a stationary subgame-perfect equilibrium outcome with the following characteristics. In each period, the constant number $x^*$ of merchants embark on new journeys and the states demand the taxes at constant rates, $t_1^*$ and $t_2^*$, respectively. I assume that $x^* \leq x^m_2$, and justify this assumption shortly.

I first show that the second state’s equilibrium tax is given by $t_2 = p(x^*) - t_1$. To see this, suppose that the equilibrium tax satisfies $t_2 < p(x^*) - t_1$. Then, the merchants would earn positive profits, so the second nation can raise the tax to increase its tax revenue. If $p(x^*) > t_2 > p(x^*) - t_1$ all the $x^*$ merchants will cross the second state but all
will suffer losses, \( p(x^*) - t_1 - t_2 < 0 \), ending all future trade. Finally, if \( t_2 > p(x^*) \), then as I explained in the previous section, only a fraction, say, \( x_2 \), of the \( x^* \) merchants will cross the second state to deliver the merchandise, where \( x_2 \) is given by \( t_2 = p(x) \), while the remainder, i.e., \( x^* - x_2 \), will end their journeys prematurely. In this case, however, all the \( x^* \) merchants suffer the negative profit \( -t_1 \) and will never come back in the future. Thus, \( t_2 = p(x^*) - t_1 \) is the only tax consistent with the equilibrium outcome in which there is trade in every period.

I have shown that the equilibrium net revenue per period to the second nation state is \([p(x^*) - t_1 - c_2]x^*\). Adding up over periods leads to the discounted sum of profits:

\[
v_2 = [p(x^*) - t_1 - c_2]x^*/(1 - \delta)
\]

where \( \delta \in (0, 1) \) is the (common) discount factor. Now, consider a deviation by state 2. Deviating from the equilibrium, state would set the tax rate equal to \( p(x^*) \) to earn the entire value, \([p(x^*) - c_2]x^*\), of the merchandise for one period and no future revenues. Thus the second state has no incentive to deviate if

\[
v_2 = [p(x^*) - t_1 - c_2]x^*/(1 - \delta) \geq [p(x^*) - c_2]x^*,
\]

which simplifies to:

\[
t_1 \leq \delta[p(x^*) - c_2].
\]

The above condition then yields state 2’s best responses to \( t_1 \):

\[
(3) \quad t_2 = p(x^*) - t_1 \quad \text{if } t_1 \leq \delta[p(x^*) - c_2]
\]

\[
t_2 = p(x^*) \quad \text{if } t_1 \geq \delta[p(x^*) - c_2]
\]
I next show that the first nation’s optimal tax is

(4) \[ t_1 = \delta[p(x^*) - c_2]. \]

To prove this, suppose contrarily the first state posts a lower tax, \( t_1 < \delta[p(x^*) - c_2] \). The merchants observe this and expect state 2 to set the tax equal to \( t_2 = p(x^*) - t_1 \), once they have gotten there. If a small group of additional merchants (of size \( \varepsilon > 0 \)) decide also to cross the first state, state 2 will adjust the tax rate to \( t_2 = p(x^* + \varepsilon) - t_1 \) for \( \varepsilon \) small enough to satisfy the inequality: \( t_1 \leq \delta[p(x^* + \varepsilon) - c_2] \). Then, all \( x^* + \varepsilon \) merchants journey to the market. This shows that \( x^* \) is not the equilibrium number of merchants. On the other hand, if the first state sets a higher tax, \( t_1 > \delta[p(x^*) - c_2] \), state 2 will act myopically, thereby disturbing the equilibrium. Thus, in the equilibrium (4) must hold. (4) thus defines a mapping from \( t_1 \) to \( x^* \).

Now, state 1 sets \( t_1 \) to maximize the net tax revenue per period, \( (t_1 - c_1)x^* \), where \( x^* \) is determined by \( t_1 \) via (4). It is more convenient to invert the above mapping and restate state 1’s problem as:

\[ \text{Max}_{x^*} \{ \delta[p(x^*) - c_2] - c_1 \}x^*. \]

The first-order condition

(5) \[ \delta[p(x^*) - c_2] - c_1 + \delta x^* p'(x^*) = 0 \]

determines the equilibrium number of traveling merchants \( x^* \). Finally, when evaluated at \( x^*_2 \) (see Eq. 2), the left-hand side of (5) equals \( -c_1 < 0 \), implying \( x^* < x^*_2 \). This justifies my focus on the case in which \( x^* \leq x^*_2 \).
Now that the equilibrium value of $x^*$ is found, the first state’s equilibrium tax rate is given by

\[ t_1^* = \delta[p(x^*) - c_2]. \tag{6} \]

(5) and (6) lead to

\[ t_1^* - c_1 = \delta[p(x^*) - c_2] - c_1 = -\delta x^* p'(x^*) > 0, \]

indicating that state 1 earns a strictly positive profit. The optimal tax for state 2, $t_2^*$, obtains from substituting for $t_1^*$ from (6) into (3). I summarize the findings of this section in the next proposition.

**Proposition 2**: In the stationary equilibrium with two nation states there is an equilibrium outcome with trade with the following features.

(A) The number of merchants who journey is $x^*$ given by (5), and $x^* < x_2^m$.

(B) The optimal tax rates of the two nation states are

\[ t_1^* = \delta[p(x^*) - c_2], \]

\[ t_2^* = (1 - \delta)p(x^*) + \delta c_2, \]

(C) Both states 1 and 2 earn positive net tax revenues.

The intuition for Result 2.C is as follows. If the tax rate the merchants paid to state 1 is not too high, state 2 can collect sufficiently large tax revenues. If the sum of such revenues over a long haul exceeds the payoff from acting myopically, state 2 will prefer that trade continues; otherwise it will act myopically. Then, state 1 can capitalize
on its first-mover advantage to raise its tax high enough to make state 2 indifferent between those two options, thereby collecting some positive net tax revenues.

In sum, in contrast to the case in which all agents interact just once, repeated interplays result in trade despite trade costs, and yield strictly positive payoffs for both nation states. Suppose that there are only $x_0$ potential merchants ($x_0 < x^*$). In this case, the proposition still holds, as can easily be confirmed, with $x_0$ replacing $x^*$ in Results 2.A and 2.B.

4. Many nations states

This section extends the above analysis to the case in which the trade route goes through multiple (more than two) nation states. A case of three nation states is sufficient to capture the essential features of such an extension. Now merchants go through nation states 1, 2 and 3 in that order.

Look again for a set of stationary equilibrium strategies that induce the same number of merchants to embark on journeys every period. Then, a procedure similar to the one employed in the previous section establishes the following best responses for state 3:

\[
(7) \quad \begin{align*}
    t_3 &= p(x^*) - t_1 - t_2 \quad \text{if } t_1 + t_2 \leq \delta[p(x^*) - c_3] \\
    t_3 &= p(x^*) \quad \text{if } t_1 + t_2 > \delta[p(x^*) - c_3],
\end{align*}
\]

where $x^*$ again denotes the (yet undetermined) number of merchants who travel the entire trade route in the equilibrium. The conditions in (7) indicate that the sum of the first two taxes holds the key to whether state 3 acts myopically or not.
I next show that the equilibrium tax rate for state 2 is

$$(8) \quad t_2^* = \delta[p(x^*) - c_3] - t_1.$$  

To prove this, observe first that state 2 will never set a tax lower than $t_2^*$, because raising the tax rate up to $\delta[p(x^*) - c_3] - t_1$ will not trigger state 3 to act myopically, and hence can increase its tax revenue. Consider next a higher tax rate, $t_2^* > \delta[p(x^*) - c_3] - t_1$.

Then, the merchants, having crossed the first state, would infer from the third state’s best responses in (7) that, once having traversed the second state, the third state will act myopically. The merchant’s net income then would be $- (t_1 + t_2^*)$, if he crosses the second state, whereas he can earn the income $- t_1$ by choosing not to cross it. Thus, no merchants would cross the second state. This proves my claim. Additionally, to be optimal, $t_2^*$ must give state 2 a non-negative payoff; i.e., we need this additional condition:

$$(9) \quad \delta[p(x^*) - c_3] - t_1 \geq c_2.$$  

Now, turning to state 1, I show that, if all the $x^*$ merchants pay $t_1$ and embark on journeys, $x^*$ must satisfy (9) with strict equality: i.e.,

$$(10) \quad t_1 = \delta[p(x^*) - c_3] - c_2.$$  

If $t_1$ is strictly less than the right-hand side of (10), more merchants are willing to travel, disturbing the equilibrium number $x^*$ of merchants. Eq. (10) thus defines a mapping from $t_1$ to $x^*$.

State 1’s problem is now stated: choose $t_1$ to maximize the net income $(t_1 - c_1)x^*$ subject to (10). Using (10), the first-order condition is written:
\[ \delta[p(x^*) + x^*p'(x^*) - c_3] - c_2 - c_1 = 0, \]

which defines \( x^* \). Now, define by

\[ x_3^m = \text{argmax} \ [p(x) - c_3]x, \]

the optimal output for the third state when it acts myopically. Then the left-hand side of (11) is negative at \( x_3^m \), implying \( x^* < x_3^m \).

By (10) state 1’s optimal tax rate is given by

\[ t_1^* = \delta[p(x^*) - c_3] - c_2. \]

(11) and (12) imply

\[ t_1^* - c_1 = \delta[p(x^*) - c_3] - c_2 - c_1 = -\delta x^*p'(x^*) > 0 \]

so state 1’s payoff is strictly positive. Substituting from (12) into (8) shows, however, that \( t_2^* = c_2 \). Thus, net tax revenue is zero for state 2. Further substitution shows a positive payoff for state 3 as given in the next proposition.

**Proposition 3**: The model with three nation states has a stationary equilibrium with the following properties.

(A) The equilibrium number \( x^* \) of merchants set out on journeys every period is determined by (11),

(B) The optimal tax rates for state \( i = 1, 2, 3 \) are

\[ t_1^* = \delta[p(x^*) - c_3] - c_2, \]

\[ t_2^* = c_2 \]

\[ t_3^* = (1 - \delta)p(x^*) + \delta c_3, \]
(C) Only states 1 and 3 earn strictly positive rents every period.

What is most surprising about this proposition is the fact that nation state 2 breaks even despite its monopoly position over the trade route. This result has the follow explanation. State 3 chooses the myopic action over the long-term equilibrium action unless the sum of two taxes the merchants pay before getting there are sufficiently low. Then, state 1 can take advantage of its first-mover position to set the tax just high enough to make state 3 indifferent between the myopic and the long-term strategy. Therefore, if state 2 sets the tax rate higher than its cost to earn positive tax revenues, no merchants will travel.

The next proposition generalizes Proposition 3 (the proof is similar and omitted).

**Proposition 4:** The model of $N (> 2)$ segmented roads has a stationary equilibrium, in which the optimal tax rates are

$$t_1^* = \delta[p(x^*) - c_N] - \sum_{k=2}^{N-1} c_k,$$

$$t_k^* = c_k; \quad k = 2, \ldots, N - 1,$$

$$t_N^* = (1 - \delta)p(x^*) + \delta c_N,$$

where $x^*$, the equilibrium number of merchants, is the solution to

$$\delta[p(x) + xp'(x) - c_N] - \sum_{k=1}^{N-1} c_k = 0.$$

5. Concluding remarks
All international trade involves the shipment of commodities from one nation to another. Many commodities, before reaching their final destinations, are transshipped through several nations, each with independent tax authorities. We find that, if trade continues over time, only the nation states occupying the first and the last segment of the trade route can extract the monopoly rent through taxation.

Our analysis leads to the following speculation. The Silk Road benefited only the Chinese and the Roman Empire as they controlled the beginning and the end of the trade route. To prosper along the Silk Road, the other nation states needed more than the monopoly power to tax commodities in transit. Similar fates may haunt the present-day nations in like positions. For example, Egypt and Panama, despite their unique positions to control the bulk of world trade, seem unable to exploit their monopoly power. Similarly, countries, though which the pipelines carry oil and natural gas to the final destinations, seem unable to capture much of the monopoly rent.
Appendix

We show that the model of Section 3 has no stationary equilibrium in which the number of merchants who travel is greater than $x^*_2$. Suppose that such an equilibrium exists so that $x^* \geq x^*_2$. Then, the optimal tax rate for state 2 is $t_2 = p(x^*) - t_1$. If it behaves myopically, state 2 would set the tax rate equal to $p(x^*_2)$ rather than at $p(x^*)$ to earn the monopoly rent $\pi^m_2$ defined in Section 2. State 2 has no incentive to behave myopically if

$$[p(x^*) - t_1 - c^*_2]x^*/(1 - \delta) \geq \pi^m_2.$$

This condition simplifies to

$$t_1 \leq p(x^*) - c^*_2 - (1 - \delta)\pi^m_2 / x^*.$$

In the equilibrium we have

$$t_1 = p(x^*) - c^*_2 - (1 - \delta)\pi^m_2 / x^*.$$

This equation maps from $t_1$ to the equilibrium $x^*$. State 1 chooses $t_1$ to maximize the net tax collection. Equivalently, it chooses $x^*$ to maximize

$$(t_1 - c^*_1)x^* = [p(x^*) - c^*_2 - c^*_1]x^* - (1 - \delta)\pi^m_2$$

The $x^*$ therefore fulfils the first-order condition:

$$p(x^*) - c^*_2 - c^*_1 + x^*p'(x^*) = 0.$$

Evaluated at $x^*_2$, the left-hand side of the above equation is negative, implying $x^* < x^*_2$. This contradicts our initial assumption. □
References

