On a Poor Country's Economic Development*

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ABSTRACT

Economic development may require labor-intensive methods of production. Using a simple Austrian model, we show that if the output elasticity of labor is high enough in a poor country whose labor productivity in the primary sector (or stage of production) is much lower than in a rich country, in the long run its per capita income (growth rate) can become strictly higher than in the rich country. For an intuitive interpretation of this claim consider the following. Since the output elasticity of labor is higher in the poor than in the rich, and labor is employed more in the poor in order to produce final consumption goods than in the rich, therefore, per capita output is larger in the poor than in the rich.

* This is a preliminary extension by this author of both his dissertation (2005) and Hiroshi Ohta's mimeo (2008) for possible collaboration.

1. Introduction

Negishi (1996, 2001) imagined the following result of economic growth as the model of an international economy, which is hidden behind Ricardo's famous numerical example. Per capita G.N.P. is higher in England than in Portugal, and so is the average productivity of labor in England.

In Bangladesh per capita income grows strictly higher than its real GDP does. In fact, its real GDP grew from 2007 to 2008 at an average annual rate of 6.21 percent, and its per capita income (in US$) grew at an average annual rate of about 14.5 percent (*Source*: Bangladesh Bureau of Statistics).

Using a simple Austrian model, we show that if the output elasticity of labor is high enough in a poor country whose labor productivity in the primary sector (or stage of production) is much lower than in a rich country, in the long run its per capita income (growth rate) can become strictly higher than in the rich country.

With this objective our inquiry proceeds as follows. First, we define the poor (rich) country as having the following production functions: the average productivity of labor in the primary (first stage) sector is low (high), but output elasticity of labor is higher (lower) than that of capital in the final goods (second stage) sector. Second, we suppose that two countries have an identical Austrian roundabout structure of production such that final consumption goods are produced from both labor and capital, and the intermediate goods (called capital) produced by labor alone. Third, pursuant to the above definition and assumption, we show that the rich chooses capital-intensive methods of production, and the poor chooses labor-intensive methods of production.

2. The Model

Let us consider a simple Austrian model.

The present section inquiry is based on the following assumptions:

The economy consists of the two sectors (final consumption goods and the intermediate goods sectors) and of one-period. The economy faces two alternative processes of production, one direct and the other indirect or roundabout method of production *a la* Böhm-Bawerk. The direct method of production requires labor input to catch fish, pick pecan, etc. or produce bundles of these and other necessary goods for direct consumption. Labor is the sole primary factor of production and a given constant. The given labor endowment is allocable as variable input to either the intermediate goods (first stage) sector or the final consumption goods (second stage) sector completely, i.e., full employment assumed. Entrepreneurs to employ workers in the first stage sector must promise the same wage rates to be offered in the second stage sector

under the pressure of free competition. With such promise the entrepreneurs must offer their intermediate output as capital on behalf of the workers in the first stage sector. The intermediate goods (called capital) are produced by labor alone. The average productivity of labor in the first stage sector is a given constant. The entrepreneurs in the second stage sector employ both the capital produced in the first stage sector and the remaining labor not used in the first stage sector in order to produce final consumption goods. The production function in the second stage sector is of the Cobb-Douglas type.

Pursuant to those assumptions above our simple model of Austrian roundabout methods of production can be summarized as follows.

The First-Stage Production Function:

1) $K = \delta L_K,\ \delta > 0,$

where a parameter δ is average product of labor.

The Second-Stage Production Function:

2) $X = \gamma K^{\alpha} L_X^{1-\alpha},\ \gamma > 0,\ 1 > \alpha > 0,$

where a parameter γ is total factor productivity and a parameter α is the output elasticity with respect to K.

The labor Market Equilibrium Condition:

3) $L = L_X + L_K,$

The Constrained Optimization Problem:

4) Max X, subject to 1) and 3).

Four variables to be determined are the intermediate output K, the final output X, the labor input L_K allocated to the production of K, the labor input L_X allocated to the production of X.

Solving the optimization problem 4) above requires the following first-order condition:

4)' $(-\partial K)/\partial L_X = \delta.$

The math model above consists of four independent equations in four unknown variables, now readily solved as follows.

$K^* = \delta\alpha L$

$L_X^* = (1 - \alpha)\, L$

$L_K^* = \alpha\, L$

$X^* = \gamma(\delta\alpha)^{\alpha}(1 - \alpha)^{1-\alpha} L$

Related to the solution set above are equilibrium values of X/L and the capital-labor ratio k in the final consumption goods sector:

5) $X^*/L = \gamma(\delta\alpha)^{\alpha}(1 - \alpha)^{1-\alpha}$,

6) $k^* = K^*/ L_X^* = \delta\alpha/(1 - \alpha)$.

3. Analysis

We are now in position to examine the model set forth above for purposes of our present inquiry.

The poor (rich) country is defined as having the following production functions: the average productivity of labor in the primary (first stage) sector δ is low (high), but output elasticity of labor $(1 - \alpha)$ is higher (lower) than that of capital in the final goods (second stage) sector α. From the definition above and 6), the rich country chooses capital-intensive methods of production, and the poor country chooses labor-intensive methods of production.

The superiority of Austrian roundabout methods of production is defined as

$X > \max K = \delta L$,

which condition requires the final output X produced in the roundabout method to be larger than the quantity of capital K that could be produced in the direct method if entire labor endowment of L were employed by the entrepreneurs.

Combining this condition above with 5) in turn yields the following technological parameters α and δ that make roundabout methods of production feasible, when γ is given:

7) $\delta < \gamma^{1/(1-\alpha)}\alpha^{\alpha/(1-\alpha)}(1 - \alpha)$,

where $\alpha^{\alpha/(1-\alpha)}(1 - \alpha)$ approaches 1, when α approaches 0. By contrast, when α approaches 1, $\alpha^{\alpha/(1-\alpha)}(1 - \alpha)$ approaches 0. Hence, if going roundabout pays, then

direct output productivity δ is required to stay below γ, or unity, when $\gamma = 1$.

4. Per-Capita GDP: X^*/L (α ; $\gamma = 1$)

Consider the relation of α and X^*/L that satisfies 7), when $\gamma = 1$. This is the relation depicted in the following figure.

[Figure here]

Suppose the poor (rich) country has the following production functions: the average productivity of labor in the primary sector δ is 0.25 (0.75) and output elasticity of capital α is 0.1 (0.6). Then the value of X^*/L in the poor is 0.628944407 and the value of X^*/L in the rich is 0.429290724. Hence, if the output elasticity of labor is high enough in a poor country whose labor productivity in the primary sector (or stage of production) is much lower than in a rich country, in the long run its per capita income (growth rate) can become strictly higher than in the rich country.

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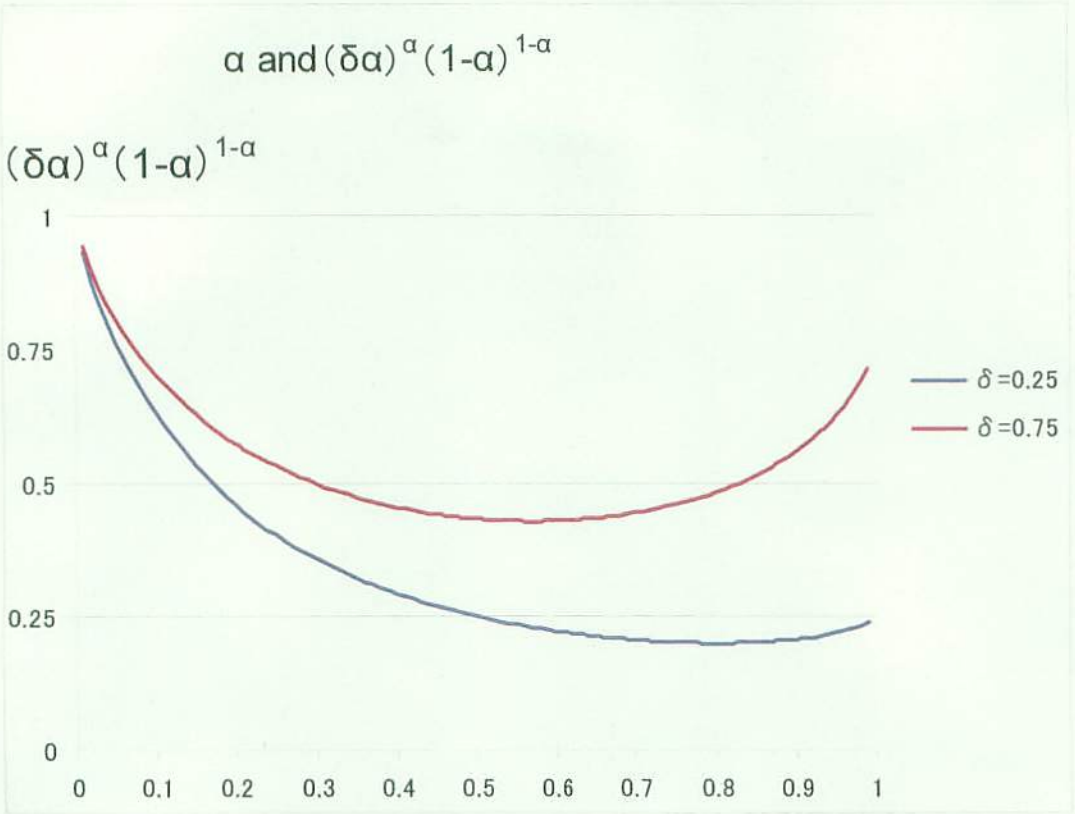
α and (δα)^α (1-α)^1-α
(δα)^α (1-α)^1-α
1
0.75
0.5
0.25
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
δ =0.25
δ =0.75