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Enforcing International Trade Agreements with Imperfect Private Monitoring: Private Trigger Strategies and a Possible Role for the WTO

by
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Abstract

International trade disputes often involve the WTO as a third party that generates impartial opinions of potential violations when countries receive *imperfect* and *private* signals of violations. To identify the role that the WTO plays in enforcing trade agreements, this paper first characterizes what countries can achieve alone in a repeated bilateral trade relationship in which they can secretly raise their protection levels through concealed trade barriers. In particular, countries adopt "*private trigger strategies* (*PTS*)" under which each country triggers a punishment phase by imposing an *explicit* tariff based on its privately observed imperfect signals of such barriers. This paper identifies the condition under which countries can restrain the use of concealed barriers based on *simple PTS*, where each country imposes its static optimal tariff in all periods under any punishment phase: The *sensitivity* of private signals rises in response to an increase in concealed protection. Any equilibrium payoff under *almost strongly symmetric PTS* will be identical to the one under *simple PTS*, as long as the *initial* punishment is triggered by a static optimal tariff, justifying the paper's focus on *simple PTS*. With countries maximizing their expected payoffs under the optimal *PTS*, they will not push down the cooperative protection level to its minimum attainable level, thus not setting it to the free trade level even when it is attainable. To analyze a possible role of the WTO, this paper considers "*third-party trigger strategies* (*TTS*)" under which the WTO allows each country to initiate a punishment phase based on the WTO's judgment (signals) about potential violations. The WTO changes the nature of punishment-triggering signals from *private* into *public*, enabling countries to use punishment phases of any length under *TTS*, which in turn facilitates a better cooperative equilibrium. The optimal *TTS* will involve an *asymmetric* and *minimum* punishment if the probability of a punishment phase being triggered is low enough, but it will entail punishments involving a permanent Nash tariff war if the probability of a punishment being triggered is high enough. A numerical comparison of the optimal *TTS* and optimal *PTS* indicates that the contribution of the WTO is likely to be significant when the signals of potential violations are relatively accurate. The WTO enables countries to adopt a more efficient punishment, such as the *asymmetric* and *minimum* punishment, which in turn enables countries to be less tolerant of potential violations and attain a higher level of cooperation as a result.

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1. Introduction

Enforcing international trade agreements often entails disputes in which countries present different opinions about potential deviations from the agreements. Differences in opinion may take various forms, such as disagreement over the existence of *concealed trade barriers* as in disputes between the U.S. and Japan during 1980s, or disagreement over the legitimacy of antidumping duties, a frequent theme in the dispute settlement procedure of the World Trade Organization (WTO). These disagreements reflect imperfectness of information about deviations from trade agreements. In addition to being *imperfect*, each country's opinion of potential violations can be *private* in the sense that the country's true opinion is not known to other countries. For example, when the United States Trade Representative (USTR) engages in a negotiation with China to curtail piracy and counterfeiting that impede the U.S. intellectual property rights, China and the USTR may not know each other's true belief regarding the Chinese government's effort level to curtail such practices, which in turn may contribute to a breakdown in the negotiation.[1]

Trade disputes typically involve the WTO as a third party that generates impartial opinions of potential violations when countries receive imperfect private signals of violations.[2] To identify the role that the WTO plays in facilitating the enforcement of trade agreements, this paper first assumes away the presence of the WTO and characterizes what countries can achieve alone in a repeated bilateral trade relationship where each country can secretly raise its protection level through concealed trade barriers. In particular, this paper explores the possibility that countries adopt "*private trigger strategies* (*PTS*)" under which each country triggers a punishment phase by imposing an *explicit* tariff based on its privately observed imperfect signals of such barriers. The analysis identifies the condition under which countries

[1] The signals that the USTR receives regarding potential deviations from trade agreements often come from the U.S. companies whose interests are affected by deviations. Such signals may involve the companies' private information. Public revelation of the private information can be costly for those companies, forcing the signals to be private. There exist many U.S. antidumping cases in which foreign companies under investigation decide not to provide costs and sales related "private" information despite the fact that such nondisclosure often leads to excessive dumping duties based on "best information available."

[2] When countries bring a disputed case to the WTO presenting different opinions about potential violations, the Dispute Settlement Procedure of the WTO encourages them to solve disputes through a consultation stage prior to initiating a panel stage where a third-party panel provides a ruling on the disputed case. Countries can appeal the panel's ruling to have the case examined by an Appellate Body. Once the case has been determined by the Appellate Body, the losing "defendant" must comply with the ruling or face the possibility of trade sanctions by the complaining side.

can restrain the use of concealed trade barriers based on *simple PTS* where each country imposes its static optimal tariff in all periods under any punishment phase. The condition is that the *sensitivity* of private signals rises in response to an increase in concealed protection. This paper then establishes that the equilibrium payoff of any *almost strongly symmetric PTS* will be identical to the one under *simple PTS*, as long as the *initial* punishment is triggered by a static optimal tariff. Given this generality result, it characterizes the optimal *PTS* that maximize symmetric countries' expected payoffs under *simple PTS*. The analysis shows that it is not optimal to push down the cooperative protection level to its minimum attainable level, thus not setting it to the free trade level even when it is attainable.

To analyze a possible role that the WTO can play in enforcing trade agreements, this paper analyzes "*third-party trigger strategies* (*TTS*)" under which the WTO decides upon whether a violation has occurred and tells each country to initiate a punishment phase based on its decision as an impartial third party. The WTO under *TTS* changes the nature of punishment-triggering signals from *private* into *public*, enabling countries to employ punishment phases of any length, which in turn can help countries to attain a better cooperative equilibrium. The comparison between the optimal *TTS* and optimal *PTS* illustrates how and to what degree that the WTO can help countries to enforce international trade agreements beyond what countries can achieve alone under *PTS*. The analysis establishes that the optimal *TTS* will involve an *asymmetric* and *minimum* punishment if the probability of a punishment phase being triggered is low enough, but it will entail punishments involving a permanent Nash tariff war if the probability of a punishment being triggered is high enough. A numerical comparison of the optimal *TTS* and optimal *PTS* indicates that the contribution of the WTO is likely to be significant when the signals of potential violations are relatively accurate. Under such circumstances, the WTO enables countries to adopt a more efficient punishment, such as the *asymmetric* and *minimum* punishment, which in turn enables countries to be less tolerant of potential violations. As a result, a higher level of cooperation is attained compared to the situation without the WTO.

This paper contributes to the literature on two levels. First, it provides a new way of understanding the role that the WTO plays in enforcing international trade agreements in the presence of potential violations of which countries receive *imperfect* and *private* signals. Because the enforcement of trade agreements ultimately rely on the threat of invoking trade sanctions against violations, previous studies have also analyzed the enforcement issue using trigger strategies in a repeated game setup.[3] Earlier models on this issue, such as Dixit (1987), Bagwell and Staiger (1990), and Riezman (1991) suggest that the WTO may serve the role of helping countries coordinate on more efficient equilibria among multiple equilibria that typically arise in a repeated game setup. To model a more explicit role of the WTO, Kovenoch and Thursby (1993) assume that the Dispute Settlement Procedure (DSP) of the WTO has an informational superiority (over trading countries) of distinguishing between true violations and mistaken perceptions, which in turn assists the workings of a reputation mechanism to support cooperation.[4] In a multilateral trading environment, Maggi (1999) shows that the WTO may facilitate cooperation enhancing third-country retaliations by disseminating information about deviations.[5] While these models introduce more specific roles for the WTO to play in coordinating a cooperative equilibrium, the literature has not resolved the question of why the WTO is *necessary* for coordination because these previous studies offer no theory of why countries could not coordinate a cooperative equilibrium in the no-WTO environment.[6]

This paper represents the emergence of the WTO as a change in the observation structure of the repeated game. The presence of the WTO changes the nature of punishment-triggering signals from *private* into *public*. This enables countries to employ punishment phases of any length, and as a result countries can attain a better cooperative equilibrium. As emphasized in the analysis of *PTS*, the private nature of signals of potential violations limits the flexibility of punishment phases that countries can use in the absence of the WTO because they need to provide countries with the incentive for truthful revelation of private signals in triggering punishment phases. The WTO can publicize its opinions of potential violations, which relaxes

[3] Bagwell and Staiger (2002) provide a comprehensive review of studies analyzing international trade agreements as a subgame perfect equilibrium in a repeated trade relationship.
[4] Hungerford (1991) develops a model where the WTO plays a negative role in enforcing trade agreements because the model assumes that the DSP of the WTO involves uninformative and costly investigation.
[5] As pointed out by a referee, third-party retaliation is rarely observed, and Maggi (1999) does not model information transmission directly and offers no theory as to why information could not be shared in the absence of the WTO.
[6] Bagwell and Staiger (2005) and more recently Bagwell (2008) analyze the issue of implementing trade agreements when each government is privately informed about its own domestic political pressure for protection. Their analysis differs from this paper's because it focuses on identifying the structure of trade agreements that can induce the truthful revelation of private political pressure rather than analyzing the enforcement of trade agreements when countries privately observe imperfect signals of potential deviations.

such a constraint in developing an optimal punishment mechanism, enabling a better cooperative equilibrium even in the absence of any informational superiority of the WTO.[7]

The other contribution of this paper is more generally toward the literature on repeated games with imperfect private monitoring. It is well known that analyzing repeated games with imperfect private monitoring is difficult because utilization of privately observed signals in determining continuation plays can destroy the recursive structure of repeated games.[8] Kandori and Matsushima (1998) and Compte (1998) demonstrate that communication can serve as a public signal that restores the recursive structure and enables players to achieve cooperation in such a repeated game.[9] In the absence of communication, *PTS* in this paper show an alternative way to restore a recursive structure to repeated games with imperfect private monitoring.[10] If players can choose explicit actions as well as concealed actions as in the case of governments' choosing their protection levels, then players can avoid confusion between punishment phases and non-punishment phases by requiring players to signal an initiation of punishments by their explicit, thus public actions.[11] This can ensure a recursive structure of the

[7] Ludema (2001) emphasizes that the DSP of the WTO may require trade agreements to be renegotiation-proof by promoting communication among countries prior to starting punishments. This negatively affects cooperation by forcing countries to rely on weaker (renegotiation-proof) punishments. In contrast to his analysis in a repeated game with perfect monitoring, an optimal trade agreement with imperfect monitoring would not typically involve the lowest levels of protection with the most severe credible threat because punishments do occur. With imperfect private monitoring, the WTO can help countries to achieve better cooperation by enabling countries to adopt weaker punishments, as shown in this paper.

[8] Kandori (2002) discusses this point and recent developments in repeated games with private monitoring in detail.

[9] In these studies, the communication among players entails no cost (so that it is "cheap talk") and each country's revealed private information does not affect its own continuation payoff in order to ensure truthful revelation of private information. As pointed out by a referee, however, they are unable to show what communication "does" though, since they were unable to show what would happen in the no-communication setting.

[10] A referee points out that communication is not illegal in the context of international trade agreements, different from communication in the context of price-fixing oligopolists. It suggests the possibility of using communication to achieve cooperation in the absence of the WTO. For example, one may consider applying the communication mechanism developed by Kandori and Matsushima (1998) to sustain international trade agreements. There are two reasons why such mechanism may not work well among countries. First, in the context of an international relationship, it is not easy to allege potential violations when violations do not affect the alleging country, especially when such allegations will negatively affect the alleged country. In fact, the DSP of the WTO reduces the burden for countries of playing the third-party role of "alleging" potential wrong doings of another country by making the DSP to be a kind of legal procedure primarily run by experts. Second, the use of transfers is rarely observed between countries, especially as compensation for potential violations of international trade agreements. If countries need to rely on imposition of tariffs in punishing potential deviations as they do in practice, then "communication" will face a similar incentive constraint as the one under *PTS* because "alleging your trading partner's wrong doings" needs to be supported by the "action" of punishing such behavior with tariffs.

[11] In the context of collusion among firms engaging in secret-price cuttings, for example, firms can employ *advertised* (thus *public*) sales to initiate a punishment phase against potential defections from collusive pricing. Similar to Green and Porter (1984), then occasional "explicit" price wars will occur as dynamic equilibrium behaviors to sustain collusion overtime. Different from the model of Green and Porter where firms would always

repeated game along the equilibrium path. The analysis of *PTS* specifies the condition under which such trigger strategies can restrain the deviatory use of concealed actions.

The use of explicit actions to initiate punishment phases, however, does not simplify everything. If each player triggers a punishment phase based on its private signals as it does under *PTS*, then in any period after a cooperative one, players need to choose their actions knowing only the probability of a punishment phase being triggered by other players. Because an action taken by each player in a current (cooperative) period affects the probability of a punishment being triggered in a next period, an optimal action in the next period depends on an action taken in the current period, and an optimal action in a period after the next period depends on an action taken in the next period, and so on until a punishment phase is triggered. Therefore, the use of *PTS* necessitates a complete characterization of optimal and potentially deviatory action *sequences* that each player may take in checking incentive compatibility for such strategies.[12] Using a dynamic programming method, this paper establishes that countries can use *simple PTS* in achieving cooperation as long as the private signals satisfy some sensitivity constraints. With regard to the possibility of proving a folk theorem result under *PTS*, this paper generates yet another anti-folk theorem result within a class of private trigger strategies, namely *almost strongly symmetric PTS*, when private monitoring is far from being perfect.[13]

The paper is organized as follows. Section 2.1 develops a bilateral trade model where each country receives *imperfect private* signals of the other country's use of concealed trade barriers and specifies *simple PTS*. Section 2.2 describes incentive constraints under *simple PTS*, providing conditions under which those incentive constraints are satisfied. Section 3.1 shows

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start a price war concurrently, each firm may unilaterally initiate a price war phase by lowering its explicit price (and gains from it in that period) under such private trigger strategies and the lengths of price war phases will be endogenously determined.

[12] This aspect of *PTS* does not allow one to apply dynamic programming techniques develops by Abreu *et al*. (1986) to characterize the set of equilibrium payoffs under *PTS* because those techniques rely on the "one-stage deviation principle." For further discussion of the "one-stage deviation principle," see footnote 21 in Section 2.2.

[13] Ely and Välimäki (2002) provide a concise discussion of why many of the strategies to prove folk theorems with public monitoring fail when monitoring is private and conditionally independent. This paper also analyses the case where monitoring is private and conditionally independent and shows that countries cannot attain the symmetric efficient frontier under *almost strongly symmetric PTS* if the monitoring is far from perfect (*Corollary 1 to Proposition 2*). This anti-folk theorem result, however, may rely on the countries' use of distortional measures like tariffs to punish potential violations. For example, Horner and Jamison (2007) show that full collusion can be approximated under minimal information in *private* strategies where punishment phases are carefully designed so that no loss occurs (collectively) for colluding firms. Such punishments are possible because

that countries can support *simple PTS* in the repeated protection-setting game, achieving a certain level of cooperation. It also establishes that the equilibrium payoff under any *almost strongly symmetric PTS* will be identical to the payoff under *simple PTS* as long as each country starts the *initial* punishment phase by imposing its static optimal tariff. Section 3.2 then characterizes optimal *simple PTS* under which countries maximize their joint expected discounted payoffs. To demonstrate a role that the WTO may play in enforcing international trade agreements, Section 4 characterizes optimal *TTS* and provides a numerical comparison between the optimal *PTS* and optimal *TTS*. Section 5 discusses some additional factors that may severely limit the use of *PTS* and summarizes results. It concludes with a discussion of a possible extension of this paper's analysis towards further understanding of the Dispute Settlement Procedure of the WTO.

2. Private Trigger Strategies

2.1. A Trade Model with Concealed Trade Barriers and Private Trigger Strategies

The basic bilateral trade model comes from Dixit (1987) with concealed trade barriers being introduced in a way similar to Riezman (1991). There exist two countries, home (H) and foreign (F), producing and trading two products, good 1 and good 2, under perfect competition. H imports good 2 and F imports good 1. In each period each country simultaneously chooses its action, $a^i \equiv (\tau^i, e^i) \in A^i$, where both elements of A^i may take any non-negative real number. Total import protection level and explicit tariff level are given by τ^i and e^i, respectively, with i = * or none. Variables with and without superscripts * denote foreign and home variables, respectively. I assume that $\tau - e \geq 0$ and $\tau^* - e^* \geq 0$, representing the concealed protection levels of H and F, respectively. The local prices p_1, p_2, p_1^*, and p_2^* are related as follows: $p_2 = p_2^*(1+\tau)$ and $p_1^* = p_1(1+\tau^*)$.[14] Given the assumption of perfect competition, I can define each country's one-period payoff function as a function of the terms

firms can avoid collective losses as long as any low-cost firm ends up selling its product at a monopoly price, a special feature that countries in a trade relationship may not replicate easily in their punishment phases.

[14] Thus, this paper does not consider the possibility of using negative or prohibitive protection.

of trade, represented by $\pi\,(\equiv p_1 / p_2^*)$, and its own total protection level, τ^i. Such a payoff function, denoted by $w^i(\pi,\tau^i)$, induces a corresponding import demand function, $m^i(\pi,\tau^i)$.

In the absence of uncertainty (no random element) in this world, each country's amount of imports is a deterministic function of its own total protection level and the terms of trade. This implies that each country may figure out the exact level of the other country's protection based on the information about the terms of trade and the amount of imports, even in the presence of concealed trade barriers. However, when I introduce uncertainty into the model as a way of representing shocks to technology or preferences, exact derivation of other countries' protection levels based on the amount of imports and the terms of trade may become impossible. Uncertainty caused by random shocks can be modeled into random components in the import demand functions as follows:

(1) $$m_t^i = m^i(\pi_t, \tau_t^i, \theta_t^i),$$

where $\theta_t^i \in \Theta^i$ denotes each country's random components affecting its import demand at period *t*, which follow a joint density function, $f(\theta_t, \theta_t^*)$ that is iid across periods. Subscript *t* denotes the variables determined in period *t*. In equilibrium, the following balance of payment condition should be satisfied:

(2) $$\pi_t \cdot m(\pi_t, \tau_t, \theta_t) = m^*(\pi_t, \tau_t^*, \theta_t^*).$$

This determines the equilibrium values for π_t, m_t, and m_t^* as functions of τ_t, τ_t^*, θ_t, and θ_t^*.

Given that each country sets its total protection level prior to the realization of random shocks, each country's one-period expected payoff, denoted by u^i, is a function of both countries' total protection levels:

(3) $$u^i(\tau_t^i, \tau_t^j) = \iint_{(\theta_t, \theta_t^*) \in (\Theta, \Theta^*)} w^i\big(\pi_t(\tau_t, \tau_t^*, \theta_t, \theta_t^*), \tau_t^i; \theta_t^i\big) f(\theta_t, \theta_t^*)\, d\theta_t\, d\theta_t^*,$$

where $w^i(\pi, \tau^i; \theta^i)$ represents each country's one-period payoff function that is affected by random shocks, θ^i, and where $i \neq j$.

This paper focuses on the analysis of symmetric equilibria of a repeated protection-setting game between symmetric countries. Thus, I assume that $u(\tau^1, \tau^2) = u^*(\tau^1, \tau^2)$ for all non-negative, real values of τ^1 and τ^2. Regarding derivatives of $u(\tau, \tau^*)$ and $u^*(\tau^*, \tau)$ with respect to τ and τ^*, I assume that the following standard trade-theoretic results continue to hold in the presence of random variables: $\partial u/\partial \tau > 0$ at $\tau = 0$ (each country has an incentive to raise its protection level above zero); $\partial u^*/\partial \tau < 0$ (such protection hurts the other country); $\partial u/\partial \tau + \partial u^*/\partial \tau < 0$ (such protection also reduces the total payoff to H and F as it creates distortional losses). For analytical simplicity, I introduce the following additional assumptions: $\partial^2 u/\partial \tau^2 < 0$ (the marginal gain from protection decreases as the protection level increases); $\partial^2 u/\partial \tau \partial \tau^* = 0$ (the marginal gain from protection is not affected by the other country's protection level).[15] These additional assumptions guarantee the existence of a unique static optimal protection level for H, which I denote by h (> 0). The one-shot protection-setting game between H and F then generates a Nash equilibrium where (τ, τ^*) = (h, h^*) with $h = h^*$ by symmetry.

Private monitoring is specified as follows. At the end of period t, each country privately observes realized values of its payoff and own random variable, (u_t^i, θ_t^i), and both countries observe a pair of explicit tariffs, (e_t, e_t^*). Denote the privately observed signal by $\omega_t^i = (u_t^i, \theta_t^i)$ $\in \Omega^i$. I assume that the probability distribution of private signal profile conditional on action profile has full support, that is $Pr(\omega_t, \omega_t^* | a_t, a_t^*) > 0$ for each $\omega_t \in \Omega$, $\omega_t^* \in \Omega^*$, $a_t \in A$ and $a_t^* \in A^*$. Note that while each country cannot infer the exact level of the other country's concealed protection even after observing its private signal (because it does not know the realized value of the other random variable), the privately observed information can serve as a measure for detecting the other country's potential use of concealed protection.[16] More specifically, H can choose a subset of its private signals, $\Omega^D \in \Omega$, so that $\partial Pr(\omega_t \in \Omega^D)/\partial \tau_t^*$ > 0, with $Pr(\omega_t \in \Omega^D) \equiv Pr(\omega_t \in \Omega^D | a_t, a_t^*)$ denoting the probability that H's private signal belongs to Ω^D conditional on an action profile. For example, H can assign values of u_t that are less than a critical value as the payoff part of Ω^D. This can induce $\partial Pr(\omega_t \in \Omega^D)/\partial \tau_t^* > 0$

[15] These properties of a social utility function can be derived from a two good, partial equilibrium model of trade with linear demand and supply curves. See Bond and Park (2002) for derivation of such properties.

[16] Once H observes u_t, θ_t, and τ_t, for example, H can calculate the probability of $\tau_t^* \leq l$ (a certain protection level) by $Pr(\tau_t^* \leq l \mid u_t, \theta_t, \tau_t) = \int_0^l \left(\int_{\theta^* \in \Theta^*(u_t, \theta_t, \tau_t \tau^*)} f(\theta_t, \theta^*) d\theta^* \right) d\tau^*$ where $\Theta^*(u_t, \theta_t, \tau_t, \tau^*) = \{\theta^* \in \Theta^* \mid u(\tau_t, \tau^*, \theta_t, \theta^*) = u_t\}$.

because $\partial u_t / \partial \tau_t^* < 0$, and the sensitivity of u_t against τ_t^* can improve once it is properly controlled for θ_t. With regard to the relationship between ω_t and ω_t^*, I assume *conditional independence*, meaning that for each action profile, countries' private signals are independently distributed of one another. This implies that each country cannot infer the other country's private signal based on its own private signal.[17] For symmetry between H and F, I also assume that $Pr(\omega_t \in \Omega^D) = Pr(\omega_t^* \in \Omega^D)$ for all $(\tau_t, e_t) = (\tau_t^*, e_t^*) \in A = A^*$ and $\Omega^D \in \Omega = \Omega^*$.

Given the stage game and associated private monitoring depicted as above, I can describe an infinitely repeated protection-setting game between H and F as follows. A strategy for each country is defined by $s^i = (s^i(t))_{t=1}^{\infty}$ with

(4) $$s^i(t) : (A^i)^{t-1} \times (\Omega^i)^{t-1} \times (E^j)^{t-1} \rightarrow A^i ,$$

where E^j denotes the set of possible explicit tariffs that each country can impose in a period with $e^j \in E^j$ and $j \neq i$. $s^i(t)$ assigns each country's current action (τ_t^i, e_t^i) based on the history of its own previous actions, $(a^i)^{t-1} \equiv (a_1^i, a_2^i \cdots, a_{t-1}^i) \in (A^i)^{t-1}$, the history of its own private information, $(\omega^i)^{t-1} \equiv (\omega_1^i, \omega_2^i \cdots, \omega_{t-1}^i) \in (\Omega^i)^{t-1}$, and the history of the other country's explicit tariffs, $(e^j)^{t-1} \equiv (e_1^j, e_2^j, \cdots, e_{t-1}^j) \in (E^j)^{t-1}$ with $j \neq i$. If each country conforms to its strategy defined in (4), then the expected discounted payoff is given by:

(5) $$V^i(s^i, s^j) = E\left[\sum_{t=1}^{\infty} u^i(\tau_t^i, \tau_t^j)(\delta^C)^{t-1} \middle| (s, s^*) \right] ,$$

where $E[\cdot \,|\, (s, s^*)]$ is the expectation with respect to the probability measure on histories induced by the strategy profile (s, s^*), and where $\delta^C \in [0, 1)$ denotes the common discount factor with $i \neq j$. Now, I define a *supergame equilibrium* in this infinitely repeated protection-setting game as follows:

[17] Matsushima (1991) analyzes repeated play of stage games with a unique static Nash equilibrium and *conditionally independent* private signals, a problem that is similar to the repeated protection-setting game of this paper, and shows that any pure-strategy equilibrium other than the static Nash equilibrium should involve conditioning on payoff-irrelevant history. As discussed by Ely and Välimäki (2002), repeated games with imperfect private monitoring, especially with conditionally independent private signals, limit the use of strategies that are often useful for repeated games with public monitoring under which each player typically has a strict incentive to follow her equilibrium strategy after every history. Private trigger strategies considered in this paper will be subject to similar constraints, but differ from previous works by considering the use of explicit actions, like tariffs, as a punishment coordination device.

Definition 1.[18] A strategy profile (s, s^*) is a *supergame equilibrium* in the repeated game between H and F, if $V(s, s^*) \geq V(s^/, s^*)$ and $V^*(s^*, s) \geq V^*(s^{*/}, s)$ for all $s^/ \neq s$ and $s^{*/} \neq s^*$.

To explore the possibility of supporting a cooperative protection level, denoted by *l*, that is lower than the one-shot Nash protection level ($h > l$) as a symmetric supergame equilibrium of the repeated game described above, I consider "*private trigger strategies*" under which each country uses its private signal, ω and ω^*, as a device to trigger a punishment phase against the other country's potential use of concealed protections.[19] Focusing on symmetric strategies with $s(t) = s^*(t)$ for all $a^{t-1} \times \omega^{t-1} \times (e^*)^{t-1} = (a^*)^{t-1} \times (\omega^*)^{t-1} \times e^{t-1}$ and $t \geq 1$, I describe H's strategy *s* (and accordingly, F's strategy s^*) as follows:

(i) Given that period $t - 1$ was a "*cooperative*" period with (e_{t-1}, e^*_{t-1}) = (0, 0), H continues cooperating by setting (τ_t, e_t) = (*l*, 0) if $\omega_{t-1} \notin \Omega^D$, but it initiates a *punishment* phase by setting (τ_t, e_t) = (*h*, *h*) if $\omega_{t-1} \in \Omega^D$.

(ii) Given that a "*punishment*" phase was initiated in period $t - 1$ with $(e_{t-1}, e^*_{t-1}) \neq$ (0, 0), H sets (τ, e) = (*h*, *h*) for the following ($T - 2$) periods and it continues to do so one more period with probability λ if $e_{t-1} > 0$ and $e^*_{t-1} = 0$; H sets (τ, e) = (*h*, *h*) for the following ($T^S - 2$) periods and it continues to do so one more period with probability λ^S if $e_{t-1} > 0$ and $e^*_{t-1} > 0$, where *T* and T^S are integer numbers that are greater than or equal to 2, and λ and λ^S belong to [0, 1]. H knows these variables (T, T^S, λ, λ^S) when it initiates a punishment phase. The actual length of a punishment phase is determined by some public randomizing device (determining λ and λ^S) after the punishment phase has been initiated.

(iii) In period 1 and other "*initial*" periods right after the end of any punishment phase, H sets (τ, e) = (*l*, 0) with probability ($1 - Pr$) but initiates a punishment phase by setting (τ^i, e^i) = (*h*, *h*) with probability *Pr*, where $Pr \equiv Pr(\omega_t \in \Omega^D)$ with (τ_t, e_t) = (*l*, 0) and (τ^*_t, e^*_t) = (*l*, 0).

[18] This definition of a *supergame equilibrium* of a repeated game with privately observed signals of other players' actions follows Matsushima (1991).
[19] One trivial supergame equilibrium strategy profile is to assign the one-shot Nash protection level for all periods because that would assign the static optimal behavior for each country.

Note that the absence or presence of explicit tariffs classifies any period into either a "*cooperative*" period (with no explicit tariffs) or a "*punishment*" period (with some positive tariffs). While H and F cannot observe each other's concealed protection levels, they use their explicit tariffs as public signals to coordinate punishment phases as described in (i) and (ii). Extending a punishment phase one more period with a certain probability as specified in (ii) is an instrument to make the expected discounted payoff from invoking a punishment phase vary smoothly so that it can be set to equal the expected discounted payoff from not invoking a punishment phase, which is an important requirement for incentive constraints considered in the following section. Also note that the actions for period 1 and other "*initial*" periods described in (iii) are designed to mimic those in a period that immediately follows a "*cooperative*" one, which in turn simplifies the analysis of the trigger strategies defined above.[20] Finally, note that the set of private signals that trigger a punishment phase (Ω^D), the lengths of different punishment phases ($T - 1$ if a single country triggers and $T^S - 1$ if H and F trigger simultaneously), and the corresponding probabilities of extending the punishment phases (λ, λ^S) characterize the strategy profile defined by (i), (ii) and (iii), together with the cooperative protection level, l. I define *simple private trigger strategies* as follows:

Definition 2. If (i), (ii), and (iii) describe a symmetric strategy profile ($\underline{s}$, $\underline{s}^*$) with $\underline{s}(t) = \underline{s}^*(t)$ for all $a^{t-1} \times \omega^{t-1} \times (e^*)^{t-1} = (a^*)^{t-1} \times (\omega^*)^{t-1} \times e^{t-1}$ and $t \geq 1$, then ($\underline{s}$, $\underline{s}^*$) are *simple* ***private trigger strategies*** **(***simple* **PTS)** with $(l, \Omega^D, T, T^S, \lambda, \lambda^S)$ as characterizing parameters.

Given this definition, I can derive H's expected discounted payoff under ($\underline{s}$, $\underline{s}^*$) with $(l, \Omega^D, T, T^S, \lambda, \lambda^S)$, denoted by $V(\underline{s}, \underline{s}^*)$, as follows:

(6)
$$\begin{aligned} V(\underline{s}, \underline{s}^*) = & \frac{(1-Pr^2)[u(l,l) - u(h,h)]}{1 - \delta^C + 2Pr(1-Pr)(\delta^C - \delta) + Pr^2(\delta^C - \delta^S)} \\ & + \frac{Pr(1-Pr)[u(l,h) - u(l,l)] + Pr(1-Pr)[u(h,l) - u(l,l)]}{1 - \delta^C + 2Pr(1-Pr)(\delta^C - \delta) + Pr^2(\delta^C - \delta^S)} + \frac{u(h,h)}{1-\delta^C}, \end{aligned}$$

[20] If, for example, $Pr = 0 \neq Pr(\omega_t \in \Omega^D)$ with $(\tau_t, e_t) = (l, 0)$ and $(\tau_t^*, e_t^*) = (l, 0)$, then the expected one-period payoffs for period 1 and other "*initial*" periods will be different from those for any period immediately following a cooperative one, making the expected discounted payoffs along the equilibrium path more complicated than those in (6). Furthermore, having actions in period 1 and in other "*initial*" periods different from those in periods immediately following a cooperative period will make deviation incentives different across these periods, which in turn complicates characterization of the optimal protection sequence in Section 2.2.2.

where $\delta^K = \lambda^K (\delta^C)^{T^K} + (1-\lambda^K)(\delta^C)^{T^K-1}$ with K = s or none. $(\delta^C - \delta)$ and $(\delta^C - \delta^S)$ respectively represent the relative length of the punishment phase initiated by H or F alone and by H and F simultaneously. Because $(T, T^S, \lambda, \lambda^S)$ uniquely defines (δ, δ^S) as shown above, I will describe *simple* ***PTS*** using $(l, \Omega^D, \delta, \delta^S)$ instead of using $(l, \Omega^D, T, T^S, \lambda, \lambda^S)$ henceforth.

Note that *simple* ***PTS*** defined above is *simple* in the sense that each country imposes its static optimal tariff in all periods under any punishment phase. More generally, *PTS* may involve more complex punishment phases such as imposing lower tariffs if the signal indicates weaker violations or/and employing a stronger punishment, such as autarky, against presumably more severe violations. As shown later, the equilibrium payoff of any ("*almost strongly*") *symmetric PTS* will be identical to the one under *simple* ***PTS*** defined above, as long as the *initial* punishment is triggered by a static optimal tariff. From now on, I abbreviate *simple* ***PTS*** to *PTS* unless it is necessary to distinguish them.

2.2. Incentive Constraints under Private Trigger Strategies

In this section, I analyze incentive constraints for *PTS* to be a supergame equilibrium in the repeated game defined in Section 2.1. The private nature of signals that trigger punishments under *PTS* makes such incentive constraints different from the incentive constraints for trigger strategies under which public signals trigger punishments in two distinctive ways. First, the private nature of signals imposes restrictions on the lengths of punishment phases under *PTS*, which contrasts with the repeated game with public information where countries can choose any length for their punishment phases. Section 2.2.1 analyzes such limits on the lengths of punishment phases under *PTS*. Second, to check the absence of $s^{/}(\neq \underline{s})$ or $s^{*/}(\neq \underline{s}^*)$ such that $V(s^{/}, \underline{s}^*) > V(\underline{s}, \underline{s}^*)$ and $V^*(s^{*/}, \underline{s}) > V^*(\underline{s}^*, \underline{s})$, one needs to check not only one-time deviations from the specified strategy, but whole deviation paths that each country may take.[21] If private signals trigger punishments as under *PTS*, any deviatory action that each country might have taken in a previous period can influence its optimal deviatory action in a current period: The previous period defection affects the probability of a

[21] When a public signal triggers a punishment phase, any deviatory actions that each country might have taken in any previous periods will not affect its optimal deviatory action in the current period for a given history of public signals up to the current period. This is because one country's defections in the previous periods affect the other country's current and future actions only through affecting the history of public signals. Therefore, we can apply

punishment phase being initiated in the current period, which in turn influences the current-period optimal action. This necessitates characterization of an optimal (potentially deviatory) protection sequence that each country may take against $(\underline{s}, \underline{s}^*)$ in analyzing the incentive constraints for *PTS*. Section 2.2.2 characterizes such a sequence for H under *PTS*, and shows that H's optimal protection sequence can be a stationary one of setting τ at l (the cooperative protection level) in all periods until a punishment phase starts, which is a prerequisite for *PTS* to be a supergame equilibrium.

2.2.1. Constraints on Lengths of Punishment Phases

In any period that immediately follows a cooperative period with $(e, e^*) = (0, 0)$ and in any *initial* periods (period 1 and a period right after the end of any punishment phase), each country faces the choice of whether or not to initiate a punishment phase by imposing its static optimal tariff. To eliminate the incentive to misrepresent private signals in such periods, the expected payoff from initiating a punishment phase should be identical to the expected payoff from not initiating it for each country. Denote the condition that equates those expected payoffs by *ICP* for H (with the same condition applying for F by symmetry). Then,

***ICP*:**

$$\begin{aligned}(1-Pr)[u(l,l)+\delta^C V_C)]+Pr[u(l,h)+(\delta^C-\delta)V_N+\delta V_C]=\\ (1-Pr)[u(h,l)+(\delta^C-\delta)V_N+\delta V_C]+Pr[u(h,h)+(\delta^C-\delta^S)V_N+\delta^S V_C],\end{aligned} \tag{7}$$

where $V_C \equiv V(\underline{s}, \underline{s}^*)$ and $V_N \equiv u(h,h)/(1-\delta^C)$. The left side of the equality in (7) represents the expected discounted payoff from not initiating a punishment phase but continuing to set $(\tau, e) = (l, 0)$. The right side of the equality represents the expected discounted payoff from initiating a punishment phase, setting $(\tau, e) = (h, h)$. In calculating these expected discounted payoffs in (7), it is assumed that the other country initiates a punishment phase with a probability that conforms *PTS*, denoted by *Pr*.

Using $u(l, l) - u(l, h) = u(h, l) - u(h, h)$ implied by $\partial u/\partial\tau\partial\tau^* = 0$, I simplify (7) into

(*ICP*) $$u(l,l)-u(h,l)+(\delta^C-\delta)(V_C-V_N)=Pr[(\delta^C-\delta)-(\delta-\delta^S)](V_C-V_N).$$

---

the logic of one-stage-deviation principle for the subgame perfect equilibrium with observable actions (Theorem 4.1. and Theorem 4.2 in Fudenberg and Tirole, 1991) to the perfect public equilibrium (with unobservable actions).

For any given cooperative protection level (l) and any given range of private signals that trigger punishment phases (Ω^D), I have two variables (δ, δ^S) to be determined with one equation (*ICP*), potentially having infinite combinations of (δ, δ^S) that satisfies *ICP*. However, *Lemma 1* (a) below establishes that $\delta^C - \delta^S = 2(\delta^C - \delta)$ and *ICP* are the necessary conditions for each country to truthfully represent its private signals under *PTS*.

Lemma 1.

(a) $\delta^C - \delta$ = $[u(h, l) - u(l, l)]/(V_C - V_N)$ and $\delta^C - \delta^S = 2(\delta^C - \delta)$ are necessary conditions for each country to truthfully represent its private signals under *PTS*, triggering a punishment phase *iff* its private signal belongs to Ω^D.

(b) If H and F value their future payoffs high enough ($\delta^C \approx 1$) and the probability of a punishment phase being triggered along the equilibrium path is low enough ($Pr \approx 0$), then, for any given combination of (l, Ω^D) with $l < h$, there exists a unique combination of (δ, δ^S) that satisfies the necessary condition for truthful revelation of private signals in *Lemma 1* (a). (See Appendix for proof.)

Recall that $\delta^C - \delta$ and $\delta^C - \delta^S$ respectively represent the length of a punishment phase that H or F can initiate alone (T, λ) and the length of a punishment phase that H and F initiate concurrently (T^S, λ^S) as $\delta = \lambda(\delta^C)^T + (1-\lambda)(\delta^C)^{T-1}$ and $\delta^S = \lambda^S(\delta^C)^{T^S} + (1-\lambda^S)(\delta^C)^{T^S-1}$. Thus, for a given combination of (l, Ω^D), *ICP* with $\delta^C - \delta^S = 2(\delta^C - \delta)$ determines $\delta^C - \delta$ = $[u(h, l) - u(l, l)]/(V_C - V_N)$. Note that the length of a punishment phase that each country initiates by itself ($\delta^C - \delta$) increases in its expected gain in the initial period of the punishment phase by imposing its static optimal tariff unilaterally ($u(h, l) - u(l, l)$) but decreases in its expected loss in the tariff-war periods that will follow ($V_C - V_N$). The expected gain in the initial period of a punishment phase provides each country with the incentive to start a punishment phase despite the expected loss from engaging in a tariff war that follows under a punishment phase. Thus, the larger the expected gain in the initial period, the longer a punishment phase that H can tolerate (without violating *ICP*) and the larger the expected loss from a tariff war, the shorter a punishment phase that H can tolerate (without violating *ICP*).

Even when *ICP* is satisfied so that each country has no (strict) incentive to untruthfully represent its private signal after a "*real*" cooperative period, it may still have an incentive to misrepresent its private signal after a "*pseudo*" cooperative period under which it deviates by setting $\tau \neq l$ (or $\tau^* \neq l$) with its explicit tariff being zero. The proof for *Lemma 1* (*a*) in the Appendix shows that $\delta^C - \delta^S = 2(\delta^C - \delta)$ is indeed a necessary condition for each country not to misrepresent its private signals in a period following a *pseudo* cooperative period. For example, if $\delta^C - \delta^S < 2(\delta^C - \delta)$ so that the length of a punishment phase that H and F initiate concurrently is shorter than what it is supposed to be, then each country will have an incentive to set its protection level higher than *l* in a cooperative period and then initiate a punishment phase in the following period regardless of its private signal. Such a deviation strategy may pay off because an increase in the protection level in a cooperative period raises the probability of a punishment phase being triggered by the other country in the following period, which would then lead to a short punishment phase ($\delta^C - \delta^S$ being small) when the deviating country initiates a punishment regardless of its private signals.

2.2.2. Optimal Protection Sequence and Existence of a Stationary Protection Level

To characterize the optimal protection sequence, I analyze the dynamic optimization problem in which H maximizes its expected discounted payoff by choosing a protection sequence $\{\tau_{d+1}\}_{d=0}^{\infty}$, given that F follows its specified strategy under *PTS*. The dynamic optimization problem for H is

$$(8) \qquad \underset{\{\tau_{d+1}\}_{d=0}^{\infty}}{Sup} \sum_{d=0}^{\infty} \left\{ (\delta^C)^d \cdot \left[\prod_{t=0}^{d-1} [1 - Pr(\tau_t)] \right] \cdot F(\tau_d, \tau_{d+1}) \right\}$$

where $\prod_{t=0}^{d-1}[1-Pr(\tau_t)] = [1-Pr(\tau_0)]\times[1-Pr(\tau_1)]\times\cdots\times[1-Pr(\tau_{d-1})]$ with $\prod_{t=0}^{-1}[1-Pr(\tau_t)] = 1$; $Pr(\tau_t) \equiv Pr(\omega_t^* \in \Omega^D)$ given (τ_t, e_t) = (*l*, 0) and (τ_t^*, e_t^*) = (*l*, 0), and $\tau_0 = l$; and $F(\tau_d, \tau_{d+1}) \equiv Pr(\tau_d)[u(\tau_{d+1}, h) + (\delta^C - \delta)V_N + \delta V_{CO}] + [1 - Pr(\tau_d)]u(\tau_{d+1}, l)$ with $V_{CO} = V_C$. Note that the protection sequence $\{\tau_{d+1}\}_{d=0}^{\infty}$ in (8) specifies protection levels only until F triggers an initial punishment phase. The optimization in (8) assumes that H will follow its specified strategy under *PTS* once F triggers an initial punishment phase with $V_{CO} = V_C \equiv V(\underline{s}, \underline{s}^*)$. The *full* optimization problem should characterize the optimal protection sequence after the end of each punishment phase that may occur in the future periods. Characteristics of the optimal protection sequence derived from (8), however, will be qualitatively identical to those of the full optimization problem. This is because the optimal sequence resulting from (8) will be identical to the one from the full optimization problem if V_{CO} in (8) is set equal to the maximized expected discounted payoff of the full problem, having H face an identical optimization problem in determining the protection sequence after the end of each punishment phase in the future.[22] Also note that the optimal protection sequence considered in (8) excludes the possibility of using explicit tariffs as a part of its path. As shown in *Lemma 4* (*b*) of this section, however, once the lengths of punishment phases satisfy the necessary conditions for truthful revelation of private signals given in *Lemma 1* (*a*), then H cannot increase its payoff by using explicit tariffs along its deviation path. Hence, there is no loss of generality in analyzing the optimal protection sequence for H through the optimization problem defined in (8).[23]

Even though the optimization problem in (8) does not take a standard form for which a dynamic programming method is typically applied, *Lemma 2* (*a*) below establishes equivalency between (8) and the following (non-standard) dynamic programming problem:[24]

(9) $$V(\tau_{-1}) = \underset{\tau\in[0,h]}{Sup}\left\{F(\tau_{-1},\tau) + \delta^C[1 - Pr(\tau_{-1})]V(\tau)\right\} \text{ for all } \tau_{-1} \in [0, h],$$

[22] The discounted payoff of the full optimization problem can be obtained by applying the following iterative process to the optimization problem in (8). Initially set V_{CO} in (8) to be V_C defined in (6) and solve the optimization in (8), obtaining a discounted payoff as an outcome of this initial optimization problem. Then, set the value of V_{CO} in (8) to have the value of this initially generated discounted payoff, supposedly higher than (or equal to) the initial V_{CO} (= V_C), which redefines the optimization problem in (8). This redefined optimization problem will generate another discounted payoff as an outcome of this second optimization problem. Then, set V_{CO} in (8) to have the value of this newly generated discounted payoff and continue this iterative process until the discounted payoff generated through this process reaches its limit. As the sequence of the discounted payoffs generated through this process is monotonically increasing and bounded, there exists such a limit. This limit will be equal to the discounted payoff of the full optimization problem.

[23] While I focus on characterizing the optimal protection sequence for H under *PTS* in this section, the same characterization can be applied to the optimal protection sequence for F.

[24] (8) is not a standard problem in the sense that the component that corresponds to the return function of a standard problem, $[\prod_{t=0}^{d-1}[1 - Pr(\tau_t)]]F(\tau_d, \tau_{d+1})$, depends not only on the current choice variable and the choice made in the immediate prior period (as in the case of a usual return function of a typical dynamic programming problem) but also on all the choices made since the initial period. The dynamic programming problem in (9) is not a standard form because the current state variable, τ_{-1}, affects not only the current return function part, $F(\tau_{-1}, \tau)$, but also the future discounted payoff part through $[1 - Pr(\tau_{-1})]$.

where τ_{-1} and τ, respectively denote a previous-period and a current-period protection level of H.[25] Given a solution $V(\cdot)$ to (9), the optimal policy correspondence G: $[0, h] \rightarrow [0, h]$ is defined by:

(10) $$G(\tau_{-1}) = \{ \tau \in [0, h]: V(\tau_{-1}) = F(\tau_{-1}, \tau) + \delta^{C}[1 - Pr(\tau_{-1})]\cdot V(\tau)\},$$

which contains values of τ that maximize $V(\tau_{-1})$ for each $\tau_{-1} \in [0, h]$. Despite the fact that the dynamic optimization problem in (8) and the corresponding dynamic programming problem in (9) take non-standard forms, *Lemma 2* establishes the following standard results on V and G:

Lemma 2.

(*a*) Define $V_S(\tau_0)$ be the supremum function that is generated by (8). Then, (*i*) V_S satisfies (9); (*ii*) the solution to (9) is $V(\tau_{-1}) = V_S(\tau_{-1})$; (*iii*) every optimal protection sequence solving (8) is generated from G in (10); (*iv*) any protection sequence generated by G in (10) is an optimal protection sequence that solves (8).

(*b*) There exists a unique continuous function V that satisfies (9).

(*c*) The optimal policy correspondence G defined by (10) is compact-valued and upper hemi-continuous. (See Appendix for Proof)

Given *Lemma 2*, I can characterize the optimal protection sequence of H by characterizing $G(\cdot)$ in (10) because any protection sequence generated by G with the initial τ_{-1} being set at l is an optimal protection sequence that solves (8). Utilizing one of the generalized envelope theorems of Milgrom and Segal (2002) and a general result on the differentiability of the value function of Cotter and Park (2006), I can characterize V and G as follows: [26]

Lemma 3. Assume that the lengths of punishment phases satisfy the conditions in *Lemma 1* (*a*).

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[25] Note that limiting H's protection choice to be equal or less than h as in (9) does not affect the generality of the optimization problem because H has no incentive to raise its protection level above its static optimal protection level, h.

[26] In characterizing V and G, I cannot use the well-known result of Benveniste and Scheinkman (1979) on the differentiability of the value function. While Benveniste and Scheinkman established that concavity of the return function on the state and choice variables is sufficient to guarantee the differentiability of the resulting value function of a typical dynamic programming problem, the dynamic problem of choosing an optimal protection sequence analyzed in this paper does not belong to the typical dynamic programming problem, as explained earlier. Recently, Milgrom and Segal (2002) developed generalized envelope theorems for arbitrary choice sets, and Cotter and Park (2006) established differentiability of the value function on the range of the optimal policy correspondence, regardless of the curvature of the return function. I apply these results in characterizing V and G, as shown in the proof of *Lemma 3*.

(a) $V(\tau_{-1})$ is strictly decreasing in $\tau_{-1} \in [0, h]$.

(b) $G(\tau_{-1})$ is strictly increasing in τ_{-1} in the sense that $g(\tau''_{-1}) > g(\tau'_{-1})$ for all $\tau''_{-1} > \tau'_{-1} \in [0, h]$ with $g(\tau''_{-1}) \in G(\tau''_{-1})$ and $g(\tau'_{-1}) \in G(\tau'_{-1})$. (See Appendix for Proof)

Because a higher τ_{-1} (a higher protection level in the cooperative previous period) implies a higher probability that F triggers a punishment phase in the current period, a higher τ_{-1} also implies a more hostile environment for H to maximize its discounted payoff. Therefore, the outcome of the maximization problem, $V(\tau_{-1})$, will get smaller as τ_{-1} increases (*Lemma 3* (a)).

To understand *Lemma 3* (b), first note that choosing τ (a current-period protection level) must balance the current period's loss from setting the protection level below h (the static optimal one) against the future periods' gain from reducing the probability of a punishment phase being triggered. Figure 1 demonstrates this. Given the pervious-period protection level τ_{-1} is equal to τ'_{-1}, setting $\tau = h$ maximizes $F(\tau'_{-1},\tau) = Pr(\tau'_{-1})u(\tau,h) + [1 - Pr(\tau'_{-1})]u(\tau,l) + Pr(\tau'_{-1})[(\delta^C - \delta)V_N + \delta V_C]$ because it maximizes the expected current period payoff, $Pr(\tau'_{-1})u(\tau,h) + [1 - Pr(\tau'_{-1})]u(\tau,l)$ and τ does not affect the future expected discounted payoff contingent upon a punishment phase being initiated in the current period, $(\delta^C - \delta)V_N + \delta V_C$. By reducing τ below h, however, H can increase its expected discounted payoff, $F(\tau'_{-1},\tau) + \delta^C[1 - Pr(\tau'_{-1})]V(\tau)$ because $V(\tau)$ strictly decreases in τ by *Lemma 3* (a). As shown in Figure 1, if H lowers τ from h, $\delta^C[1 - Pr(\tau'_{-1})]V(\tau)$ strictly increases. Therefore, $g(\tau'_{-1})$, the optimal current-period protection with τ'_{-1} being the previous-period protection level, is lower than h.

Given this understanding of the optimal choice over τ, I can explain why $G(\tau_{-1})$ strictly increases in τ_{-1} using Figure 1. When τ_{-1} increases from τ'_{-1} to τ''_{-1}, it may shift $F(\tau_{-1},\tau)$ upwards as shown in Figure 1 but it will not affect $\partial F(\tau_{-1},\tau)/\partial\tau = \partial u(\tau,l^*)/\partial\tau$, implying that the static incentive to raise τ closer to h stays the same; for example, $F(\tau''_{-1},h) - F(\tau''_{-1},g(\tau'_{-1})) = F(\tau'_{-1},h) - F(\tau'_{-1},g(\tau'_{-1}))$ in Figure 1. An increase in τ_{-1}, however, weakens the dynamic incentive for lowering τ to avoid a punishment phase in a future period because it increases the likelihood of a punishment phase starting in the current period. Figure 1 illustrates this: $[1 - Pr(\tau''_{-1})][V(g(\tau'_{-1})) - V(h)] < [1 - \Pr(\tau'_{-1})][V(g(\tau'_{-1}) - V(h)]$ with $Pr(\tau''_{-1}) >$

$Pr(\tau^{/}_{-1})$. The dynamic gains from reducing τ from h to $g(\tau^{/}_{-1})$ decreases as τ_{-1} increases from $\tau^{/}_{-1}$ to $\tau^{//}_{-1}$. As a result, a higher τ_{-1} moves the balance for choosing an optimal τ towards a higher current period protection level so that $g(\tau^{//}_{-1}) > g(\tau^{/}_{-1})$ as shown in Figure 1.

The fact that $G(\tau_{-1})$ is strictly increasing in τ_{-1} may entail both an increasing protection sequence and a decreasing one as shown in Figure 2; if $\tau_0 = \tau^{/}_0$, then the optimal protection sequence will be increasing with $\tau^{/}_0 < \tau^{/}_1 < \tau^{/}_2 < \cdots$; and if $\tau_0 = \tau^{//}_0$, then the optimal protection sequence will be decreasing with $\tau^{//}_0 > \tau^{//}_1 > \tau^{//}_2 > \cdots$.[27] If $\tau_0 = \tau_S$, however, the resulting optimal protection sequence will be stationary with $\tau_0 = \tau_1 = \tau_2 = \cdots$. If there exists such a stationary protection level, $\tau_S \in [0, h)$ under *PTS* with $G(\tau_S) = \tau_S$ and $l = \tau_S$, then H would continue to set its protection level at l until a punishment phase begins. Therefore, the existence of such a stationary protection level, τ_S, is a prerequisite for *PTS* to be a supergame equilibrium of the repeated game. An increasing optimal policy correspondence (*Lemma 3* (*b*)) itself, however, does not rule out the possibility that the only stationary protection level of the dynamic problem in (9) is h, as demonstrated by $G^{/}(\tau_{-1})$ in Figure 2.

To address the existence issue of a stationary protection level $\tau_S \in [0, h)$ with $G(\tau_S) = \tau_S$, I analyze a necessary condition for such τ_S. If $V(\tau)$ is differentiable with respect to τ, then τ_S should satisfy the following first order condition for a stationary equilibrium, denoted by *IC*:

(11) ***IC*:** $\partial F(\tau_S, \tau_S)/\partial \tau + \delta^C[1 - Pr(\tau_S)]\cdot[\partial V(\tau_S)/\partial \tau] = 0$,

where $\partial F(\tau_S, \tau_S)/\partial \tau = \partial u(\tau_S, l)/\partial \tau$ and $\partial V(\tau_S)/\partial \tau = -[\partial Pr(\tau_S)/\partial \tau]\{u(\tau_S, l) + \delta^C V(\tau_S) - [u(\tau_S, h) + (\delta^C - \delta)V_N + \delta V_C]\}$. While I cannot assume differentiability of $V(\tau)$ on $\tau \in [0, h]$ as explained earlier, $V(\tau)$ is differentiable on any $\tau \in G(\tau_{-1})$ and $\tau \in (0, h)$ for each $\tau_{-1} \in [0, h]$, according to a generalized differentiability result of Cotter and Park (2006). Therefore, (11) is indeed a necessary condition for any stationary protection level that belongs to (0, h). Thus it serves as an incentive constraint (*IC*) for H to sustain the cooperative protection level, $l = \tau_S$ under *PTS*.

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[27] If the cooperative protection level is set too low under *PTS* with $l = \tau_0^{/}$, then H would keep raising the protection level above the cooperative one until it reaches a stationary level, τ_S, and the opposite is true if the cooperative protection level is too high with $l = \tau_0^{//}$. Blonigan and Park (2004) identify that a similar dynamic behavior emerges in the context of an exporting firm's dynamic pricing problem in the presence of antidumping policy; once an exporting firm becomes subject to an antidumping duty, it would either continue to decrease its export price (thus, having the duty increase over time) or continue to increase its export price (thus, having the duty lowered over time) depending on whether the initial export pricing is higher or lower than a stationary pricing.

For τ_S to be a stationary protection level for H, the static incentive to raise the protection level, $\partial F(\tau_S, \tau_S)/\partial\tau > 0$ in (11), needs be balanced by the dynamic incentive to avoid a costly punishment phase in the future, $\delta^C[1 - Pr(\tau_S)]\cdot[\partial V(\tau_S)/\partial\tau] < 0$ in (11). *Lemma 4* (*a*) below provides a sufficient condition for the existence of such $\tau_S \in (0, h)$ with $G(\tau_S) = \tau_S$, and *Lemma 4* (*b*) shows that H does not have any incentive to utilize explicit tariffs as part of its deviation path if $l = \tau_S$.

Lemma 4. Assume that the lengths of punishment phases satisfy the conditions in *Lemma 1* (*a*).

(*a*) If $\partial^2 Pr(\tau)/(\partial\tau)^2 > 0$ with $[\partial^2 Pr(\tau)/(\partial\tau)^2][1 - Pr(\tau)] - \{1 + \delta^C[1 - Pr(\tau)]\}[\partial Pr(\tau)/\partial\tau]^2 > 0$ for all $\tau \in [0, h]$ and $\partial Pr(\tau)/\partial\tau \approx 0$ at $\tau = 0$, then there exists a unique stationary equilibrium protection level $\tau_S \in (0, \mathrm{h})$ with $G(\tau_S) = \tau_S$. τ_S is also a globally stable equilibrium with $G(\tau) > \tau$ for $\tau \in [0, \tau_S)$ and $G(\tau) < \tau$ for $\tau \in (\tau_S, h)$.[28]

(*b*) If $l = \tau_S$, then H cannot increase its discounted payoff above $V(\underline{s}, \underline{s}^*)$ by taking any (deviatory) protection sequence that involves initiating punishment phases by imposing explicit tariffs. (See Appendix for Proof)

According to *Lemma 4* (*a*), it is possible to have *IC* in (11) satisfied for some $\tau_S < h$ if the sensitivity of F's private information in detecting a rise in H's concealed protection, $\partial Pr(\tau_S)/\partial\tau$, increases as H's concealed protection level rises with $\partial^2 Pr(\tau)/(\partial\tau)^2 > 0$. On the one hand, H's static incentive to raise its protection level, $\partial F(\tau_S, \tau_S)/\partial\tau = \partial u(\tau_S, l)/\partial\tau$ in (11), diminishes as τ_S increases with $\partial^2 u(\tau_S, l)/\partial\tau^2 < 0$, reaching zero at $\tau_S = h$. On the other hand, H's dynamic incentive to avoid a future punishment phase, $\delta^C[1 - Pr(\tau_S)]\cdot[\partial V(\tau_S)/\partial\tau]$ in (11), may diminish or intensify in response to an increase in τ_S, depending on the value that $\partial^2 Pr(\tau_S)/\partial\tau^2$ takes. A higher τ_S reduces H's weight on its dynamic incentive to avoid a punishment phase, $1 - Pr(\tau_S)$, by increasing the probability of a punishment phase being triggered in the current period. If $\partial^2 Pr(\tau_S)/\partial\tau^2 > 0$, an enhanced sensitivity of F's private information in detecting a rise in H's protection can offset such a reduction in H's incentive to avoid a punishment phase. The absolute value of $\delta^C[1 - Pr(\tau_S)]\cdot[\partial V(\tau_S)/\partial\tau]$ rises in response to a rise in τ_S if $[\partial^2 Pr(\tau)/(\partial\tau)^2][1 -$

[28] τ_S being a globally stable protection level is a contributing factor to the stability of *PTS* as an equilibrium of the repeated game. This is because H will eventually return to its globally stable behavior of setting $\tau = \tau_S$ (= l) after any arbitrary perturbations (possibly caused by errors) in its protection level choices.

$Pr(\tau)] - \{1 + \delta^C[1 - Pr(\tau)]\}[\partial Pr(\tau)/\partial\tau]^2 > 0$ for all $\tau \in [0, h]$, as assumed in *Lemma 4* (*a*). This in turn guarantees the existence of a unique $\tau_S \in (0, h)$ that satisfies *IC* in (11) with $\partial Pr(\tau)/\partial\tau \approx 0$ at $\tau = 0$.

Having the sensitivity of private information rise against increasing concealed protection can be crucial in discouraging the use of concealed protection under *PTS*. If $\partial^2 Pr(\tau)/(\partial\tau)^2 = 0$, for example, the dynamic incentive for lowering τ below h to avoid a tariff war in a future punishment phase, $\delta^C[1 - Pr(\tau_S)]\cdot[\partial V(\tau_S)/\partial\tau]$ in (11), decreases as τ_S gets higher, entailing the possibility of *IC* in (11) not being satisfied for any $\tau_S < h$.

While *Lemma 4* specifies the condition under which H (and F) would follow *PTS* by keeping its protection at a cooperative level until a punishment phase is triggered, note that *Lemma 4* "assumes" that the lengths of punishment phases satisfy the conditions in *Lemma 1* (*a*). Because such lengths of punishment phases "vary" with the cooperative protection level to sustain under *PTS*, it still remains to be shown whether there exist *PTS* that satisfy the conditions in *Lemma 1* (*a*) and *IC* simultaneously. The following section provides an affirmative answer.

3. Optimal Private Trigger Strategies

This section establishes that symmetric countries can sustain a symmetric cooperative protection level under *simple PTS* defined in the previous section if the sensitivity of their private information satisfies certain conditions. In addition, this section proves that any equilibrium payoff under ("*almost strongly*") symmetric trigger strategies that start an *initial* punishment phase by imposition of a static optimal tariff based on each country's imperfect private signal should be identical to the payoff under *simple PTS*. [29] After proving the existence and the uniqueness (in terms of payoffs, at least among a certain class of trigger strategies) of symmetric *PTS* as a supergame equilibrium in Section 3.1, I characterize optimal symmetric *PTS* under which H and F maximize their joint expected discounted payoffs in Section 3.2.

3.1. Private Trigger Strategies and Uniqueness Results

This section first proves the existence of *simple PTS* that satisfy the conditions in *Lemma 1* (*a*) and *IC* simultaneously. Assume that there exists τ_S that satisfies *IC* in (11) with $\tau_S = l$. This implies that $V(\tau_S) = V_C$, and I can rewrite *IC* in (11) as follows:

(12) $\partial u(\tau_S, l)/\partial\tau = \delta^C[\partial Pr(\tau_S)/\partial\tau][1 - Pr(\tau_S)][u(\tau_S, l) - u(\tau_S, h) + (\delta^C - \delta)(V_C - V_N)]$.

As discussed in the previous section, (12) is a necessary condition for H to have no incentive to change its protection level away from the cooperative one until a punishment phase starts. I also assume that the lengths of punishment phases are determined by the conditions in *Lemma* 1 (*a*); $\delta^C - \delta = [u(h, l) - u(l, l)]/(V_C - V_N)$ and $\delta^C - \delta^S = 2(\delta^C - \delta)$. *IC* in (12) then can be rewritten into the following implicit function, $I(l)$:

(13) $I(l) \equiv \partial u(l, l)/\partial\tau - \delta^C[\partial Pr(l)/\partial\tau][1 - Pr(l)][u(h, l) - u(l, h)] = 0$,

by substituting $\delta^C - \delta$ with $[u(h, l) - u(l, l)]/(V_C - V_N)$. Using $I(l)$, *Proposition 1* provides a sufficient condition for the existence of *simple PTS* that countries can sustain as a supergame equilibrium of their repeated protection-setting game:

Proposition 1. If $\partial^2 Pr(l)/(\partial l)^2 > 0$ with $[\partial^2 Pr(l)/(\partial l)^2][1 - Pr(l)] - \{1 + \delta^C[1 - Pr(l)]\}[\partial Pr(l)/\partial l]^2 > 0$ for all $l \in [0, h]$, $\partial Pr(l)/\partial l \approx 0$ at $l = 0$, and there exists at least one protection level, $l_S < h$ such that $I(l_S) = 0$, then, H and F can employ *simple PTS* with $l = l_S$, $\delta^C - \delta = [u(h, l_S) - u(l_S, l_S)]/(V_C - V_N)$, and $\delta^C - \delta^S = 2(\delta^C - \delta)$ as a supergame equilibrium of the repeated protection-setting game. (See Appendix for Proof)

Proposition 1 assumes the same condition regarding the sensitivity of private information as the one in *Lemma 4* (*a*), ensuring that there exists a unique stationary equilibrium protection level $\tau_S \in (0, h)$ with $G(\tau_S) = \tau_S$. In addition, it requires $I(l) = 0$ for at least one value of $l < h$, denoting it by l_S. With $l = l_S$, $\delta^C - \delta = [u(h, l_S) - u(l_S, l_S)]/(V_C - V_N)$ and $\delta^C - \delta^S = 2(\delta^C - \delta)$, $I(l_S) = 0$ guarantees that *IC* and the conditions in *Lemma 1* (*a*) are simultaneously satisfied under such *PTS*. According to *Lemma 4*, l_S is the unique stationary protection level with $G(l_S) = l_S$ and countries have no incentive to deviate from such *PTS*.

[29] *Definition 3* below provides a formal definition of "almost strongly symmetric private trigger strategies."

The sufficient condition in *Lemma 4* (*a*) does not necessarily imply that the second term of $I(l)$ in (13), $\delta^C[\partial Pr(l)/\partial \tau][1 - Pr(l)][u(h, l) - u(l, h)]$, representing H's dynamic incentive to avoid a tariff war, increases in response to a rise in l.[30] Thus, one may consider the case where multiple values of l satisfy $I(l) = 0$ as illustrated in Figure 3; $l = l_{\max}$ as well as $l = l_{min}$ satisfy $I(l) = 0$. Denoting the minimum of such l by l_{min}, then *simple PTS* with $l = l_{min}$ will Pareto-dominate the others when $Pr(l)$ is small enough.[31]

While the above result establishes that symmetric countries may employ *simple PTS* characterized by *Proposition 1* (and *Definition 2*) in restraining the use of concealed trade barriers, one may wonder whether there exist other (symmetric) private trigger strategies that may outperform this simple one. Surprisingly, the following result shows that there is no loss of generality in focusing on this *simple PTS* to characterize the optimal *symmetric private trigger strategies* as long as the explicit tariff that starts an *initial* punishment phase is the static optimal tariff of each country. The first part of the following result applies to a larger class of trigger strategies: any *symmetric private trigger strategies* where each country can start an *initial* punishment phase with an explicit tariff of any level.

Denote the level of τ (total protection) that initiates the first (or *initial*) punishment phase with e (explicit tariff) > 0 by d_0 and the cooperative protection level for the *initial* cooperative periods (prior to any punishment being triggered) by l_0, thus focusing on the *symmetric private trigger strategies* where the cooperative protection level and the protection level that starts an *initial* punishment phase are stationary at least prior to an *initial* punishment phase. Then, I can represent the expected discounted payoff of H of employing such *symmetric private trigger strategies* as follows, denoting it by $V(l_0; d_0)$:

(14)
$$V(l_0;d_0) = (1-Pr)(1-Pr)[u(l_0,l_0)+\delta^C V_C] + Pr(1-Pr)[u(l_0,d_0)+\delta^C V_{D^*}] \\ + Pr(1-Pr)[u(d_0,l_0)+\delta^C V_D] + Pr^2[u(d_0,d_0)+\delta^C V_{D^S}],$$

where $Pr \equiv Pr(l_0) = Pr(\omega \in \Omega^D)$ given $(\tau, e) = (l_0, 0)$ and $(\tau^*, e^*) = (l_0, 0)$, and $V_C \equiv V(l_0; d_0)$. V_D, V_{D^*} and V_{D^S} denote the expected discounted payoff of H after an *initial* punishment phase has been by triggered, respectively by H alone, by F alone, and by H and F simultaneously in a

[30] For the proof of this claim, see the proof for *Proposition 1* in Appendix.
[31] Note that $u(l_{min}, l_{min}) > u(l_{max}, l_{max})$ and $Pr(l_{min}) < Pr(l_{max})$ imply a higher cooperative-period payoff and a lower probability of punishment phases with $l = l_{min}$ than with $l = l_{max}$. While the lengths of punishment phases may be longer with $l = l_{min}$ than with $l = l_{max}$, an increase in l will lower the expected discounted payoff under *simple PTS* if $Pr(l)$ is close enough to 0, as shown in (18) of the following section.

previous period. Given these notations, the following definition defines "*almost strongly symmetric PTS*":

Definition 3. Among the set of symmetric strategies with $s(t) = s^*(t)$ for all $a^{t-1} \times \omega^{t-1} \times (e^*)^{t-1} = (a^*)^{t-1} \times (\omega^*)^{t-1} \times e^{t-1}$ and $t \geq 1$, *almost strongly symmetric PTS* with (l_0; d_0) are private trigger strategies under which each country starts its *initial* punishment phase by imposing an explicit tariff ($e > 0$) with its $\tau = d_0$, and the *initial* cooperative protection level is l_0 with $V_D = V_{D^*}$.

It is "*almost strongly symmetric*" strategies rather than "*strongly symmetric*" strategies under which $s(t) = s^*(t)$ for all $t \geq 1$ because $s(t) \neq s^*(t)$ may occur when $\omega_{t-1} \in \Omega^D$ or $\omega^*_{t-1} \in \Omega^D$, and $V_D = V_{D^*}$ does not necessarily entail $s(t) = s^*(t)$ after such contingencies. The payoff function in (14) implicitly assumes that each country sets its initial cooperative explicit tariff to be zero. Relaxing this assumption by allowing $e > 0$ in the initial cooperative periods would not raise the payoff in (14) as long as the sensitivity of private information of concealed trade barriers improves with a higher level of concealed trade barriers.[32] For simplicity, I will abbreviate *almost strongly symmetric PTS* by *symmetric PTS* henceforward.

For *symmetric PTS* defined by *Definition 3* to be incentive-compatible (so that they can be supported as equilibrium behaviors), the following analysis establishes that the payoff in (14) needs to be equal to the following expression:

$$(15) \qquad V(l_0; d_0) = \frac{u(l_0, l_0)}{1-\delta^C} - Pr(l_0)\frac{u(d_0, l_0) - u(l_0, d_0)}{1-\delta^C}.$$

Note that the payoff in (15) depends only on the values of l_0 and d_0. Once established, the above result remarkably simplifies the job of characterizing the payoff frontier attainable under any *symmetric PTS* that rely on triggering a punishment phase with some explicit tariffs: one only needs to find l_0 and d_0 that are incentive-compatible, which in turn maximize the payoff in (15).

[32] $e = e^* = 0$ forces each country to raise its protection level all through concealed trade barriers. If the sensitivity of private information improves with a higher level of such barriers, the effectiveness of private trigger strategies against the incentive to raise protection levels should improve with such constraints of setting $e = e^* = 0$ in the (initial) cooperative periods.

To prove (15), I use three incentive-compatibility conditions: (i) a generalized version of *ICP*, (ii) *IC* for setting the initial cooperative protection level to be l_0 and continuing to set $\tau = l_0$ in the following period upon the contingency of no punishment phase being initiated, and (iii) *IC* for setting the initial cooperative protection level to be l_0 and starting an initial punishment phase by setting $\tau = d_0$ in the following period upon the contingency of no punishment phase being initiated. First, note that any *symmetric PTS* triggering the initial punishment phase with $\tau = d_0$ and $e > 0$ should satisfy the following ICP^G:

(ICP^G)
$$(1-Pr)[u(l_0,l_0)+\delta^C V_C]+Pr[u(l_0,d_0)+\delta^C V_{D^*}]$$
$$=(1-Pr)[u(d_0,l_0)+\delta^C V_D]+Pr^*[u(d_0,d_0)+\delta^C V_{D^S}],$$

which equalizes the payoff of initiating the (initial) punishment phase with the payoff of not initiating it, similarly to *ICP* in the previous section. Using $u(l_0, l_0) - u(l_0, d_0) = u(d_0, l_0) - u(d_0, d_0)$ implied by $\partial^2 u/\partial\tau\partial\tau^* = 0$, I can simplify ($ICP^G$) as follows:

(ICP^G) $$u(l_0,l_0)-u(d_0,l_0)+\delta^C(V_C-V_{D^*})=\delta^C Pr[(V_C-V_D)+(V_C-V_{D^*})-(V_C-V_{D^S})].$$

To identify incentive constraints for setting the initial cooperative protection level to be l_0, I can write the expected discounted payoff of H setting τ to be τ_I in an initial period (prior to any punishment phase being triggered) as

$$Pr[u(\tau_I,d_0)+\delta^C V_D]$$
$$+(1-Pr)\{u(\tau_I,l_0)+\delta^C Pr(\tau_I)[u(l_0,d_0)+\delta^C V_{D^*}]+\delta^C[1-Pr(\tau_I)][u(l_0,l_0)+\delta^C V_C]\},$$

or

$$Pr[u(\tau_I,d_0)+\delta^C V_D]$$
$$+(1-Pr)\{u(\tau_I,l_0)+\delta^C Pr(\tau_I)[u(d_0,d_0)+\delta^C V_{D^S}]+\delta^C[1-Pr(\tau_I)][u(d_0,l_0)+\delta^C V_D]\}$$

depending on whether H sets $\tau = l_0$ or $\tau = d_0$, respectively, in the following period upon the contingency of no punishment phase being initiated after setting $\tau = \tau_I$ as the initial cooperative protection level. To be able to support $\tau_I = l_0$ as an equilibrium behavior, the following IC^G need to be satisfied:

(IC^G)
$$\frac{\partial u(l_0,l_0)}{\partial\tau}=\delta^C(1-Pr)\frac{\partial Pr(l_0)}{\partial l_0}[u(l_0,l_0)-u(l_0,d_0)+\delta^C(V_C-V_{D^*})],$$
and
$$\frac{\partial u(l_0,l_0)}{\partial\tau}=\delta^C(1-Pr)\frac{\partial Pr(l_0)}{\partial l_0}[u(d_0,l_0)-u(d_0,d_0)+\delta^C(V_D-V_{D^S})],$$

implying that $u(l_0,l_0)-u(d_0,l_0)+\delta^C(V_C-V_{D^*})=u(d_0,l_0)-u(d_0,d_0)+\delta^C(V_D-V_{D^S})$. Using this last equality together with the simplified ICP^G and $V_D=V_{D^*}$, one can show that $(V_C-V_D)+(V_C-V_{D^*})-(V_C-V_{D^S})=0$, thus $u(d_0,l_0)-u(l_0,l_0)=\delta^C(V_C-V_D)$. Given these equalities, one can rewrite $V(l_0; d_0)$ in (14) into the one in (15) using the following steps:

$$
\begin{aligned}
V(l_0;d_0) &= u(l_0,l_0)+\delta^C V_C + Pr[u(l_0,d_0)-u(l_0,l_0)-\delta^C(V_C-V_D)] \\
&\quad + Pr[u(d_0,l_0)-u(l_0,l_0)-\delta^C(V_C-V_{D^*})] \\
&\quad + Pr^2\{[u(d_0,d_0)-u(l_0,d_0)]-[u(d_0,l_0)-u(l_0,l_0)]\} \\
&\quad + \delta^C Pr^2[(V_C-V_D)+(V_C-V_{D^*})-(V_C-V_{D^S})] \\
&= u(l_0,l_0)+\delta^C V_C + Pr[u(l_0,d_0)-u(d_0,d_0)-u(d_0,l_0)+u(l_0,l_0)],
\end{aligned} \tag{16}
$$

where the second line through the forth in (16) all take zero values and the last line in (16) generates (15) using $V(l_0;d_0)=V_C$. I can summarize this result in the following proposition:

Proposition 2. The equilibrium payoff of any *symmetric PTS* (defined by *Definition 3*) with the initial cooperative protection level being l_0 and the level of total protection that starts the initial punishment phase being d_0, denoted by $V(l_0; d_0)$, is a function of only l_0 and d_0 with

$$V(l_0;d_0)=\frac{u(l_0,l_0)}{1-\delta^C}-Pr(l_0)\frac{u(d_0,l_0)-u(l_0,d_0)}{1-\delta^C},$$

where $Pr(l_0) = Pr(\omega_t^* \in \Omega^D)$ given $(\tau_t, e_t) = (l_0, 0)$ and $(\tau_t^*, e_t^*) = (l_0, 0)$.

The above proposition establishes that one can fully characterize the equilibrium payoff of any *symmetric PTS* only with the information about l_0 and d_0 that are incentive-compatible. Given $u(d_0,l_0)-u(l_0,l_0)=\delta^C(V_C-V_D)$, the necessary condition for an incentive-compatible choice of l_0 is

$$\textbf{(}IC^G\textbf{)} \qquad \frac{\partial u(l_0,l_0)}{\partial \tau}=\delta^C[1-Pr(l_0)]\frac{\partial Pr(l_0)}{\partial l_0}[u(d_0,l_0)-u(l_0,d_0)].$$

This condition is identical to *IC* for *simple PTS* characterized in (13) if $d_0 = h$, the static optimal protection level. IC^G and *Proposition 2* together imply that countries cannot attain the symmetric efficient frontier where $V_C = u(l_0, l_0)/(1 - \delta^C)$ with $l_0 = 0$ as their equilibrium payoffs under any *symmetric PTS* if their private signals entail non-negligible errors in detecting the use of concealed trade barriers. If $l_0 = 0$, then $\partial u(l_0,l_0)/\partial\tau > 0$, which in turn requires

$u(d_0, l_0) - u(l_0, d_0) > 0$ to satisfy IC^G. Given that $Pr(l_0) > 0$ due to non-negligible errors in the private information, *Proposition 2* implies that $u(l_0, l_0)/(1-\delta^C) - V(l_0; d_0) = Pr(l_0)[u(d_0, l_0) - u(l_0, d_0)]/(1-\delta^C) > 0$ with $l_0 = 0$. The following corollary to *Proposition 2* states this finding:

Corollary 1 to Proposition 2. Under any *symmetric PTS*, countries cannot attain the symmetric efficient frontier where $V_C = u(l_0, l_0)/(1 - \delta^C)$ with $l_0 = 0$ as their equilibrium payoffs if their private signals entail non-negligible errors in detecting concealed trade barriers with $Pr(l_0) > 0$.

This anti-folk theorem result under *symmetric PTS* demonstrates an aspect of private trigger strategies considered in this paper: To have each country properly trigger a punishment phase under which it may gain in the initial punishment period, such a punishment-initiating country needs to be penalized later in the punishment phase, a costly process for all countries involved. Also note that this anti-folk theorem result is attained under *symmetric PTS*, a subset of private trigger strategies that countries can employ, thus it is still an open question whether one can obtain a folk theorem result under a more general private trigger strategies.

Beyond proving the above anti-folk theorem result under *symmetric PTS*, a further characterization of *symmetric PTS* is not a simple matter. While *Proposition 1* guarantees the existence of incentive-compatible *symmetric PTS* with $d_0 = h = e$, characterizing the necessary condition for an incentive compatible choice of $d_0 \neq h$ is far from being an easy task. Once $d_0 \neq h$, each country would have an incentive to start an initial punishment phase by choosing $\tau \neq d_0$ such as $\tau = h$, necessitating a punishment scheme against such a deviation incentive. Note that the punishment scheme against a deviatory initiation of an initial punishment phase generates yet another private monitoring problem, which can be different from the one for the initial periods. This process of having an additional and different private monitoring issue against deviatory uses of punishment phases may continue forever, making a general characterization of it a very difficult task.[33]

[33] If *PTS* are *strongly symmetric* with $s(t) = s^*(t)$ for all $t \geq 1$, except for unilateral initiations of punishment phases against potential violations, and if *PTS* do not allow each country to start another (new) punishment phase right after its initiation of an *initial* punishment phase, then one can show that the only incentive compatible choice of d_0 is h. This is possibly another way of justifying the paper's focus on *simple PTS*, but this approach seems to impose rather stringent constraints on *PTS*.

Even when one ignores the issue of finding an incentive-compatible d_0 and pretends that one can choose any value for d_0, it is not clear whether choosing $d_0 < h$ would increase the expected discounted payoff in comparison with the choice of setting $d_0 = h$. For example, setting $d_0 < h$ would raise $V(l_0; d_0)$ by decreasing $u(d_0, l_0) - u(l_0, d_0)$ with $\partial[u(d_0, l_0) - u(l_0, d_0)]/\partial d_0 > 0$. However, lowering d_0 weakens the IC^G by lowering the right hand side value of IC^G shown above, thus decreasing $V(l_0; d_0)$ by raising the value for l_0. The optimality of choosing $d_0 \neq h$, even when it is incentive-compatible, therefore, depends on the trade-off between its direct effect on the payoff through changing $u(d_0, l_0) - u(l_0, d_0)$ and its indirect effect through changing the incentive-compatible l_0, which in turn requires further characterization of the private information of concealed trade barriers.

In the following characterization of the optimal *symmetric PTS*, I will focus on the optimal *symmetric PTS* with $d_0 = h$. Note that this constraint of setting $d_0 = h$ still allows full flexibility over the choice of strategies that each government can take once an initial punishment phase starts. With regard to the issue of characterizing the efficient frontier among this subset of *symmetric PTS* with $d_0 = h$, one can focus on *simple PTS* characterized in *Proposition 1* as the following corollary clarifies.

Corollary 2 to Proposition 2. The equilibrium payoff of any *symmetric PTS* (defined by *Definition 3*) with the initial cooperative protection level being l and the level of total protection that starts the initial punishment phase being h, is identical to the payoff of *simple PTS* characterized in *Proposition 1* with

$$V(l;h) = \frac{u(l,l)}{1-\delta^C} - Pr(l)\frac{u(h,l)-u(l,h)}{1-\delta^C},$$

where $Pr(l) = Pr(\omega_t^* \in \Omega^D)$ given $(\tau_t, e_t) = (l, 0)$ and $(\tau_t^*, e_t^*) = (l, 0)$.[34]

3.2 Optimal Private Trigger Strategies

Up to this point, I have assumed that the range of private signals that trigger a punishment phase, Ω^D, is fixed. Countries can change the (initial) cooperative protection level by changing the range of punishment-phase-triggering private signals, Ω^D, because it affects the probability of a punishment phase being triggered against the potential use of concealed trade barriers. This section characterizes the optimal *simple PTS*, or equivalently the optimal *symmetric PTS* with $l_0 = l$ and $d_0 = h$ (*Corollary 2* to *Proposition 2*), focusing its analysis on the choice of Ω^D that maximizes the expected discounted payoffs of countries. Once again I abbreviate optimal *simple PTS* by optimal *PTS* hereafter, unless it is necessary to distinguish them.

The private signal $\omega \in \Omega$ has two distinctive yet related quality dimensions as a measure that detects the potential use of concealed protection. One is the *sensitivity* of the signal in detecting possible defections, which links a higher protection to a higher probability of a punishment phase being triggered. The other is the *stability* of the signal that rewards cooperative behaviors with a lower probability of a punishment phase. I can represent the sensitivity by $Pr^{/}(\tau) \equiv \partial Pr(\tau)/\partial\tau > 0$ and the stability by $1 - Pr(\tau)$ measured at $\tau = l$.

A change in the range of private signals that trigger a punishment phase may affect these qualities of signals in different directions. In particular, countries may raise the sensitivity by properly expanding the range of punishment-phase-triggering private signals, Ω^D , but at the cost of undermining the stability. By denoting the degree of such expansion with a parameter ω^D, to be termed "a *trigger control variable*," I can formalize this trade-off that countries face in choosing ω^D by assuming $\partial Pr^{/}(\tau)/\partial\omega^D > 0$ and $\partial Pr(\tau)/\partial\omega^D > 0$.

The analysis of optimality in this section focuses on *simple PTS* identified in *Proposition 1*, with the cooperative protection level being determined by a choice over ω^D. Assuming that ω^D uniquely determines l with $I(l) = 0$, I can represents l as a function of ω^D; $l = l(\omega^D)$. Then, as shown by *Corollary 2 to Proposition 2*,

(17) $$V_C \equiv V(\underline{s}, \underline{s}^*) = \frac{u(l,l)}{1-\delta^C} - Pr(l)\frac{u(h,l)-u(l,h)}{1-\delta^C},$$

where ($\underline{s}$, $\underline{s}^*$) are *simple PTS* defined in *Definition 2*. Note that the expected discounted payoff in (17) is no longer depending on the lengths of punishment phases. Therefore, I can describe the optimal choice for ω^D using the following first order condition:

(18) $$\frac{\partial V_C}{\partial\omega^D} = \frac{\partial V_C}{\partial l}\frac{\partial l(\omega^D)}{\partial\omega^D} + \frac{\partial V_C}{\partial Pr}\frac{\partial Pr(l)}{\partial\omega^D} = 0, \text{ with}$$

[34] One can derive H's expected discounted payoff under *symmetric PTS* shown in the above corollary, from (6), using $\delta^C - \delta = [u(h, l) - u(l, l)]/(V_C - V_N)$ and $\delta^C - \delta^S = 2(\delta^C - \delta)$.

$$\frac{\partial V_C}{\partial l} = \frac{\partial u(l,l)}{\partial l}\frac{1}{1-\delta^C} - Pr^{/}(l)\frac{u(h,l)-u(l,h)}{1-\delta^C}$$
$$- Pr(l)\frac{\{\partial u(l,l)/\partial\tau^* - \partial u(l,l)/\partial\tau\}}{1-\delta^C} < 0 \text{ for } Pr(l) \text{ being close to 0,}$$
$$\frac{\partial l(\omega^D)}{\partial\omega^D} = -\frac{\partial I/\partial\omega^D}{\partial I/\partial l} < 0 \text{ iff } \frac{\partial Pr^{/}(l)}{\partial\omega^D}[1-Pr(l)] - \frac{\partial Pr(l)}{\partial\omega^D}Pr^{/}(l) > 0, \text{ and}$$
$$\frac{\partial V_C}{\partial\omega^D}\frac{\partial Pr(l)}{\partial\omega^D} = -\frac{\partial Pr(l)}{\partial\omega^D}\frac{[u(h,l)-u(l,h)]}{1-\delta^C} < 0,$$

where $I = I(l)$ is the implicit function defined in (13). The first order condition is informative about the trade-off that countries face in choosing an optimal ω^D. Raising the trigger control variable (ω^D) will have a positive effect on the expected discounted payoff (V_C) by lowering the cooperative protection level (l) since $\partial l/\partial\omega^D < 0$ and $\partial V_C/\partial l < 0$, but it also has a negative effect on the expected payoff by raising the probability of a punishment phase being invoked, as shown by $\partial V_C/\partial\omega^D < 0$ in (18). Thus, the optimal ω^D should balance the gain from raising the sensitivity of the private signal (thus achieving a lower l) against the loss from reducing the stability of the cooperative equilibrium with a higher punishment phase probability.

When the initial ω^D is at a very low level, then, it is generally possible to lower l by raising the trigger control variable. For example, if $\Omega^D = \varnothing$, then $l = h$ and $Pr(l) = Pr^{/}(l) = 0$, implying $\partial l/\partial\omega^D < 0$ with $\partial Pr^{/}(l)/\partial\omega^D > 0$ from (18). If countries continue to raise ω^D, the marginal increase in the sensitivity of private signals in response to an increase in ω^D is likely to get smaller. To formalize this decreasing return to raising the trigger control variable, I assume that $\partial^2 Pr^{/}(l)/\partial(\omega^D)^2 < 0$ and $\partial^2 Pr(l)/\partial(\omega^D)^2 = 0$, with the latter assumption making the effect of a higher ω^D on $Pr(l)$ to be constant. Then, it is possible to have $\partial^2 l/\partial(\omega^D)^2 > 0$ and $\partial l/\partial\omega^D = 0$ for a high enough ω^D.

Even when it is possible to raise ω^D to such a point that the countries would no longer be able to lower the cooperative protection level any further ($\partial l/\partial\omega^D = 0$), note that it is never optimal to do so. If countries were to raise ω^D in this way, then the first order condition for the optimal ω^D in (18) will be violated with $\partial V_C/\partial\omega^D = (\partial V_C/\partial Pr)(\partial Pr(l)/\partial\omega^D) < 0$, implying that countries can increase their payoffs by lowering the trigger control variable. One can use a similar argument to show that setting $l = 0$ cannot be optimal when $\partial u(l, l)/\partial l = 0$ at $l = 0$ and

$\partial Pr(l)/\partial l \approx 0$ at $l = 0$, as assumed in *Proposition 1*. I summarize these characterizations of *optimal simple PTS* in the following proposition.

Proposition 3. Assume that the sufficient conditions for the existence of equilibrium *simple PTS* in *Proposition 1* are satisfied. In addition, assume that $\partial Pr^{/}(l)/\partial \omega^D > 0$, $\partial Pr(\tau)/\partial \omega^D > 0$, $\partial^2 Pr^{/}(l)/\partial(\omega^D)^2 < 0$, and $\partial^2 Pr(l)/\partial(\omega^D)^2 = 0$ where ω^D denotes the *trigger control variable* associated with an expansion of Ω^D. Then, under the optimal *PTS*, countries do not raise the trigger control variable to the level that pushes down the cooperative protection level to its minimum attainable level where $\partial l/\partial \omega^D = 0$. In particular, the optimal *PTS* will not set $l = 0$ with $\partial u(l, l)/\partial l = 0$ at $l = 0$.[35]

The characterization of optimal *PTS* in Proposition 3 emphasizes the need for tolerating some level of concealed trade barriers under *PTS*. For example, setting the concealed trade barriers to zero in the cooperative period is not optimal: a slightly higher cooperative protection level (by choosing a slightly lower ω^D) would cause no first order loss as free trade is efficient with $\partial u(l, l)/\partial l = 0$ at $l = 0$ and would decrease the likelihood of a costly punishment phase being triggered. One cannot directly apply *PTS* for understanding the working of Section 301 of the U.S. under which the United States Trade Representative (USTR) follows an elaborate procedure prior initiating a punishment against potential deviatory actions of other countries. However, the following practice of Special Section 301 to protect U.S. intellectual property rights (IPR) in foreign markets does illustrate the U.S. government's willingness to tolerate some level of deviations from agreements, reserving retaliatory sanctions mainly against considerable deviations. In applying Special Section 301, the USTR specifies not only "Priority Foreign Countries" who are "pursuing the most onerous or egregious policies that have the greatest adverse impact on U.S. right holders or products, and are subject to accelerated investigations and possible sanctions," but also "Priority Watch List" of countries "who do not provide an adequate level of IPR protection or enforcement, or market access for persons relying on intellectual property protection."[36] Such a practice may not lead to the maximal protection of the U.S. IPR, but may reduce the probability of costly tariff wars invoked by Special Section 301.

[35] A similar characterization has been drawn for optimal cartel trigger price strategies by Porter (1983).

4. A Possible Role for the WTO: Optimal Third Party Trigger Strategies

Regarding the issue of enforcing international trade agreements, this paper focuses on a phenomenon that the trade literature has not fully explored; countries may form different opinions about potential violations of trade agreements. In the absence of a third party like the WTO that can generate supposedly impartial opinions about such violations, Section 2 and 3 of this paper explore the possibility of countries' adopting *private trigger strategies*, under which each country initiates punishment phases based on its own imperfect private signals of the other country's potential use of concealed trade barriers. In particular, this paper characterizes the optimal *PTS* as an attempt to describe what countries can achieve with regard to trade policy coordination in the absence of the WTO, a prerequisite for analyzing how the WTO can facilitate improved coordination, especially when the WTO can simply generate its opinion of potential violations without any coercive power to impose its opinions upon countries.

To understand a possible role that the WTO can play under imperfect private monitoring of potential violations of trade agreements, this section analyzes "*third-party trigger strategies*" under which a third party, such as the WTO, decides upon whether a violation has occurred and allows each country to initiate a punishment phase based on its decision. Given the characterization of optimal *PTS* of the previous section, the comparison between the optimal *third-party trigger strategies* and optimal *PTS* will illustrate how and to what degree the WTO can help countries to enforce international trade agreements beyond what countries can do alone.

This paper, however, does not attempt to build a model that can proxy the actual operation of the WTO in dealing with potential violations and associated trade disputes: Though as I discuss in the conclusion, this in itself would be a meaningful research direction. Instead, this section will consider *third-party trigger strategies* under which the only role that the WTO plays is providing an impartial third-party (thus, *public*) opinion of violations so that trigger strategies are no longer subject to constraints imposed by the *private* nature of countries' signals of violations under *private triggers strategies*, such as *ICP*. This analysis thus

[36] These quoted definitions come from the USTR website (http://www.ustr.gov).

illustrates the minimum role that the WTO can play in facilitating countries to improve their trade policy coordination.

To make a direct comparison between *third-party trigger strategies* and *PTS* characterized in Section 3, I make the following assumptions in this section. The stage-game payoffs and action variables of H and F are the same as those described in Section 2. In addition to these two players, there exists the WTO, a third party supposedly neutral with regard to the issue of enforcing international trade agreements. At the end of period *t*, the WTO obtains $\omega_t \in \Omega$ and $\omega_t^* \in \Omega^*$, the same private signals that each country receives of the other country's potential violations. One may model a mechanism under which each country truthfully reports its private signals to the WTO in a non-public manner if the WTO can verify the reported signals. For simplicity, this section simply assumes that the WTO has an access to such signals. Given the setup of Section 2, then the WTO would have complete information of τ_t and τ_t^* because the WTO knows all the random components of the model. Even when one introduces additional random components into the model, the WTO may still have an informational superiority over countries given the access to private signals of both countries. The analysis of how the WTO may utilize such an informational superiority, which itself is attributable to the WTO's neutrality, is an interesting topic. As mentioned earlier, this paper assumes away such a possibility, simply focusing on the possible role of the WTO in relaxing the constraints on the lengths of punishment phases imposed by the private nature of signals that trigger punishments, namely the conditions specified in *Lemma 1* (*a*). Therefore, the following analysis will characterize how changing "*private*" trigger strategies into "*third-party*" ones through the WTO may improve the enforcement of international trade agreements, controlling the quality of available information about potential deviations.

Once again Ω^D denotes the range of private signals that triggers H (F) to initiate a punishment phase by imposing an explicit tariff, but it is the WTO that tells each country to initiate such a punishment phase in *third-party trigger strategies*. The infinitely repeated protection-setting game between H and F stays the same as before, except that now the WTO tells or does not tell each country to initiate a punishment phase by imposing an explicit tariff based on its own (the WTO's) signals of potential deviations. Note that these signals remain "not public" unless the WTO decides to make them "public." For simplicity, I denote the

WTO's decision to tell H to initiate a punishment phase in period t based on its signals received at the end of period $t-1$ by $\mu_{t\text{-}1} \in M \equiv \{1, 0\}$, with $\mu_{t\text{-}1}$ being 1 iff $\omega_{t\text{-}1} \in \Omega^D$, denoting its similar decision for F by $\mu^*_{t\text{-}1} \in M^* \equiv \{1, 0\}$. Then, a strategy for each country is defined by $s^{W\,i} = (s^{W\,i}(t))_{t=1}^{\infty}$, similarly to the ones in Section 2, with

(19) $$s^W(t): A^{t-1} \times M^{t-1} \times M^{*\,t-2} \times E^{*\,t-1} \rightarrow A \text{ and } s^{W^*}(t): A^{*\,t-1} \times M^{*\,t-1} \times M^{t-2} \times E^{t-1} \rightarrow A^*$$

where M^{t-1} and $M^{*\,t-1}$, respectively denote the history of the WTO's decision of telling H and F to initiate a punishment phase up to period $t-1$. Note that strategies defined in (19) allow each country to observe the WTO's decision for the other country to initiate a punishment phase only afterwards. This strategy specification under which each country chooses its current action without knowing the WTO's current decision on the other country's initiation of a punishment phase may seem unnatural. This specification, however, enables a direct comparison between *third-party trigger strategies* and *PTS* of Section 2 by making these two types of strategies differ only in their ability in selecting the lengths of punishment phases. Henceforth, the analysis will focus on *third-party trigger strategies* defined in *Definition 4* below.

(i) Given that period $t-1$ was a "*cooperative*" period with (e_{t-1}, e^*_{t-1}) = (0, 0), each country keeps cooperating by setting (τ^i_t, e^i_t) = (l, 0) as long as the WTO does not tell it to initiate to a punishment phase by having $\mu_{t\text{-}1}{}^i = 0$ with i = * or none.

(ii) Given that period $t-1$ was a "*cooperative*" period with (e_{t-1}, e^*_{t-1}) = (0, 0), the WTO tells H to initiate a punishment phase by setting (τ_t, e_t) = (h, 0) iff $\omega_{t-1} \in \Omega^D$ and it tells F to initiate a punishment phase by setting (τ^*_t, e^*_t) = (h, 0) iff $\omega^*_{t-1} \in \Omega^D$.

(iii) Given that a "*punishment phase*" was initiated in period $t-1$ by only one country, countries set (τ, e) = (h, h) and (τ^*, e^*) = (h, h) for the following ($T-2$) periods and they continue to do so one more period with probability λ. Given that a "*punishment phase*" was initiated in period $t-1$ simultaneously by both countries, countries set (τ, e) = (h, h) and (τ^*, e^*) = (h, h) for the following ($T^S - 2$) periods and they continue to do so one more period with probability λ^S. T and T^S are integers that are greater than or equal to 2 with λ and λ^S belonging to [0, 1]. Each country knows these variables (T, T^S, λ, λ^S) when it initiates a punishment phase and the actual length of a punishment phase is determined by some public randomizing device (determining λ and λ^S) after a punishment phase being initiated.

(iv) In period 1 and other "*initial*" periods right after the end of any punishment phase, with probability *Pr* the WTO tells each country to initiate a punishment phase by setting $(\tau^i, e^i) = (h, h)$, and with probability $(1 - Pr)$ the WTO does not tell each country to initiate a punishment phase so that it sets $(\tau^i, e^i) = (l, 0)$, where $Pr = Pr(\omega_t^i \in \Omega^D)$ with $(\tau_t, e_t) = (l, 0)$, $(\tau_t^*, e_t^*) = (l, 0)$, and i = * or none.

Definition 4. If (i), (ii), (iii), (iv) describe $(\underline{s}^W, \underline{s}^{W*})$, then $(\underline{s}^W, \underline{s}^{W*})$ are ***third-party trigger strategies (TTS)*** with $(l, \Omega^D, T, T^S, \lambda, \lambda^S)$ as characterizing parameters.

Given this definition, it is easy to check that the expected discounted payoff under $(\underline{s}^W, \underline{s}^{W*})$ with $(l, \Omega^D, T, T^S, \lambda\ \lambda^S)$, denoted by $V^W(\underline{s}^W, \underline{s}^{W*})$, is identical to $V(\underline{s}, \underline{s}^*)$ in (6). Once again, I have $(\delta^C - \delta)$ and $(\delta^C - \delta^S)$ respectively represent the (relative) length of the punishment phase initiated by one country and by both countries simultaneously.

While the expression for the expected discounted payoff is same under *TTS* defined above and under *PTS* defined in *Definition 2*, there exists an important distinction between these two types of trigger strategies: The WTO has no incentive to lie about its private signals so that *TTS* are not subject to the *ICP*. This implies that one can choose any values for the lengths of punishment phases, $(\delta^C - \delta)$ and $(\delta^C - \delta^S) \in [0, \delta^C]$. Recall that $\delta^C - \delta = [u(h, l^c) - u(l^c, l^c)]/(V_C - V_N)$ and $(\delta^C - \delta^S) = 2(\delta^C - \delta)$ under *PTS*. To make the comparison between the *TTS* and the *PTS* even simpler, I make one more assumption that $(\delta^C - \delta^S) = 2(\delta^C - \delta)$ holds under *TTS*, thus allowing full flexibility only over the choice of $(\delta^C - \delta)$, the length of a single-country-initiated punishment phase. This assumption enables one to tell whether the lengths of punishment phases under the optimal *PTS* are too short or too long (compared with the optimal *TTS*) by comparing the endogenously determined value for $(\delta^C - \delta)$ under the optimal *PTS* with the optimal choice of $(\delta^C - \delta)$ under the *TTS*. Given this assumption of $(\delta^C - \delta^S) =$

$2(\delta^C - \delta)$, one can simplify $V^W(\underline{s}^W, \underline{s}^{W*})$ into $V_C{}^W \equiv V^W(\underline{s}^W, \underline{s}^{W*}) = (1 - Pr)[u(l, l) - u(h, h)]/[1 - \delta^C + 2Pr(\delta^C - \delta)] + V_N$ with $V_N = u(h, h)/(1 - \delta^C)$.

To be able to support *TTS* as an equilibrium of the repeated protection-setting game between H and F, *TTS* need to satisfy the following incentive constraint, denoted by IC^W:

(IC^W) $\quad I^W(l) \equiv \partial u(l, l)/\partial\tau - \{\delta^C[\partial Pr(l)/\partial\tau][1-Pr(l)][u(l, l) - u(l, h) + (\delta^C - \delta)(V_C{}^W - V_N)]\} = 0.$

Note that IC^W is identical to *IC* in (12) under *PTS* as long as δ under *TTS* is the same as under *PTS*. This equivalence results from constructing *TTS* in the way that it may only differ from *PTS* in its flexibility to choose the single-country-initiated punishment phase to last for any length. The intuition behind this equivalence between *IC* under *PTS* and IC^W under *TTS* is quite simple: Each country chooses its cooperative-period protection level, knowing that raising the protection level increases the probability of a punishment phase being triggered in the same manner under both trigger strategies.

In addition to IC^W, there is one more incentive constraint that *TTS* needs to satisfy: Each country has an incentive to follow the WTO's decision on initiating a punishment phase. Because the WTO's decision becomes *public* (known to all players) with a one period lag, one may construct a (off-equilibrium-path) punishment strategy, such as a permanent Nash tariff war, against the behavior of not following the WTO's decision on triggering a punishment phase. Given that the expected discounted payoff under *TTS* is strictly greater than the discounted payoff of playing the static Nash tariff war forever, it is easy (and standard in the literature) to show that each country has an incentive to follow the WTO's decision as long as the discount factor (δ^C) is high enough. I assume that this standard result is valid for the following analysis with a high enough value for δ^C.

For the analytical simplicity, one can represent a choice of (T, λ) by a real number $T^W \in [1, \infty)$ with $\delta^C - (\delta^C)^{T^W} = (\delta^C - \delta)$. $T^W = 1$ (equivalent to the case of $T = 2$ and $\lambda = 0$) is the case where any country's initiation of a punishment phase by imposing its static optimal tariff is not followed by any punishment period where countries play a Nash tariff war of setting their tariffs to be the static optimal ones, representing the shortest possible punishment phase. $T^W \to \infty$ is the case where a permanent Nash tariff war is followed by an initiation of a punishment phase, representing the longest possible punishment phase.[37] Then, the problem of finding the optimal *TTS* is solving the following maximization problem:

(20)
$$\underset{\omega^D \text{ and } T^W \in [1,\infty)}{Max} \left\{ \frac{[1-Pr(l)][u(l,l)-u(h,h)]}{1-\delta^C + 2Pr(l^c)[\delta^C - (\delta^C)^{T^W}]} + V_N \right\} \text{ subject to}$$
$$\frac{\partial u(l,l)}{\partial \tau} = \delta^C \frac{\partial Pr(l)}{\partial \tau}[1-Pr(l)]\{u(l,l)-u(h,h)+[\delta^C - (\delta^C)^{T^W}](V_C^W - V_N)\},$$

where ω^D represents a *trigger control variable*, defined in the same way as in Section 3.2.

Because the problem of finding the optimal *PTS* in Section 3.2 is to choose only ω^D to maximize the same payoff function as in (20) subject to the same incentive compatibility condition, but with T^W (or equivalently, corresponding T and λ) being determined by $\delta^C - (\delta^C)^{T^W} = [u(h, l^c) - u(l^c, l^c)]/(V_C^W - V_N)$, it is obvious that the optimal *TTS* of solving the maximization problem in (20) will yield an expected discounted payoff that is greater than (or at least equal to) that under the optimal *PTS*. The question is how and to what degree the less-constrained optimal *TTS* will outperform the optimal *PTS*.

Analyzing the first order conditions of the maximization problem for the optimal *TTS* in (20) can provide some insight into the factors that determine the optimal choice of ω^D and T^W:

$$\frac{dV_C^W}{d\omega^D} = \frac{\partial V_C^W}{\partial l}\frac{\partial l}{\partial \omega^D} + \frac{\partial V_C^W}{\partial Pr}\frac{\partial Pr(l)}{\partial \omega^D} = \left(-\frac{\partial V_C^W / \partial l}{\partial I^W(l)/\partial l}\right)\frac{\partial I^w(l)}{\partial \omega^D} + \frac{\partial V_C^W}{\partial Pr}\frac{\partial Pr(l)}{\partial \omega^D} = 0,$$
$$\frac{dV_C^W}{dT^w} = \frac{\partial V_C^W}{\partial l}\frac{\partial l}{\partial T^w} + \frac{\partial V_C^W}{\partial T^W} = \left(-\frac{\partial V_C^W / \partial l}{\partial I^W(l)/\partial l}\right)\frac{\partial I^W(l)}{\partial T^W} + \frac{\partial V_C^W}{\partial T^W} = 0$$

with

(21)
$$\frac{\partial V_C^W}{\partial Pr} = -\frac{\{[1-\delta^C + 2[\delta^C - (\delta^C)^{T^W}]\}[u(l,l)-u(h,h)]}{\{1-\delta^C + 2Pr[\delta^C - (\delta^C)^{T^W}]\}^2} < 0 \text{ and}$$
$$\frac{\partial V_C^W}{\partial T^w} = \frac{2\ln(\delta^C)(\delta^C)^{T^W} Pr(1-Pr)[u(l,l)-u(h,h)]}{\{1-\delta^C + 2Pr[\delta^C - (\delta^C)^{T^W}]\}^2}$$
$$= \frac{-2\ln(\delta^C)(\delta^C)^{T^W} Pr(1-Pr)}{1-\delta^C + 2[\delta^C - (\delta^C)^{T^W}]}\frac{\partial V_C^W}{\partial Pr} < 0,$$

[37] Under *TTS*, it is not impossible to choose $T^W \in (0, 1)$ by setting $T = 1$ and $\lambda \in (0, 1)$. For example, the WTO uses its own randomizing device in determining whether to tell each country to impose its static optimal tariff for one period or not with probability λ if $\omega \in \Omega^D$ or $\omega^* \in \Omega^D$. To make a direct comparison between *PTS* and *TTS*, once again I limit the choices of T^W with $T^W \in [1, \infty)$.

where $I^W(l)$ represents the implicit function defining the IC^W above, $\partial l/\partial\omega^D = -(\partial I^W/\partial\omega^D)/(\partial I^W/\partial l)$ and $\partial l/\partial T^W = -(\partial I^W/\partial T^W)/(\partial I^W/\partial l)$ generate the second equality for $dV_C^W/d\omega^D$ and dV_C^W/dT^W, respectively. The expression after the second equality for $\partial V_C^W/\partial T^W$ is obtained using the expression for $\partial V_C^W/\partial Pr$ in (21). As explained in Section 3.2, the optimal choice of ω^D involves the balance between its positive effect of lowering the cooperative protection and its negative effect of increasing the probability of costly punishment phases. Similarly, increasing the length of a punishment phase has a positive effect of lowering the cooperative protection by strengthening the punishment but also entails a negative effect of increasing the cost of punishment with the costly punishment phase being longer. The optimal choice of T^W also involves balancing between these counteracting forces.

This section focuses on the analysis of an optimal choice of T^W because Section 3.2 provides an analysis of the optimal choice over ω^D and a similar characterization should apply to the one under *TTS*.[38] For further characterization of an optimal choice of T^W, I assume that the optimal ω^D is an interior solution, thus $dV_C^W/d\omega^D = 0$. It is reasonable to assume that $dV_C^W/d\omega^D = 0$ for any *TTS* that attains improvement over one-shot Nash equilibrium because a corner solution for ω^D implies either no punishment for any contingency ($\Omega^D = \varnothing$) or punishment for all contingencies ($\Omega^D = \Omega$). Using $dV_C^W/d\omega^D = 0$ together with the second expression for $\partial V_C^W/\partial T^W$ in (21), I can rewrite dV_C^W/dT^W as follows:

(22)
$$\frac{dV_C^W}{dT^W} = \left(-\frac{\dfrac{\partial V_C^W}{\partial l}}{\dfrac{\partial I^W}{\partial l}} \right) \left[\frac{\partial I^W}{\partial T^W} + A\frac{\partial I^W}{\partial \omega^D} \right], \text{ where } A = \frac{2\ln(\delta^C)(\delta^C)^{T^W} Pr(1-Pr)}{1-\delta^C+2[\delta^C-(\delta^C)^{T^W}]} \left(\frac{\partial Pr(l)}{\partial \omega^D} \right)^{-1},$$
$$\text{with } \left(-\frac{\dfrac{\partial V_C^W}{\partial l}}{\dfrac{\partial I^W}{\partial l}} \right) < 0, \frac{\partial I^W}{\partial T^W} < 0, \frac{\partial I^W}{\partial \omega^D} < 0, \text{ and } A\frac{\partial I^W}{\partial \omega^D} > 0.$$

The above first order condition for an optimal choice of T^W, which also embodies the first order condition for the choice of ω^D, reveals a potentially "competing" nature of these two choice variables in restraining the use of concealed trade barriers. $\partial I^W/\partial T^W < 0$ and $\partial I^W/\partial\omega^D < 0$

[38] For any given level of T^W, the optimal choice over ω^D under *TTS* should be the same kind of balancing choice as the one under *PTS*. See the above discussion on the choice of ω^D in relation with (21). Therefore, the characterization of an optimal ω^D of *Proposition 2* should apply to the optimal ω^D under *TTS*.

demonstrate that both of these choice variables can relax IC^W, which in turn enable countries to lower the cooperative protection level, l. For example, if the effectiveness of ω^D in relaxing IC^W rises so that the absolute value of $\partial I^W/\partial\omega^D$ (and $A\partial I^W/\partial\omega^D$) increases, then the optimal choice of T^W may involve a decrease in T^W and an increase in ω^D to sustain $dV_C{}^W/dT^W = 0$ if $\partial^2 I^W/(\partial T^W)^2 > 0$ and $\partial(A\partial I^W/\partial\omega^D)/\partial\omega^D < 0$.[39] In fact, the following result establishes that the optimal T^W may take corner solutions depending on the probability of a punishment being triggered in the equilibrium, which in turn may depend on the accuracy of information about potential deviations, as shown through a numerical analysis that follows this analytical result:

Proposition 4. Given that $\partial Pr^/(l)/\partial\omega^D > 0$, $\partial Pr(\tau)/\partial\omega^D > 0$, $\partial^2 Pr^/(l)/\partial(\omega^D)^2 < 0$, and $\partial^2 Pr(l)/\partial(\omega^D)^2 = 0$ as assumed in *Proposition 3* for the characterization of optimal *PTS*,

(a) the length of a single-country initiated punishment phase, T^W, equals 1 under the optimal *TTS* if $Pr(l) < \overline{Pr}$, where

$$\overline{Pr} = \frac{-3(1-\delta^C) + \sqrt{[3(1-\delta^C)]^2 + 16\delta^C(1-\delta^C)}}{8\delta^C},$$

with $\partial\overline{Pr}/\partial\delta^C < 0$ and $\lim_{\delta^C \to 0} \overline{Pr}/\partial\delta^C = 1/3$ so that $\overline{Pr} \in (0,1/3)$ for $\delta^C \in (0,1)$, and

(b) the length of a single-country initiated punishment phase, T^W, goes to ∞ under the optimal *TTS* if $Pr(l) > \underline{Pr}$, where

$$\underline{Pr} = \frac{2 - [u(l,l) - u(h,h)]/[u(l,l) - u(l,h)]}{4 - [u(l,l) - u(h,h)]/[(u(l,l) - u(l,h)]},$$

with $[u(l,l) - u(h,h)]/[u(l,l) - u(l,h)] \in (0,1)$ for $l \in [0,h)$ so that $\underline{Pr} \in (1/3,1/2)$.

(See Appendix for Proof)

According to *Proposition 4*, the length of a single-country initiated punishment phase under the optimal *TTS* takes its minimum value of $T^W = 1$ if the probability of a punishment phase being triggered is below a critical level, denoted by $\overline{Pr}$. With $T^W = 1$, note that no tariff war period (under which both countries impose their static optimal tariffs) will follow an initiation of any punishment phase. This implies an *asymmetric* (in the sense that only the potential deviator is punished with the punishing country being rewarded by imposing its static

[39] One can show that $\partial^2 I^W/(\partial T^W)^2 > 0$ but it seems to be very difficult prove that $\partial(A\partial I^W/\partial\omega^D)/\partial\omega^D < 0$ given the

optimal tariff) and *minimum* (in the sense that the punishment length is taking its minimum value) punishment against potential violations. Note also that countries cannot use such an *asymmetric* and *minimum* punishment under *PTS* because countries will have an incentive to initiate such a punishment phase regardless of their private signals. The presence of the WTO, a third party who *impartially* judges whether a country (might have) violated a trade agreement allows countries to use an *asymmetric* and *minimum* punishment, facilitating countries to realize higher expected payoffs beyond what they can do by themselves under *PTS*.[40]

This *asymmetric* and *minimum* punishment ($T^W = 1$) is optimal when the probability of a punishment phase being triggered, $Pr(l)$ is less than a critical level, $\overline{Pr}$. As briefly discussed with regard to the first order condition for choosing T^W in (22), one can understand this (sufficient) condition for $T^W = 1$ by looking at how a change in $Pr(l)$ affects the relative effectiveness of T^W and ω^D in relaxing the incentive constraint, IC^W. In fact, one can show that the (relative) effectiveness of ω^D increases faster than the effectiveness of T^W in response to a decrease in $Pr(l)$ so that the effectiveness of ω^D is greater than that of T^W for all values of T^W when $Pr(l) < \overline{Pr}$, thus having $dV_C^W/dT^W < 0$ for all $T^W \in [1, \infty)$. The effectiveness of ω^D relative to T^W is measured by the absolute value of $A(\partial I^W/\partial \omega^D)$ relative to the absolute value of $\partial I^W/\partial T^W$ in the second bracket of (22).

$\partial \overline{Pr} / \partial \delta^C < 0$ in *Proposition 4* implies that the optimal *TTS* is less likely to involve $T^W = 1$ when countries' relative valuations of future payoffs increase with higher values of δ^C. Once again, one can understand this result by examining how a change in δ^C affects the relative effectiveness of T^W and ω^D in relaxing the incentive constraint, IC^W: One can show that the effectiveness of T^W increases faster than that of ω^D in response to an increase in δ^C so that the optimal *TTS* is less likely to set $T^W = 1$ when δ^C is higher. If $\delta^C = 1/2$, for example, $\overline{Pr} = 1/4$, implying that $T^W = 1$ is optimal under *TTS* if the probability of a punishment being triggered is less than 1/4, and $\overline{Pr}$ decreases toward zero as δ^C approaches 1.

Proposition 4 (*b*) shows that $T^W \to \infty$ may also emerge as an optimal punishment length choice under *TTS* if the probability of a punishment phase being triggered is above a critical

highly non-linear nature of A in ω^D, unless one introduces stringent assumptions on Pr.

[40] This kind of asymmetric action is often one of important characteristics of optimal strategies of repeated games under various applications, such as in Kandori and Matsushima (1998), Compte (1998), and Athey and Bagwell (2001), because such asymmetry allows players to avoid actions with (at least heavy) dead-weight losses.

level, denoted by $\underline{Pr}$. This maximum punishment of playing the Nash tariff war forever once a punishment is triggered is a surprising result because the main reason for countries to coordinate their trade policies is to avoid playing the Nash tariff war, and because they can choose any length for their punishment phase under *TTS*. Again, it is possible to understand this sufficient condition for $T^W \rightarrow \infty$ by looking at how a change in $Pr(l)$ affects the relative effectiveness of T^W and ω^D in relaxing the incentive constraint, IC^W. The effectiveness of ω^D decreases faster than that of T^W in response to an increase in $Pr(l)$ so that the effectiveness of ω^D is smaller than that of T^W even when $T^W \rightarrow \infty$ if $Pr(l) > \underline{Pr}$, thus having $dV_C^W/dT^W > 0$ even when $T^W \rightarrow \infty$.

Proposition 4 provides a characterization of the optimal *TTS*, which depends on the probability of a punishment phase being triggered right after a cooperative period. One may find that such a characterization is not satisfactory because the characterization relies on $Pr(l)$, a variable that countries choose indirectly by choosing ω^D.[41] One may also wonder about the possibility of more directly comparing the optimal *PTS* and the *TTS*, thus finding when they will differ from each other and how they will differ.[42] In response to such demands, one may try to introduce more structures to the private signals, thus making $Pr(l)$ depend on some accuracy measure of private signals, then characterize the optimal *TTS* (and the optimal *PTS*) depending on such a fundamental variable. Because of the highly non-linear nature of the maximization problem involving two choice variables (T^W and ω^D) as shown through the first order condition in (22), pursuing such a characterization is extremely difficult, if not infeasible.[43]

While it might not be possible to derive complete analytical results regarding the characterization of the optimal *TTS* and optimal *PTS* in the way the preceding paragraph discusses, one can conduct a numerical analysis for such characterization. The following numerical analysis does just that and reveals several interesting (numerical) results. To conduct

[41] A positive side of the characterization of optimal *TTS* in *Proposition 4* is that it imposes relatively weak assumptions on private signals and is still able to drive a relatively sharp prediction of when the corner solutions will emerge as an optimal choice for T^W, depending on the equilibrium values of $Pr(l)$.

[42] *Proposition 4* does provide results that show how and when the optimal *TTS* would differ from the optimal *PTS* because neither $T^W = 1$ nor $T^W \rightarrow \infty$ occur under *PTS*. What is missing is a more continuous comparison of the two strategies, possibly depending on some fundamental variables, such as a measure for accuracy of signals.

[43] As shown in the proof of *Proposition 4* in the Appendix, proving the results in *Proposition 4* itself is not a trivial exercise given the highly nonlinear nature of the optimization problem to solve.

a numerical analysis, I use the same partial equilibrium trade model as the one in Bond and Park (2002) where H exports good 1 and F exports good 2, with $\sigma \in [1, \infty)$ denoting the size of H's markets relative to F's.[44] Demand for good i in H is $D_i = \sigma(A - Bp_i)$ and supply of good i in H is $X_i = \sigma(\alpha_i + \beta p_i)$, where p_i is the price of good i in H with i = 1 or 2. For F, demand and supply are given by $D_i^* = A - Bp_i^*$ and $X_i^* = \alpha_i^* + \beta p_i^*$. To ensure that H will export good 1 and import good 2 and that the countries will be symmetric when $\sigma = 1$, $\alpha_1 - \alpha_1^* = \alpha_2^* - \alpha_2 > 0$ and $\alpha_1 = \alpha_2^*$. In addition, I assume that $Pr(l)$ takes the following functional form:

(23)
$$\begin{aligned} Pr(l) = Pr(l \mid \omega^D; \rho, \chi) &= \omega^D[l^2/(2\chi) + \rho] \text{ for } l \leq \underline{l}/2, \\ &= \omega^D[(\underline{l} \times l)/\chi - l^2/(2\chi) + \rho - \underline{l}^2/(4\chi)] \text{ for } \underline{l}/2 < l \leq \underline{l}, \\ &= 1 \text{ for } l > \underline{l}, \end{aligned}$$

where $\bar{l} \equiv 2\sqrt{\chi/\omega^D - \chi\rho}$, $1/\chi \in (0,\infty)$ represents the sensitivity of the signal in detecting an increase in the level of concealed trade barriers, and $\rho \in [0, \infty)$ represents the level of errors in detecting concealed trade barriers (thus, the "in-"stability of the signals), making $Pr(l) > 0$ even when $l = 0$ with $\rho > 0$ and ω^D ($\in [0, 1/\rho)$) > 0. While the complicated expression for $Pr(l)$ with $l > \underline{l}/2$ is used to make the probability density function to be symmetric around $\underline{l}/2$ and $Pr(l) = 1$ when $l = \underline{l}$, the equilibrium values for l are all less than $\underline{l}/2$ in the following numerical analysis, thus making this part of the probability definition be redundant. $Pr(l)$ defined in (23) is one of simplest functional forms for $Pr(l)$ with parameters representing both the sensitivity and (in)stability of private signal and also having $\partial Pr(l)/\partial l > 0$, $\partial^2 Pr(l)/\partial(l)^2 > 0$, $\partial Pr'(l)/\partial\omega^D > 0$, $\partial Pr(\tau)/\partial\omega^D > 0$, $\partial^2 Pr'(l)/\partial(\omega^D)^2 < 0$, and $\partial^2 Pr(l)/\partial(\omega^D)^2 = 0$ for $l \leq \underline{l}/2$, as assumed in *Proposition 4*.[45]

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[44] In a previous version of this paper, there was a section that provides an analysis of *PTS* in the presence of asymmetry in the size of trading countries. The following concluding section briefly discusses the effect of introducing such asymmetry on *PTS* as a factor that may limit the use of *PTS* in restraining concealed trade barriers.

[45] One may find $Pr(l) = 1$ for $l > \underline{l}$ not satisfying, especially when $\underline{l} < h$. Thus, one can consider using an adjusted Rayleigh distribution, $Pr(l) = Pr(l \mid \omega^D; \rho, \chi) = 1 - \exp[-(\omega^D l)^2/(2\chi^2) - \rho\omega^D]$ for the numerical analysis because $Pr(l) < 1$ for all $l \in [0, \infty)$. The problem associated with using this Rayleigh distribution is that $\partial^2 Pr(l)/\partial(\omega^D)^2 = 0$ is no longer true, and this assumption is what enables simplification of the first order condition for T^W in (22), which in turn leads to the analytical results in *Proposition 4*. As a robustness check, I have done a numerical analysis using this probability function and found that characteristics of the optimal *PTS* and optimal *TTS* are qualitatively identical to those shown in the numerical analysis of this section using $Pr(l)$ in (23).

I assume that $\sigma = 1$ to analyze the case of symmetric countries, and also assume that $\alpha_1 - \alpha_1^* = 3$, $\beta + B = 1$, which induces $h = 1$ for simplicity.[46] To illustrate how the optimal *TTS* change as the instability of the private signal (measured by ρ) changes, Figure 4 shows the outcome of the numerical analysis with $\chi = 1$ and $\delta^C = 0.5$. It indicates how each of the following changes in response to an increase in the instability of the signal, ρ, from 80(×0.00005) to 130.2(×0.00005): (i) the expected percentage payoff gain under the optimal *TTS* compared with playing the static Nash tariff war forever, $(V_C^W - V_N)/V_N$; (ii) the cooperative protection level, l; (iii) the probability of a punishment phase being triggered, $Pr(l)$; (iv) the length of a punishment phase, T^W; and (v) the trigger control variable choice, ω^D.

As predicted by *Proposition 4*, $T^W = 1$ when $Pr(l) < \overline{Pr} = 1/4$ (using $\delta^C = 0.5$) and $T^W \to \infty$ when $Pr(l) > 4/9$, using the fact that the maximum value that $\underline{Pr}$ can take is 4/9 as $[u(l, l) - u(h, h)]/[\mathrm{u}(l, l) - \mathrm{u}(l, h)]$ reaches its minimum at 2/5 with $l = 0$ given the parameter values of the trade model under consideration. It also confirms the conjecture that the probability of a punishment phase being triggered in the equilibrium would depend on the accuracy of information about potential deviations (at least in the limits), thus having $T^W = 1$ for low enough values of ρ and $T^W \to \infty$ for high enough values of ρ. Another notable aspect of this numerical result is that $Pr(l)$ decreases in response to an increase in ρ, the instability (or inaccuracy) measure of private signals, when optimal *TTS* utilize both ω^D and T^W (> 1). A possible explanation for this phenomenon once again can be based on the relative effectiveness of ω^D and T^W in relaxing IC^W: If the effectiveness of T^W relative to ω^D improves as ρ increases, then countries will substitute ω^D with T^W, implying a lower ω^D and a higher T^W as shown in the bottom two graphs in Figure 4, which in turn may lead to a decrease in $Pr(l)$ because $\partial Pr(l)/\partial\omega^D > 0$.[47]

[46] In deriving this result, I assume that each country's welfare function (as a function of τ and τ^*) derived from demand and supply functions with no uncertainties is identical to the ones derived with uncertainties described in Section 2.1. This is a strong assumption but justifiable given the fact that what one really needs are $u(\tau, \tau^*)$ and $u^*(\tau^*, \tau)$ with $\partial u(\tau, \tau^*)/\partial\tau > 0$ at $\tau = 0$, $\partial u^*(\tau^*, \tau)/\partial\tau < 0$, $\partial[u(\tau, \tau^*) + u^*(\tau^*, \tau)]/\partial\tau < 0$, $\partial^2 u(\tau, \tau^*)/\partial\tau^2 < 0$, and $\partial^2 u(\tau, \tau^*)/\partial\tau\partial\tau^* = 0$, properties of welfare functions of the trade model of Bond and Park (2002).

[47] This explanation of $\partial Pr(l)/\partial\rho < 0$ for internal values of T^W seams to be in conflict with the following explanation for *Proposition 4* (*a*) given earlier, "the (relative) effectiveness of ω^D increases faster than that of T^W in response to a decrease in $Pr(l)$," because ω^D decreases and T^W increases when $Pr(l)$ decreases in response to an increase in ρ in the bottom 3 graphs of Figure 4. However, these are not contradictory explanations because the explanation for *Proposition 4* (*a*) is explaining how the corner solution of $T^W = 1$ may rise for small values of $Pr(l)$ by changing

Another interesting exercise one can do with this numerical analysis is to compare the optimal *TTS* with optimal *PTS*. Continuing to assume the same parameter values, except for χ being 100 instead of being 1 (thus, the sensitivity of private signals being lower), Figure 5 compares the optimal *TTS* and the optimal *PTS* in all the same 5 variables as in Figure 4 when ρ increases from 30(×0.000005) to 61.9(×0.000005). Note that the bold lines represent variables for the optimal *TTS* and the dotted lines depict variables for the optimal *PT*S. The graphs on the right column in Figure 5 provide zoomed graphs of the same 5 variables for high values of ρ, from 59(×0.000005) to 61.9(×0.000005) because the variable for the *TTS* and the *PTS* are very similar for these high values of ρ. One obvious result is that the gains from cooperation under the *TTS* are higher than those under *PTS* (being identical only when $\rho = 60.5$ in Figure 5 with all other variables being identical as well, as they should be). One less obvious but potentially important result is that the gains from moving from the optimal *PTS* to the optimal *TTS* are significant when the signals are relatively accurate with low values for ρ. As one can easily tell from the top graphs in Figure 5, such gains can become negligible for high values of ρ. It is important to note that the significant gains from moving from the optimal *PTS* to the optimal *TTS* come from countries' ability to reduce the length of punishment phase and substitute it with a higher value for ω^D under *TTS*. The probability of a punishment phase being triggered is higher under *TTS* than under *PTS* for all $\rho < 60.5$ due to a higher value for ω^D. This higher value for ω^D enables to countries to support a lower protection level under *TTS* than under *PTS*, as shown in Figure 5 for $\rho < 60.5$.

Given the analytical results in *Proposition 4* as well as the numerical ones shown in Figure 4 and 5, I can highlight the main potential benefit of the WTO's presence in enforcing international trade agreements as follows. Even when the (private) signals of violations are relatively accurate, it might be hard for countries to be responsive against potential violations (choosing a higher value for ω^D) under *PTS* because initiating a punishment should and will accompany a rather long and costly tariff-war phase between countries (to eliminate the incentive to abuse the punishment). Once countries can utilize opinions of an impartial third party, such as the WTO, then countries can employ a more effective punishment, possibly the

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Pr(*l*) as if it is an exogenous variable, even though it is in fact an endogenous variable affected by the optimal choices of ω^D and T^W.

asymmetric and *minimum* punishment with $T^W = 1$, which in turn enables countries to be less tolerant of potential violations, attaining a higher level of cooperation!

5. Concluding Remarks

In the presence of concealed trade barriers of which each country has imperfect private signals, the WTO can facilitate a better cooperative equilibrium in the repeated trade relationship. This is established by comparing the optimal *PTS* (*private trigger strategies*) in which each country triggers a punishment phase based on its own private signals with the optimal *TTS* (*third-party trigger strategies*) in which the WTO tells who should start a punishment phase based on its (the WTO's) signals, abstracting away from any informational advantage or disadvantage of the WTO over trading countries. Prior to discussing the role of the WTO, the analysis first establishes that symmetric countries may restrain the use of concealed trade barriers under *simple PTS* if the sensitivity of their private signals rises in response to an increase in such barriers. It also shows that any equilibrium payoff under *almost strongly symmetric PTS* will be identical to the one under *simple PTS* as long as the initial punishment is triggered by a static optimal tariff, justifying the focus on *simple PTS*. The analysis of optimal (*simple*) *PTS* reveals that it is not optimal to push down the cooperative protection level to its minimum attainable level (such as free trade) due to the cost associated with increasing the probability of costly punishments.

To illustrate how and by what degree the WTO may facilitate countries in enforcing international trade agreements beyond what they can achieve alone under *PTS*, this paper conducts both an analytical analysis of the optimal *TTS* and a numerical comparison of the optimal *PTS* and optimal *TTS*. If the probability of a punishment phase being triggered is low enough, possibly because of accurate enough signals of potential violations, the analytical analysis establishes that the optimal *TTS* entail an *asymmetric* and *minimum* punishment. The punishment is *asymmetric* in the sense that only the potential deviator is punished with the punishing country being rewarded by imposing its static optimal tariff and *minimum* in the sense that the punishment length is taking its minimum value. Just the opposite result of using a punishment involving a permanent Nash tariff war will emerge under the optimal *TTS* if the probability of a punishment being triggered is high enough, possibly because of inaccurate signals of violations. The presence of the WTO under *TTS* changes the nature of signals that trigger punishments from *private* into *public*, enabling countries to employ punishment phases of any length, which in turn can help countries to attain a better cooperative equilibrium. The numerical analysis illustrates that the WTO's contribution is likely to be more significant when its private signals are relatively accurate so that the lengths of punishment phases are shorter than those under the optimal *PTS*, possibly involving the *asymmetric* and *minimum* punishment.

With regard to the effectiveness of *PTS*, there exist other factors that may severely limit the use of *PTS* so that countries cannot support any level of cooperation, as analyzed in a previous version of this paper. One is a reduction in each country's time lag in readjusting its tariff protection level in response to the other country's initiation of a punishment phase by imposing an explicit tariff. The other is asymmetry among countries. Both of these factors may limit the level of cooperation attainable under *PTS* by reducing the lengths of punishment phases that countries can employ against potential deviations.

Recall that each country is willing to initiate a punishment phase involving costly tariff war periods under *PTS* because it can realize some gains in the initial period of a punishment phase by imposing its static optimal tariff unilaterally. If countries can readjust their tariff levels faster so that countries play the static Nash tariff war (almost) instantaneously in response to an initiation of a punishment, then no length of a punishment phase would satisfy the incentive compatibility condition for truthful revelation of private information (*ICP*).[48] This is because countries will only lose from initiating a punishment, thus making it impossible to support any cooperation under *PTS*.[49] If there exists a large enough asymmetry among trading countries, a similar problem will rise under *PTS*. When one of two trading countries gets very small compared to the other one, then the small country's static optimal tariff goes to zero because its

[48] It is sometimes argued that enforcement constraints cannot be relevant in the trade policy setting, since a government can retaliate almost immediately whenever another government defects. This result suggests that such an argument is based on a public-action model and requires substantial modification in a private monitoring setting, as pointed out by a referee of this paper.

[49] Abreu, Milgrom and Pearce (1991) and more recently Sannikov and Skrzypacz (2007) show that shortening the period over which actions are held fixed can hurt the possibilities for cooperation under imperfect public monitoring, possibly leading to the impossibility of cooperation. While their impossibility of cooperation outcome from shortening the period over which actions are held fixed is similar to the one under *PTS*, the driving forces behind these impossibility results are different. Under imperfect public monitoring, shorter periods of fixed action multiply the ways that player can deviate from the equilibrium, leading to the impossibility. Under *PTS*, the impossibility of cooperation arises not because countries can deviate more effectively (the period over which concealed trade barriers are held fixed remains constant and only the period of readjusting tariff levels in response

ability to change the terms of trade by imposing tariff becomes negligible.[50] This implies that there will be no length of a punishment phase satisfying the incentive compatibility condition for the small country, thus eliminating the possibility of supporting any cooperation under *PTS*.[51]

In the presence of factors that may limit the credibility of initiating strong punishments against potential deviations under *PTS*, once again the WTO may facilitate cooperation by changing the nature of information that triggers punishments from *private* into *public*, which in turn restores the credibility of punishments. For example, the WTO mandates a regular review of its members under the Trade Policy Review Mechanism (TPRM), generating "*public*" reports which consist of detailed chapters examining the trade policies and practices of the members. According to the WTO's website, "Surveillance of national trade policies is a fundamentally important activity running throughout the work of the WTO. At the centre of this work is the TPRM."

Another activity that the WTO does in enforcing trade agreements is settling disputes through its Dispute Settlement Procedure (DSP). When countries form different opinions of potential violations based on their *imperfect* and *private* information, the DSP of the WTO may generate third-party rulings on disputed cases, thus *public* signals about potential deviations. As emphasized in this paper through the analysis of the optimal *TTS*, the availability of an impartial third party's opinion may enable countries to adopt a more efficient punishment, such as the *asymmetric* and *minimum* punishment. This in turn enables countries to be more

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to initiations of punishment phases shortens) but because the punishments that countries can use against deviations weaken.

[50] McLaren (1999) and Park (2000) analyze trade agreements between countries of asymmetric size where a small country has no ability to change the terms of trade by its tariff so that its static optimal tariff is zero.

[51] Formal proofs for these results can be found in an earlier version of this paper, "Private Trigger Strategies in the Presence of Concealed Trade Barriers." As correctly pointed out by one of referees of this paper, a proper way to introduce a change in the speed of readjusting tariff protection levels is to make the model into one in which information arrives continuously over time and to shorten the period under which tariff levels are held fixed. The *ad-hoc* approach of changing the payoff function to some convex combination of the payoff before and after the readjustment of tariffs is adopted to introduce a change in the readjustment speed of tariffs without any change in the basic structure of the model and without any change in the readjustment speed of concealed trade barriers. This reflects that the readjustment of concealed trade barriers may take longer than readjusting tariffs because concealed trade barriers often rely on customary practices or implicit agreements but each country may readjust its tariff level by simply issuing an executive order. Given the logic of the proof, however, the impossibility of cooperation result should be still valid under a proper modeling of a change in the readjustment speed of tariffs. A referee's questioning the focus on the symmetry in the triggering event ($\Omega^D = \Omega^{D^*}$) in the presence of asymmetry among countries is also legitimate, but the impossibility of cooperation result under a large enough asymmetry among countries should be valid even when one considers asymmetric triggering events with $\Omega^D \neq \Omega^{D^*}$.

responsive of potential violations and as a result attain a higher level of cooperation compared to the situation with no DSP.

While this paper provides a new way of understanding the role that the WTO plays in enforcing international trade agreements, there is still much to be done for a more complete understanding of its role in dispute settlements.[52] For example, the DSP of the WTO encourages settlements through consultations among disputing parties as a preferred way to settle trade disputes. According to the official website of the WTO, "The priority is to settle disputes, through consultations if possible. By July 2005, only 130 of the nearly 332 WTO's dispute cases had reached the full panel process. Most of the rest have either been notified as settled "out of court" or remain in a prolonged consultation phase — some since 1995."[53] This indicates that the DSP plays a role that goes beyond simply generating public signals of potential deviations. Carefully analyzing the role that the DSP of the WTO plays in the context of imperfect private monitoring of potential violations, especially regarding settlements through consultations, would be a meaningful extension of this paper.

[52] Maggi and Stagier (2008) analyze the possible role that the DSP of the WTO plays in completing an incomplete contract, characterizing the optimal choice of contractual incompleteness and the DSP design.
[53] This quote comes from the following website: http://www.wto.org/english/thewto_e/whatis_e/tif_e/disp1_e.htm.

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Appendix

Proof for Lemma 1 (a)

It is obvious that *ICP* is a necessary condition and *ICP* becomes $\delta^C - \delta = [u(h, l) - u(l, l)]/(V_C - V_N)]$ if $\delta^C - \delta^S = 2(\delta^C - \delta)$. Therefore, I only need to show that $\delta^C - \delta^S = 2(\delta^C - \delta)$ is also a necessary condition for each country to truthfully represent its private signals under *PTS*. Note that *ICP* only provides the incentive for each country to truthfully initiate a punishment phase given that it was following the equilibrium strategy of setting $\tau = l$ in a pervious (cooperative) period. Even when *ICP* is satisfied, there is a deviation possibility of setting $\tau \neq l$ in a current period and starting a punishment phase in a following period regardless of its private signal, upon the contingency of no punishment phase being initiated in that current period. In an equilibrium of the repeated game, there should be no such deviation incentive and the following argument will prove that $\delta^C - \delta^S = 2(\delta^C - \delta)$ is necessary for eliminating such an incentive.

For *PTS* defined in *Definition 2* to be equilibrium strategies, each country should have no incentive to set $\tau \neq l$ in any period following a cooperative one (or in any "initial" period) unless it desires to initiate a punishment phase by setting $\tau = e = h$, regardless of whether it would initiate a punishment or continue cooperating in a following period, upon the contingency of no punishment phase being initiated. To derive the (necessary) condition for such an equilibrium behavior, first note that the expected discounted payoff of setting its total protection level to equal τ in any period following a cooperative one is

$$Pr[u(\tau,h) + (\delta^C - \delta)V_N + \delta V_C] \\ + (1-Pr)\{u(\tau,l) + \delta^C Pr(\tau)[u(l,h) + (\delta^C - \delta)V_N + \delta V_C] + \delta^C[1-Pr(\tau)][u(l,l) + \delta^C V_C]\},$$

or

$$Pr[u(\tau,h) + (\delta^C - \delta)V_N + \delta V_C] \\ + (1-Pr)\{u(\tau,l) + \delta^C Pr(\tau)[u(h,h) + (\delta^C - \delta^S)V_N + \delta^S V_C] + \delta^C[1-Pr(\tau)][u(h,l) + (\delta^C - \delta)V_N + \delta V_C]\},$$

depending on whether H continues to cooperate (by setting its total protection level to equal *l*) or initiate a punishment phase (by setting its total protection level to equal *h*), respectively, in the following period upon the contingency of no punishment phase being initiated after setting its total protection level to equal τ. To be able to support $\tau = l$, the following first order conditions need to be satisfied for each of the above expected discounted payoff expressions: $\partial u(l, l)/\partial\tau = \delta^C (1 - Pr)[\partial Pr(l)/\partial l][u(l, l) - u(l, h) + (\delta^C - \delta)(V_C - V_N)]$ for the first expression, and $\partial u(l, l)/\partial\tau = \delta^C (1 - Pr)[\partial Pr(l)/\partial l][u(h, l) - u(h, h) + (\delta - \delta^S)(V_C - V_N)]$ for the second one. Using $u(l, l) - u(l, h) = u(h, l) - u(h, h)$, these two first order conditions imply that $\delta^C - \delta = \delta - \delta^S$, or equivalently $\delta^C - \delta^S = 2(\delta^C - \delta)$.

Proof for Lemma 1 (b)

I will prove *Lemma 1* (*b*) in the following way. First, I will assume the existence of (δ, δ^S) that satisfies $\delta^C - \delta = [u(h, l) - u(l, l)]/(V_C - V_N)]$ and $\delta^C - \delta^S = 2(\delta^C - \delta)$ so that $V_C \equiv V(\underline{s}, \underline{s}^*)$ in (6) can be rewritten into a simpler form. Given $\delta^C \approx 1$ and $Pr \approx 0$, then I will show that there exists indeed a unique combination of (δ, δ^S) that satisfies these necessary conditions.

Using $\delta^C - \delta^S = 2(\delta^C - \delta)$, I can simplify $V(\underline{s}, \underline{s}^*)$ in (6) into $V_C = k/[1-\delta^C + 2Pr(\delta^C - \delta)] + V_N$ with $k = (1-Pr^2)[u(l,l) - u(h,h)] + Pr(1-Pr)[u(l,h) - u(l,l)] + Pr(1-Pr)[u(h,l) - u(l,l)]$. To denote the value of δ that satisfies *ICP* with $V_C = V_C(\delta_0) \equiv k/[1-\delta^C + 2Pr(\delta^C - \delta_0)] + V_N$, define $\delta_e(\delta_0) \equiv$ $\delta^C - (1-\delta^C)[u(h,l) - u(l,l)]/[(1-\delta^C)V_C(\delta_0) - u(h,h)] = \delta^C - [u(h,l) - u(l,l)][1-\delta^C + 2Pr(\delta^C - \delta_0)]/k$. If there exists a unique value of $\delta_0 \in (0, \delta^C)$ such that $\delta_e(\delta_0) = \delta_0$ and $\delta^S = 2\delta_0 - \delta^C \in (0, \delta^C)$ when $\delta^C \approx 1$ and $Pr \approx 0$, then proof is done for *Lemma 1* (*b*). First, note that $\partial\delta_e(\delta_0)/\partial\delta_0$ $= 2Pr[u(h,l) - u(l,l)]/k > 0$ approaches zero if $Pr \approx 0$. Second, note that $\delta_e(\delta_0)$ approaches δ^C with $\delta_e(\delta_0) < \delta^C$ when $\delta^C \approx 1$ and $Pr \approx 0$, including the case with $\delta_0 = 0$. These two facts together imply that there exists a unique value of $\delta_0 \in (0, \delta^C)$ such that $\delta_e(\delta_0) = \delta_0$ when $\delta^C \approx 1$ and $Pr \approx 0$. Because $\delta_0 < \delta^C$ and $\delta_0 \approx \delta^C$ for δ_0 satisfying $\delta_e(\delta_0) = \delta_0$ when $\delta^C \approx 1$ and $Pr \approx 0$, $\delta^S = 2\delta_0 - \delta^C$ $\in (0, \delta^C)$.

Proof for Lemma 2

Proofs for the results in *Lemma 2* follow the same logics as the proofs for the corresponding results in Stokey and Lucas (1989). More specifically, Theorem 4.2, 4.3, 4.4, and 4.5 in Stokey and Lucas correspond to (i), (ii), (iii), and (iv) of *Lemma 2* (*a*), respectively. One may also find corresponding proofs for *Lemma 2* (*b*) and *Lemma 2* (*c*) in Theorem 4.6 in Stokey and Lucus. To save the space, I discuss how one can adjust the corresponding proofs in Stockey and Lucas to prove the results in *Lemma 2*. A complete proof for *Lemma 2* is available upon request.

For Lemma 2 (a):

Let $\Gamma: X \to X$ denote the correspondence describing the feasibility constraints with X =[0, h]. Given $x_0 \in X$, let $\Pi(x_0) = \{ \{x_t\}_{t=0}^{\infty} : x_{t+1} \in \Gamma(x_t), t = 0, 1, \ldots\}$ be the set of plan that are feasible from x_0. Define $F(x_t, x_{t+1})$ as $F(\cdot)$ in (8). Then, Assumption 4.1 in Stokey and Lucas is satisfied. I modify Assumption

4.2 with $\lim_{n\to\infty}\sum_{t=0}^{n}(\delta^{C})^{t}[\prod_{i=0}^{t-1}(1-Pr(x_i))]F(x_t,x_{t+1})$ existing for all $x_0 \in X$ and $\underline{x}\in\prod(x_0)$, then it is also satisfied. For each n = 0, 1, ⋯, define u_n: $\Pi(x_0) \to R$ by $u_n(\underline{x})=\sum_{t=0}^{n}(\delta^{C})^{t}[\prod_{i=0}^{t-1}(1-Pr(x_i))]F(x_t,x_{t+1})$. Define u: $\prod(x_0)\to\overline{R}$ by $u(\underline{x})=\lim_{n\to\infty}u_n(\underline{x})$. Then, it is easy to show that Lemma 4.1 in Stocky and Lucas holds when one replaces $u(\underline{x})=F(x_0,x_1)+\delta^{C}u(\underline{x}^{/})$ with $u(\underline{x})=F(x_0,x_1)+\delta^{C}(1-Pr(x_0))u(\underline{x}^{/})$. Having v^* and v in Stocky and Lucas representing V_S and V in *Lemma 2*, I can also show that Theorem 4.2, 4.3, 4.4, and 4.5 hold for these newly defined variables, replacing $v^*(x_t^*)=F(x_t^*,x_{t+1}^*)+\delta^{C}v(x_{t+1}^*)$ of (9) in Stocky and Lucas with $v^*(x_t^*)=F(x_t^*,x_{t+1}^*)+\delta^{C}(1-Pr(x_i))v(x_{t+1}^*)$. While one needs to modify some lines of proofs in Stocky and Lucas, it is a pretty straightforward extension of the logics of their proofs, as mentioned earlier.

***For Lemma 2* (*b*) and (*c*):**

First note that *Lemma 2* (*b*) and *Lemma 2* (*c*) correspond to Theorem 4.6 of Stocky and Lucas. Also note that Theorem 4.6 basically uses the Contraction Mapping Theorem (Theorem 3.2) and the Theorem of Maximum (Theorem 3.6) to prove the results. To show that the proof in Theorem 4.6 works for proving *Lemma 2* (*b*) and *Lemma 2* (*c*), I establish the following result. Define an operator T by $(Tv)(x) = \underset{y\in[0,h]}{Max}\{F(x, y) + \delta^{C}[1 - Pr(x)]v(y)\}$. T satisfies Blackwell's sufficient condition for contraction mapping as it satisfies both "Monotonicity" and "Discounting" criteria:

(Monotonicity)

If $v(y) \le w(y)$ for all values of y, then $Tv(y) \le Tw(y)$ because $[1 - Pr(x)] \ge 0$ by definition.

(Discounting)

$T(v + a)(x) = \underset{y\in[0,h]}{Max}\{F(x, y) + \delta^{C}[1 - Pr(x)][v(y) + a]\} = \underset{y\in[0,h]}{Max}\{F(x, y) + \delta^{C}[1 - Pr(x)]v(y) + \delta^{C}[1 - Pr(x)]a\}\} = (Tv)(x) + \delta^{C}[1 - Pr(x)]a \le (Tv)(x) + \delta^{C}a$ because $[1 - Pr(x)] \in [0,1]$.

In addition, T: $C(X) \to C(X)$ from the Theorem of Maximum with $C(X)$ denoting the set of bounded continuous functions f: $X \to R$. Thus, T: $C(X) \to C(X)$ is a contraction mapping with modulus δ^{C}, implying that I can apply the Contraction Mapping Theorem to T. Thus, I can show that *Lemma 2* (*b*) and (*c*) hold using the Theorem of Maximum as in Theorem 4.6.

<u>Proof for Lemma 3</u>

***For Lemma 3* (*a*):**

Define $f(\tau_{-1}, \tau) \equiv F(\tau_{-1}, \tau) + \delta^C[1 - Pr(\tau_{-1})]V(\tau)$. Note that $f(\tau_{-1}, \tau)$ is everywhere differentiable w.r.t. τ_{-1} for all $\tau \in [0, h]$ and $\partial f(\tau_{-1}, \tau)/\partial\tau_{-1} = -[\partial Pr(\tau_{-1})/\partial\tau_{-1}]\{u(\tau, l) + \delta^C V(\tau) - u(\tau, h) - (\delta^C - \delta) V_N - \delta V_C\}$ is bounded for all $\tau \in [0, h]$. This implies that $f(\tau_{-1}, \tau)$ is absolutely continuous w.r.t. τ_{-1} for all $\tau \in [0, h]$. Therefore, I can use Theorem 2 of Milgrom and Segal (2002) in deriving the following expression

(A1) $$V(\tau_{-1}) = V(0) + \int_0^{\tau_{-1}} [\partial f(m, g(m))/\partial m]dm,$$

where $g(m) \in G(m)$ and $\partial f(m,g(m))/\partial m = -[\partial Pr^*(m)/\partial m]\{u(g(m), l) + \delta^C V(g(m)) - u(g(m), h) - (\delta^C - \delta)V_N - \delta V_C\}$.

(A1) implies that $V(\tau_{-1})$ will be strictly decreasing in $\tau_{-1} \in [0, h]$, if $u[g(m), l] + \delta^C V(g(m)) - u(g(m), h) - (\delta^C - \delta)V_N - \delta V_C > 0$ for all $m \in [0, h]$, because $\partial Pr(m)/\partial m > 0$ by assumption. To show that $u(g(m), l) + \delta^C V(g(m)) - u(g(m), h) - (\delta^C - \delta)V_N - \delta V_C > 0$ for all $m \in [0, h] > 0$, I first establish that the inequality holds for any $g(m) \le l$, and then show that the inequality holds for any $g(m) > l$.

First, assume that $g(m) \le l$. To have $u(g(m), l) + \delta^C V(g(m)) \le u(g(m), h) + (\delta^C - \delta) V_N + \delta V_C$, $V_C > V(g(m))$ because $u(g(m), l) > u(g(m), h)$ with $l < h$ and $V(g(m)) \ge V_N$. The last inequality is obvious because the strategy of always setting $\tau = h$ will generate a discounted expected payoff at least as good as V_N, regardless of $g(m)$ taking any feasible values. $V(g(m)) \ge [1 - Pr(g(m))][u(l, l) + \delta^C V_C] + Pr(g(m))[u(l, h) + (\delta^C - \delta)V_N + \delta^* V_C] \ge [1 - Pr(l)][u(l, l) + \delta^C V_C] + Pr(l)[u(l, h) + (\delta^C - \delta)V_N + \delta V_C]$, where the last inequality comes from $g(m) \le l$ and $[u(l, l) + \delta^C V_C] \ge [u(l, h) + (\delta^C - \delta)V_N + \delta V_C]$, and the first inequality comes from the fact that $[1 - Pr(g(m))][u(l, l) + \delta^C V_C] + Pr(g(m))[u(l, h) + (\delta^C - \delta)V_N + \delta V_C]$ represents a discounted expected payoff of playing a potentially suboptimal strategy of setting $\tau = l$ with $\tau_{-1} = g(m)$. From *ICP*, $V_C = [1 - Pr(l)][u(l, l) + \delta^C V_C] + Pr(l)[u(l, h) + (\delta^C - \delta)V_N + \delta V_C]$, which implies that $V_C \le V(g(m))$, thus a contradiction. Therefore, $u(g(m), l) + \delta^C V(g(m)) > u(g(m), h) + (\delta^C - \delta)V_N + \delta V_C$ if $g(m) \le l$.

Now, I will show that $u(g(m), l) + \delta^C V(g(m)) > u(g(m), h) + (\delta^C - \delta)V_N + \delta V_C$ if $g(m) > l$. Define $K \equiv u(g(m), h) + (\delta^C - \delta)V_N + \delta V_C$. Then, $V(g(m)) \ge [1 - Pr(g(m))]u(g(m), l)/\{1 - \delta^C[1 - Pr(g(m))]\} + Pr(g(m))K/\{1 - \delta^C[1 - Pr(g(m))]\}$ because the right-hand side of the inequality represents a discounted expected payoff from playing a potentially suboptimal strategy of setting the current and all the future protection level at $g(m)$ with $\tau_{-1} = g(m)$. This implies that $u(g(m), l) + \delta^C V(g(m)) - K \ge u(g(m), l) + \delta^C[1 - Pr(g(m))]u(g(m), l)/\{1 - \delta^C[1 - Pr(g(m))]\} + \delta^C Pr(g(m))K/\{1 - \delta^C[1 - Pr(g(m))]\} - K = (1 - \delta^C)\{u(g(m), l)/(1 - \delta^C) - [u(g(m), h) + (\delta^C - \delta)V_N + \delta V_C]\}/\{1 - \delta^C[1 - Pr(g(m))]\}$. Note that the last term has a positive sign because $u(g(m), l)/(1 - \delta^C) > [u(g(m), h) + (\delta^C - \delta)V_N + \delta V_C]$ with $u(g(m), l)/(1 - \delta^C) > V_C$ as $g(m) > l$. This implies that $u(g(m), l) + \delta^C V(g(m)) > K$.

***For Lemma 3* (*b*):**

To prove that $G(\tau_{-1})$ is strictly increasing in τ_{-1}, I first show that $\tau^{//} \geq \tau^{/}$ for all $\tau^{//}_{-1} > \tau^{/}_{-1} \in [0, h]$ with $\tau^{//} \in G(\tau^{//}_{-1})$ and $\tau^{/} \in G(\tau^{/}_{-1})$. Then, I show that $\tau^{//} = \tau^{/}$ will lead to a contradiction using a result in Cotter and Park (2006). Consider $\tau^{//}_{-1} > \tau^{/}_{-1}$, having $V(\tau^{/}_{-1}) = F(\tau^{/}_{-1}, \tau^{/}) + \delta^{C}[1 - Pr(\tau^{/}_{-1})]V(\tau^{/})$ and $V(\tau^{//}_{-1}) = F(\tau^{//}_{-1}, \tau^{//}) + \delta^{C}[1 - Pr(\tau^{//}_{-1})]V(\tau^{//})$. Then, $F(\tau^{/}_{-1}, \tau^{/}) + \delta^{C}[1 - Pr(\tau^{/}_{-1})]V(\tau^{/}) \geq F(\tau^{/}_{-1}, \tau^{//}) + \delta^{C}[1 - Pr(\tau^{/}_{-1})]V(\tau^{//})$ and $F(\tau^{//}_{-1}, \tau^{//}) + \delta^{C}[1 - Pr(\tau^{//}_{-1})]V(\tau^{//}) \geq F(\tau^{//}_{-1}, \tau^{/}) + \delta^{C}[1 - Pr(\tau^{//}_{-1})]V(\tau^{/})$ because the terms of the right-hand sides of these inequalities represent discounted expected payoffs from playing potentially suboptimal strategies. These two inequalities together imply that

(A2) $[F(\tau^{/}_{-1}, \tau^{/}) - F(\tau^{//}_{-1}, \tau^{/})] - [F(\tau^{/}_{-1}, \tau^{//}) - F(\tau^{//}_{-1}, \tau^{//})] \geq \delta^{C}[Pr(\tau^{//}_{-1}) - Pr(\tau^{/}_{-1})][V(\tau^{//}) - V(\tau^{/})]$. Define $E(\tau; \tau^{/}_{-1}, \tau^{//}_{-1}) = F(\tau^{/}_{-1}, \tau) - F(\tau^{//}_{-1}, \tau)$. According to the mean value theorem (using the fact that $E(\tau; \tau^{/}_{-1}, \tau^{//}_{-1})$ is continuous and differentiable w.r.t. τ, then $\exists\ \bar{\tau} \in [\mathrm{Min}(\tau^{/}, \tau^{//}), \mathrm{Max}(\tau^{/}, \tau^{//})]$ such that

(A3)
$$E(\tau^{/}; \tau^{/}_{-1}, \tau^{//}_{-1}) - E(\tau^{//}; \tau^{/}_{-1}, \tau^{//}_{-1}) = (\tau^{/} - \tau^{//})[\partial E(\bar{\tau}; \tau^{/}_{-1}, \tau^{//}_{-1}) / \partial\tau]$$
$$\geq \delta^{C}[Pr(\tau^{//}_{-1}) - Pr(\tau^{/}_{-1})][V(\tau^{//}) - V(\tau^{/})]$$

with the inequality coming from (A2). Note that $[\partial E(\bar{\tau}; \tau^{/}_{-1}, \tau^{//}_{-1})/\partial\tau] = [\partial u(\bar{\tau}, l)/\partial\tau - \partial u(\bar{\tau}, l)/\partial\tau] = 0$ as $\partial^{2}u(\tau, \tau^{*})/\partial\tau\,\partial\tau^{*} = 0$. Now, I will show that $\tau^{//} < \tau^{/}$ leads to a contradiction. If $\tau^{//} < \tau^{/}$, $\delta^{C}[Pr(\tau^{//}_{-1}) - Pr(\tau^{/}_{-1})][V(\tau^{//}) - V(\tau^{/})] > 0$ because $Pr(\tau^{//}_{-1}) - Pr(\tau^{/}_{-1})) > 0$ and $[V(\tau^{//}) - V(\tau^{/})] > 0$ from *Lemma 3* (*a*). This contradicts $\delta^{C}[Pr(\tau^{//}_{-1}) - Pr(\tau^{/}_{-1})][V(\tau^{//}) - V(\tau^{/})] \leq 0$ in (A3), thus $\tau^{//} \geq \tau^{/}$ for all $\tau^{//}_{-1} > \tau^{/}_{-1} \in [0, h]$.

Now, it remains to prove that $\tau^{//} = \tau^{/}$ leads to a contraction. From Theorem 2 of Cotter and Park (2006), $V(\tau)$ is differentiable for $\tau \in G(\tau_{-1})$ for all $\tau_{-1} \in [0, h]$. Therefore,

(A4)
$$\partial F(\tau^{/}_{-1}, \tau^{/}) / \partial\tau + \delta^{C}[1 - Pr(\tau^{/}_{-1})][\partial V(\tau^{/}) / \partial\tau] = 0 \text{ and}$$
$$\partial F(\tau^{//}_{-1}, \tau^{//}) / \partial\tau + \delta^{C}[1 - Pr(\tau^{//}_{-1})][\partial V(\tau^{//}) / \partial\tau] = 0.$$

If $\tau^{//} = \tau^{/}$, $\partial F(\tau^{/}_{-1}, \tau^{/}) / \partial\tau - \partial F(\tau^{//}_{-1}, \tau^{/}) / \partial\tau = -\delta^{C}[Pr(\tau^{//}_{-1}) - Pr(\tau^{/}_{-1})][\partial V(\tau^{/}) / \partial\tau]$ from (A4), contradicting $\partial[F(\tau^{/}_{-1}, \tau^{/}) - F(\tau^{//}_{-1}, \tau^{/})] / \partial\tau = 0$, $[Pr(\tau^{//}_{-1}) - Pr(\tau^{/}_{-1})] > 0$, and $\partial V(\tau^{/}) / \partial\tau > 0$.

<u>Proof for Lemma 4</u>

For Lemma 4 (a):

In proving *Lemma 4* (*a*), I use Theorem 4 in Cotter and Park (2006). According to the theorem, if there exists a unique $\tau_S \in (0, h)$ that satisfies *IC* defined in (11): $\partial F(\tau_S, \tau_S)/\partial\tau + \delta^C[1 - Pr(\tau_S)][\partial V(\tau_S)/\partial\tau] = 0$ and $\tau \in (0, \text{h})$ for every $\tau_{-1} \in [0, h]$ and $\tau \in G(\tau_{-1})$, then $G(\tau_S) = \{\tau_S\}$ and τ_S is a strongly stable protection level in the sense that for every $\tau_{-1} > \tau_S$ and $\tau \in G(\tau_{-1}), \tau < \tau_{-1}$, and for every $\tau_{-1} < \tau_S$ and $\tau \in G(\tau_{-1})$, $\tau > \tau_{-1}$. To prove *Lemma 4* (*a*), therefore, I first show that there exists a unique $\tau_S \in (0, h)$ such that $\partial F(\tau_S, \tau_S)/\partial\tau + \delta^C[1 - Pr(\tau_S)][\partial V(\tau_S)/\partial\tau] = 0$ if $[\partial^2 Pr(\tau)/(\partial\tau)^2][1 - Pr(\tau)] - \{1 + \delta^C[1 - Pr(\tau)]\}[\partial Pr(\tau)/\partial\tau]^2 > 0$ for all $\tau \in [0, h]$ and $\partial Pr(\tau)/\partial\tau \approx 0$ at $\tau = 0$, then establish that $\tau \in (0, \text{h})$ for every $\tau_{-1} \in [0, h]$ and $\tau \in G(\tau_{-1})$.

First note that $\partial F(\tau_S, \tau_S)/\partial\tau = \partial u(\tau_S, l)/\partial\tau > 0$ at $\tau_S = 0$ and $\partial^2 F(\tau_S, \tau_S)/\partial\tau^2 < 0$ with $\partial F(\tau_S, \tau_S)/\partial\tau = \partial u(\tau_S, l)/\partial\tau = 0$ at $\tau_S = h$ from the assumptions on the derivatives of $u(\tau, \tau)$ w.r.t. τ. Because $\partial V(\tau_S)/\partial\tau = -[\partial Pr(\tau_S)/\partial\tau]\{u(\tau_S, l) + \delta^C V(\tau_S) - [u(\tau_S, h) + (\delta^C - \delta)V_N + \delta V_C]\} \approx 0$ at $\tau_S = 0$ from the assumption of $\partial Pr(\tau)/\partial\tau \approx 0$ at $\tau = 0$, $F(\tau_S, \tau_S)/\partial\tau > 0$ at $\tau_S = 0$ implies that *IC* in (11) will not be satisfied at $\tau_S = 0$. Now, define $A(\tau_S) \equiv u(\tau_S, l) + \delta^C V(\tau_S) - [u(\tau_S, h) + (\delta^C - \delta)V_N + \delta V_C]$ and $B(\tau_S) \equiv \delta^C[1 - Pr(\tau_S)][\partial Pr(\tau_S)/\partial\tau]A(\tau_S)$, thus $\delta^C[1 - Pr(\tau_S)][\partial V(\tau_S)/\partial\tau] = -B(\tau_S)$. Then, $\partial B(\tau_S)/\tau_S = \delta^C A(\tau_S)\langle[\partial^2 Pr(\tau)/(\partial\tau)^2][1 - Pr(\tau)] - \{1 + \delta^C[1 - Pr(\tau)]\}[\partial Pr(\tau)/\partial\tau]^2\rangle > 0$ for all $\tau_S \in [0, h]$ because $[\partial^2 Pr(\tau)/(\partial\tau)^2][1 - Pr(\tau)] - \{1 + \delta^C[1 - Pr(\tau)]\}[\partial Pr(\tau)/\partial\tau]^2 > 0$ for all $\tau_S \in [0, h]$ by assumption and $A(\tau_S) > 0$ as shown in the proof for *Lemma 3* (*a*). This implies that there exists a unique $\tau_S \in (0, h)$ such that $\partial F(\tau_S, \tau_S)/\partial\tau + \delta^C[1 - Pr(\tau_S)][\partial V(\tau_S)/\partial\tau] = 0$.

Now, I only need to prove that $\tau \in (0, \text{h})$ for every $\tau_{-1} \in [0, h]$ and $\tau \in G(\tau_{-1})$. Because $G(\tau_{-1})$ is strictly increasing in τ_{-1} as proved in *Lemma 3* (*b*), it suffices to prove that $0 \notin G(0)$ and $h \notin G(h)$. Note that $0 \notin G(0)$ is already proven above: "*IC* in (11) will not be satisfied at $\tau_S = 0$." Because *IC* in (11) is a necessary condition for any stationary protection level, *IC* in (11) being not satisfied at $\tau_S = 0$ implies that $0 \notin G(0)$.

I can show that $h \notin G(h)$ by contradiction. First, assume that $h = G(h)$, implying that $V(h) = \sum_{d=0}^{\infty}\left\{(\delta^C)^d\left[\prod_{i=0}^{d-1}[1 - Pr(\tau_i)]\right]F(\tau_d, \tau_{d+1})\right\}$ with $\{\tau_d = h\}_{d=0}^{\infty}$. Consider an alternative protection sequence with $\tau_0 = h$, $\tau_1 = h - \varepsilon$, and $\{\tau_d = h\}_{d=2}^{\infty}$, which defines a corresponding discounted expected payoff, denoted by $V_A(h)$. Then, I can show that $V_A(h) - V(h) = \{Pr(h)u(h - \varepsilon, h) + [1 - Pr(h)]u(h - \varepsilon, l) + Pr(h)[(\delta^C - \delta)V_N + \delta V_C]\} - \{Pr(h)u(h, h) + [1 - Pr(h)]u(h, l) + Pr(h)[(\delta^C - \delta)V_N + \delta V_C]\} + \delta^C[1 - Pr(h)][Pr(h - \varepsilon) - Pr(h)]\{u(h, h) - u(h, l) + [(\delta^C - \delta)V_N + \delta V_C]\} - \delta^C[Pr(h) - Pr(h - \varepsilon)]F(h, h)\delta^C[1 -$

$Pr(h)]/\{1 - \delta^C[1 - Pr(h)]\}$. $\lim_{\varepsilon\to 0}[V_A(h) - V(h)]/(\varepsilon - 0) = -\delta^C[\partial Pr(h)/\partial\tau][1 - Pr(h)]\{(1 - \delta^C)[u(h, h) + (\delta^C - \delta)V_N + \delta V_C] - u(h, l)\}/\{1 - \delta^C[1 - Pr(h)]\} > 0$ where the last inequality comes from $\partial Pr(h)/\partial\tau > 0$ and $u(h, l)/(1 - \delta^C) > u(h, h) + (\delta^C - \delta)V_N + \delta V_C$ as shown in *Lemma 3* (a). This implies that $h \notin G(h)$.

For Lemma 4 (b):

To prove *Lemma 4* (*b*), I will show that H cannot strictly increase its discounted payoff by initiating a punishment phase in a period that that follows a cooperative period during which H set its protection level at $l' \neq l = \tau_S$, as long as the lengths of punishment phases satisfy the necessary conditions in *Lemma 1* (*a*). Once I prove this result, this implies that H cannot increase its discounted expected payoff by initiating a punishment phase along any (deviatory) protection sequence, thus *Lemma 4* (*b*).

Suppose that H sets its protection level at l in a period that follows a cooperative period during which H sets its protection level at $l' \neq l = \tau_S$, then chooses its optimal protection sequence from the next period on. Denote the discounted expected payoff from taking this potentially suboptimal action by $C(l')$, then

(A5) $$C(l') = Pr(l')[u(l,h) + (\delta^C - \delta)V_N + \delta V_C] + [1 - Pr(l')][u(l,l) + \delta^C V_C].$$

Now suppose that H initiates a tariff war phase by setting tariff level at h in a period that follows a cooperative period where H set its protection level $l' \neq l = \tau_S$, then follows its specified strategy once the tariff war phase is over. Denote the discounted expected payoff from taking this potentially suboptimal action by $D(l')$, then

(A6) $$\begin{aligned} D(l') = {} & Pr(l')[u(h,h) + (\delta^C - \delta^S)V_N + \delta^S V_C] + \\ & [1 - Pr(l')][u(h,l) + (\delta^C - \delta)V_N + \delta V_C]. \end{aligned}$$

I can rewrite $C(l')$ and $D(l')$ into

(A7) $$\begin{aligned} C(l') &= u(l,l) + \delta^C V_C - Pr(l')[u(l,l) - u(l,h) + (\delta^C - \delta)(V_C - V_N)] \\ D(l') &= u(h,l) + (\delta^C - \delta)V_N + \delta V_C - Pr(l')[u(h,l) - u(h,h) + (\delta - \delta^S)(V_C - V_N)]. \end{aligned}$$

Now, note that $C(l') - D(l') = [u(l, l) - u(h, l)] + (\delta^C - \delta)(V_C - V_N) - Pr(l')\{[u(l, l) - u(l, h)] - [u(h, l) - u(h, h)] + [(\delta^C - \delta) - (\delta - \delta^S)](V_C - V_N)\} = 0$ from $[u(l, l) - u(l, h)] = [u(h, l) - u(h, h)]$ and the sufficient condition for *ICP* and *ICP** in *Lemma* 1 (*a*): $\delta^C - \delta = [u(h, l) - u(l, l)]/(V_C - V_N)$ and $\delta^C + \delta^S = (\delta + \delta)$. Because $C(l')$ is equal or possibly lower than a discounted expected payoff from choosing an optimal protection sequence of not involving an initiation of a punishment phase, this implies that H cannot strictly increase its discounted payoff by initiating a punishment phase in a period that follows a cooperative period during which H sets its protection level at $l' \neq l = \tau_S$.

Proof for Proposition 1

With $\delta = \delta^C - [u(h, l_S) - u(l_S, l_S)]/(V_C - V_N)$, and $\delta^C - \delta^S = 2(\delta^C - \delta)$, note that setting $\tau_S = l_S$ satisfies *IC* in (11), thus l_S is the unique stationary protection level from which H does not have any incentive to deviate from, as described in *Lemma 4*. By symmetry, l_S is also such a protection level for F. If $l = l_S$, then *PTS* satisfy *ICP* and $\delta^C - \delta^S = 2(\delta^C - \delta)$ as well as *IC*, thus becoming a supergame equilibrium of the protection setting game between H and F from which no country has any unilateral incentive to change its specified strategy.

What is the relationship between the condition for *Lemma 4* (*a*) and the existence of l ($< h$) that satisfies $I(l) = 0$ in (13)? For example, does the condition for *Lemma 4* (*a*) guarantee the existence of such l? To address this issue, I show that the second term of $I(l)$ in (13), $\delta^C[\partial Pr(l)/\partial \tau][1 - Pr(l)][u(h, l) - u(l, h)]$, representing H's dynamic incentive to avoid a punishment phase, may not necessarily increase in l when the condition for *Lemma 4*(*a*) is satisfied. $\partial\{[\partial Pr(l)/\partial l][1 - Pr(l)][u(h, l) - u(l, h)]\}/\partial l = \langle[\partial^2 Pr(l)/(\partial l)^2][1 - Pr(l)] - [\partial Pr(l)/\partial l]^2\rangle[u(h, l) - u(l, h)] + [\partial Pr(l)/\partial l][1 - Pr(l)]\{\partial[u(h, l) - u(l, h)]/\partial l\} = \langle[\partial^2 Pr(l)/(\partial l)^2][1 - Pr(l)] - \{1 + \delta^C[1 - Pr(l)]\}[\partial Pr(l)/\partial l]^2\rangle[u(h, l) - u(l, h)] + \langle\{\delta^C[1 - Pr(l)]\}[\partial Pr(l)/\partial l]^2[u(h, l) - u(l, h)] + [\partial Pr(l)/\partial l][1 - Pr(l)]\{\partial[u(h, l) - u(l, h)]/\partial l\}\rangle$. Because $[\partial Pr(l)/\partial l][1- Pr(l)]\{\partial[u(h, l) - u(l, h)]/\partial l\} < 0$, once cannot rule out the possibility of having $\{\delta^C[1 - Pr(l)]\}[\partial Pr(l)/\partial l]^2[u(h, l) - u(l, h)] + [\partial Pr(l)/\partial l][1 - Pr(l)]\{\partial[u(h, l) - u(l, h)]/\partial l\} < 0$, thus $\partial\{[\partial Pr(l)/\partial l][1 - Pr(l)][u(h, l) - u(l, h)]\}/\partial l < 0$ even when $[\partial^2 Pr(l)/(\partial l)^2][1 - Pr(l)] - \{1 + \delta^C[1 - Pr(l)]\}[\partial Pr(l)/\partial l]^2 > 0$. Therefore, the condition for *Lemma 4* (*a*) does not necessarily guarantee the existence of l ($< h$) that satisfies $I(l) = 0$, validating the insertion of an additional condition to guarantee the existence of such l in *Proposition 1*.

Proof for Proposition 4

***For (a)*:** It is sufficient to show that dV_C^W/dT^W in (22) is less than 0 for all values of $T^W \geq 1$ if $Pr(l) < \overline{Pr}$. Using

$$\frac{\partial I^W}{\partial T^W} = \delta^C \delta^{C^{T^W}} \ln(\delta^C)\frac{\partial Pr}{\partial l}(1-Pr)\left\{(V_C^W - V_N) - \frac{2(\delta^C - \delta^{C^{T^W}})Pr(1-Pr)[u(l,l)-u(h,h)]}{[1-\delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})]^2}\right\}, \text{ and}$$

(A8) $$\frac{dI^W}{d\omega^D} = \delta^C\left[-\frac{\partial^2 Pr}{\partial l \partial \omega^D}(1-Pr) + \frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial \omega^D}\right]\left[u(l,l) - u(l,h) + (\delta^C - \delta^{C^{T^W}})(V_C^W - V_N)\right]$$
$$+ \frac{\delta^C(\delta^C - \delta^{C^{T^W}})(\partial Pr/\partial l)(1-Pr)}{[1-\delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})]^2}\frac{\partial Pr}{\partial \omega^D}[1-\delta^C + 2(\delta^C - \delta^{C^{T^W}})][u(l,l) - u(h,h)],$$

I can rewrite dV_C^W/dT^W in (22) into

$$\frac{dV_C^W}{dT^W} = \left(-\frac{\frac{\partial V_C^W}{\partial l}}{\frac{\partial I^W}{\partial l}} \right) \left\{ \begin{array}{l} \delta^C \delta^{C^{T^W}} \ln(\delta^C) \frac{\partial Pr}{\partial l}(1-Pr)(V_C^W - V_N) \\ + \delta^C A \left[-\frac{\partial^2 Pr}{\partial l \partial \omega^D}(1-Pr) + \frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial \omega^D} \right] \left[\begin{array}{l} u(l,l) - u(l,h) \\ + (\delta^C - \delta^{C^{T^W}})(V_C^W - V_N) \end{array} \right] \end{array} \right\}, \text{ with}$$

$\left[-\frac{\partial^2 Pr}{\partial l \partial \omega^D}(1-Pr) + \frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial \omega^D} \right] < 0,$ to have $dV_C{}^W/d\omega^D = 0$. Using $V_C^W - V_N = \frac{(1-Pr)[u(l,l) - u(h,h)]}{1 - \delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})}$,

$$\frac{dV_C^W}{dT^W} =$$

$$B \left\langle \begin{array}{l} \frac{\partial Pr}{\partial l} \frac{(1-Pr)[u(l,l) - u(h,h)]}{1 - \delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})} + \\ 2Pr\left(\frac{\partial Pr}{\partial \omega^D}\right)^{-1} \left[-\frac{\partial^2 Pr}{\partial l \partial \omega^D}(1-Pr) + \frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial \omega^D} \right] \left\{ \begin{array}{l} \frac{[1-\delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})][u(l,l) - u(l,h)]}{[1-\delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})][1-\delta^C + 2(\delta^C - \delta^{C^{T^W}})]} \\ + \frac{(1-Pr)(\delta^C - \delta^{C^{T^W}})[u(l,l) - u(h,h)]}{[1-\delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})][1-\delta^C + 2(\delta^C - \delta^{C^{T^W}})]} \end{array} \right\} \end{array} \right\rangle$$

with $B \equiv \left(-\frac{\frac{\partial V_C^W}{\partial l}}{\frac{\partial I^W}{\partial l}} \right) \delta^C \delta^{C^{T^W}} \ln(\delta^C)(1-Pr).$

By replacing $u(l, l) - u(l, h)$ with $u(l, l) - u(h, h)$ in the above expression and using $u(l, l) - u(l, h) > u(l, l) - u(h, h)$, I obtain the first inequality in the following expressions:

$$\frac{dV_C^W}{dT^W}$$

$$< BC\left\{ \frac{\partial Pr}{\partial l}(1-Pr) + 2Pr\left(\frac{\partial Pr}{\partial \omega^D}\right)^{-1} \left[-\frac{\partial^2 Pr}{\partial l \partial \omega^D}(1-Pr) + \frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial \omega^D} \right] \left[\frac{1-\delta^C + (1+Pr)(\delta^C - \delta^{C^{T^W}})}{1-\delta^C + 2(\delta^C - \delta^{C^{T^W}})} \right] \right\}$$

$$< BC\left\{ \frac{\partial Pr}{\partial l}(1-Pr) + 2Pr\left(\frac{\partial Pr}{\partial \omega^D}\right)^{-1} \left[-\frac{\partial^2 Pr}{\partial l \partial \omega^D}(1-Pr) + \frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial \omega^D} \right] \left(\frac{1+Pr\delta^C}{1+\delta^C} \right) \right\}$$

$$= BC\left(\frac{\partial Pr}{\partial \omega^D}\right)^{-1} \frac{\partial (Pr/\omega^D)}{\partial l} \frac{Pr}{1+\delta^C} \left[(1-Pr)(1+\delta^C) + 2(2Pr-1)(1+Pr\delta^C) \right],$$

with $C \equiv \frac{u(l,l) - u(h,h)}{1 - \delta^C + 2Pr(\delta^C - \delta^{C^{T^W}})}$,

where $\partial\{[1-\delta^C + (1+Pr)(\delta^C - \delta^{C^{T^W}})]/[1-\delta^C + 2(\delta^C - \delta^{C^{T^W}})]\}/\partial T^W < 0$ and setting $T^W \to \infty$ for the last bracketed term, $[1-\delta^C + (1+Pr)(\delta^C - \delta^{C^{T^W}})]/[1-\delta^C + 2(\delta^C - \delta^{C^{T^W}})]$ in the preceding expression to the second inequality are used to obtain the second inequality. To obtain the (last) equality in the above expressions, I use the assumption of $\partial^2 Pr(l)/\partial(\omega^D)^2 = 0$, thus $Pr(l)$ being linear in ω^D. With this assumption, I can rewrite $Pr(l) = \omega^D Pr^D(l)$, which in turn implies that $\partial Pr(l)/\partial l = \omega^D[\partial Pr^D(l)/\partial l]$, $\partial Pr(l)/\partial\omega^D = Pr^D(l)$, $\partial^2 Pr(l)/\partial l\partial\omega^D = \partial Pr^D(l)/\partial l$, and $[\partial Pr(l)/\partial l][\partial Pr(l)/\partial\omega^D] = Pr[\partial Pr^D(l)/\partial l]$. Once I rewrite the corresponding terms in the expression preceding to the last equality in this way, I can obtain the last expression (or equivalently, the last equality). As a result of these transformations, I obtain the following inequalities:

(A9) $$\frac{dV_C^W}{dT^W} < BC\left(\frac{\partial Pr}{\partial\omega^D}\right)^{-1}\frac{\partial(Pr/\omega^D)}{\partial l}\frac{Pr}{1+\delta^C}\Big[(1-Pr)(1+\delta^C)+2(2Pr-1)(1+Pr\delta^C)\Big] < 0$$

with the last inequality holding for all values of $T^W \geq 1$ if $(1-Pr)(1+\delta^C)+2(2Pr-1)(1+Pr\delta^C) < 0$, which in turn holds if $Pr(l) < \overline{Pr}$.

For (b): It is sufficient to show that dV_C^W/dT^W in (22) is greater than 0 for all values of $T^W \geq 1$ if $Pr(l) > \underline{Pr}$. As shown above, I can rewrite dV_C^W/dT^W (22) into the following expression, using (A8):

$$\frac{dV_C^W}{dT^W} = B\left\langle \begin{array}{l} \dfrac{\partial Pr}{\partial l}\dfrac{(1-Pr)[u(l,l)-u(h,h)]}{1-\delta^C+2Pr(\delta^C-\delta^{CT^W})} + \\ 2Pr\left(\dfrac{\partial Pr}{\partial\omega^D}\right)^{-1}\left[-\dfrac{\partial^2 Pr}{\partial l\partial\omega^D}(1-Pr)+\dfrac{\partial Pr}{\partial l}\dfrac{\partial Pr}{\partial\omega^D}\right]\left\{\begin{array}{l}\dfrac{[1-\delta^C+2Pr(\delta^C-\delta^{CT^W})][u(l,l)-u(l,h)]}{[1-\delta^C+2Pr(\delta^C-\delta^{CT^W})][1-\delta^C+2(\delta^C-\delta^{CT^W})]} \\ +\dfrac{(1-Pr)(\delta^C-\delta^{CT^W})[u(l,l)-u(h,h)]}{[1-\delta^C+2Pr(\delta^C-\delta^{CT^W})][1-\delta^C+2(\delta^C-\delta^{CT^W})]}\end{array}\right\}\end{array}\right\rangle.$$

By replacing $u(l, l) - u(h, h)$ with $u(l, l) - u(l, h)$ in the above expression and using $u(l, l) - u(l, h) > u(l, l) - u(h, h)$, I obtain the first inequality in the following expressions

$$\begin{aligned}\frac{dV_C^W}{dT^W} &> BD\left\{\frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial\omega^D}(1-Pr)\frac{u(l,l)-u(h,h)}{u(l,l)-u(l,h)}+2Pr\left[-\frac{\partial^2 Pr}{\partial l\partial\omega^D}(1-Pr)+\frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial\omega^D}\right]\left[\frac{1-\delta^C+(1+Pr)(\delta^C-\delta^{CT^W})}{1-\delta^C+2(\delta^C-\delta^{CT^W})}\right]\right\} \\ &> BD\left\{\frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial\omega^D}(1-Pr)\frac{u(l,l)-u(h,h)}{u(l,l)-u(l,h)}+2Pr\left[-\frac{\partial^2 Pr}{\partial l\partial\omega^D}(1-Pr)+\frac{\partial Pr}{\partial l}\frac{\partial Pr}{\partial\omega^D}\right]\right\} \\ &= BD\frac{\partial(Pr/\omega^D)}{\partial l}Pr\left[(1-Pr)\frac{u(l,l)-u(h,h)}{u(l,l)-u(l,h)}+2(2Pr-1)\right],\end{aligned}$$

with $D \equiv \dfrac{u(l,l)-u(l,h)}{1-\delta^C+2Pr(\delta^C-\delta^{CT^W})}\left(\dfrac{\partial Pr}{\partial\omega^D}\right)^{-1}$,

where $[1-\delta^C+(1+Pr)(\delta^C-\delta^{CT^W})]/[1-\delta^C+2(\delta^C-\delta^{CT^W})] < 1$ for the last bracketed term in the preceding expression to the second inequality is used to obtain the second inequality. To obtain the (last) equality in the above expressions, once again I use the assumption of $\partial^2 Pr(l)/\partial(\omega^D)^2 = 0$ in the same manner that I used it to obtain the last equality in the corresponding expressions in the proof for *Proposition 4* (*a*). As a result of these transformations, I obtain the following inequalities:

$$\text{(A10)} \qquad \frac{dV_C^W}{dT^W} > BD\frac{\partial(Pr/\omega^D)}{\partial l}Pr\left[(1-Pr)\frac{u(l,l)-u(h,h)}{u(l,l)-u(l,h)}+2(2Pr-1)\right] > 0$$

with the last inequality holding for all values of $T^W \geq 1$ if $(1 - Pr)[u(l, l) - u(h, h)]/[u(l, l) - u(l, h)] + 2(2Pr - 1) > 0$, which in turn holds if $Pr(l) > \underline{Pr}$.

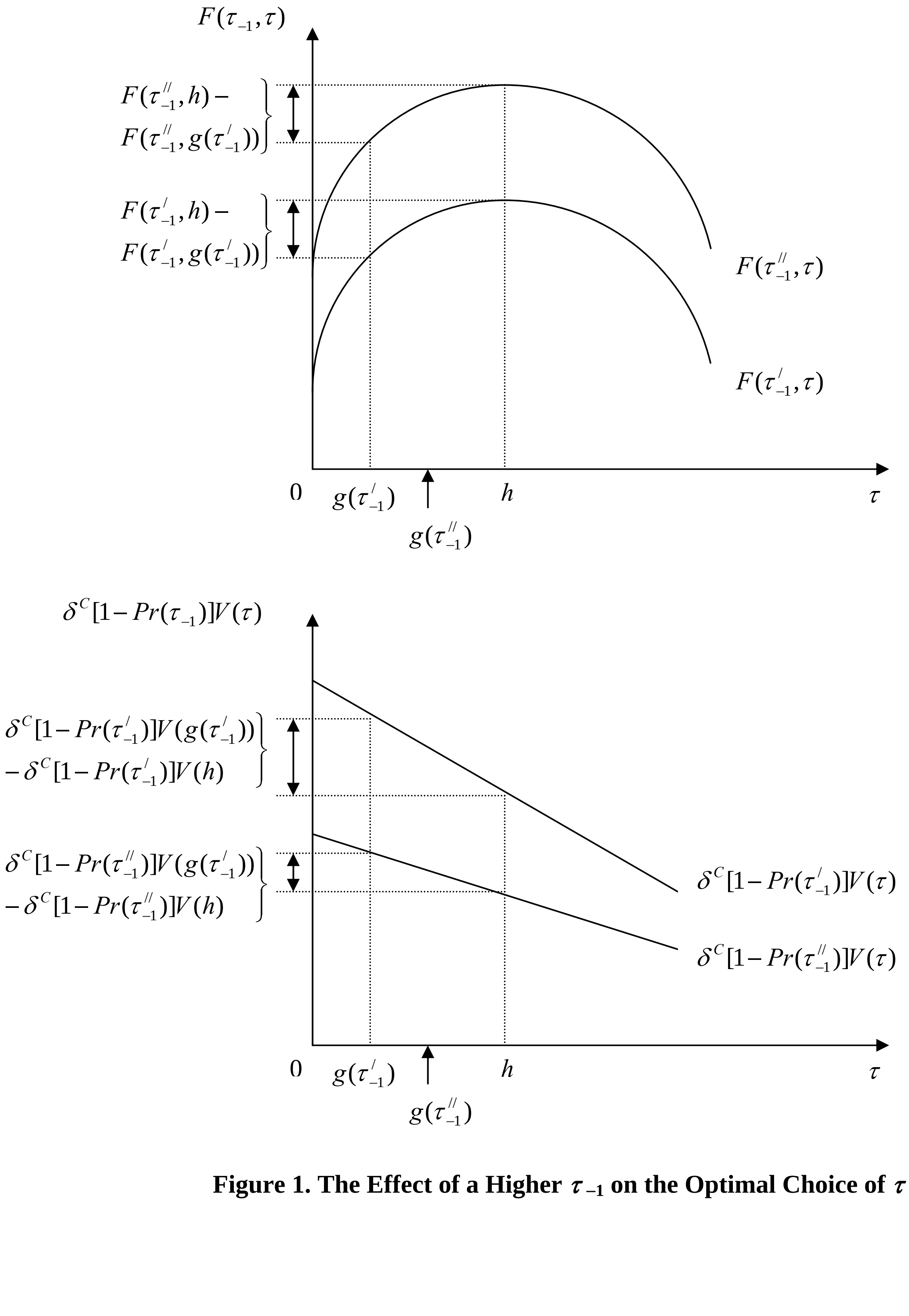

Figure 1. The Effect of a Higher τ_{-1} on the Optimal Choice of τ

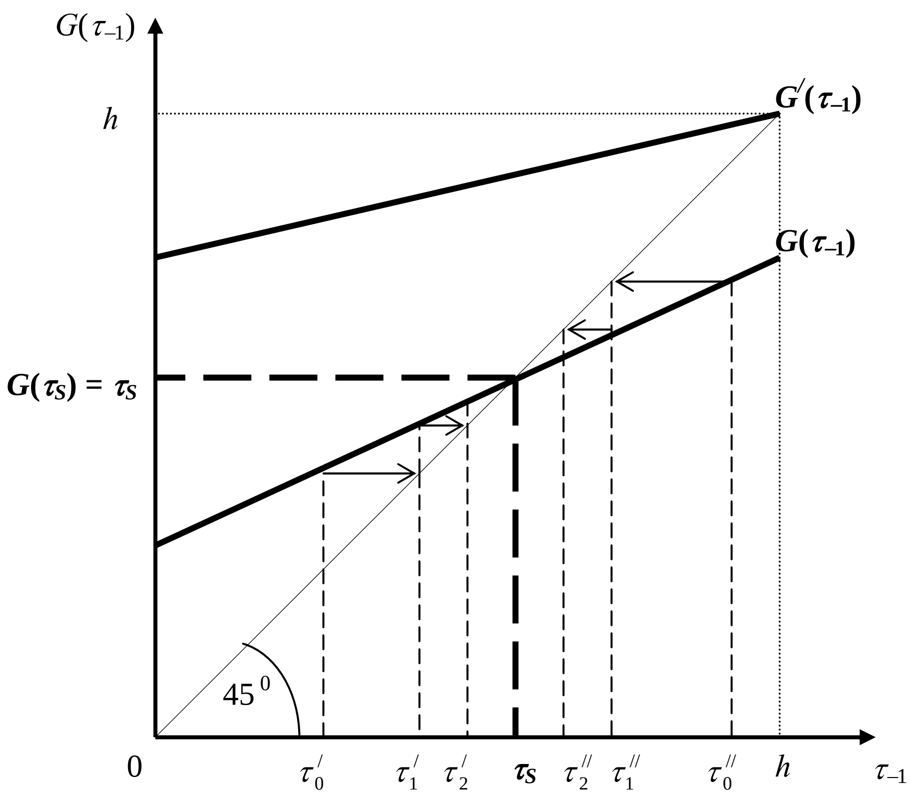

Figure 2. The Existence of a Stationary Protection Sequence at τ_S

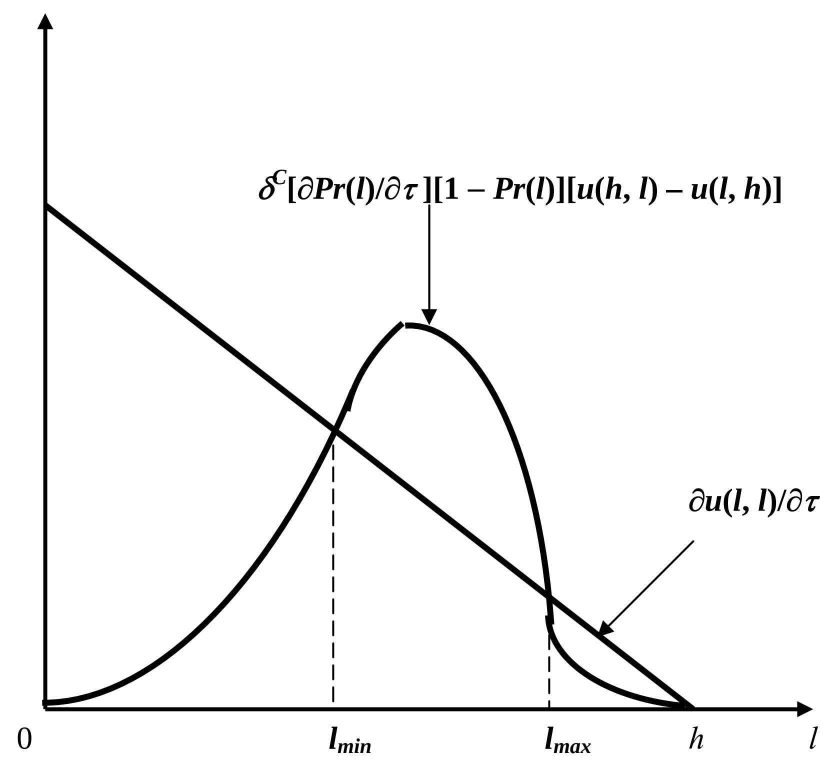

Figure 3. Multiple *l* satisfying *I*(*l*) = 0

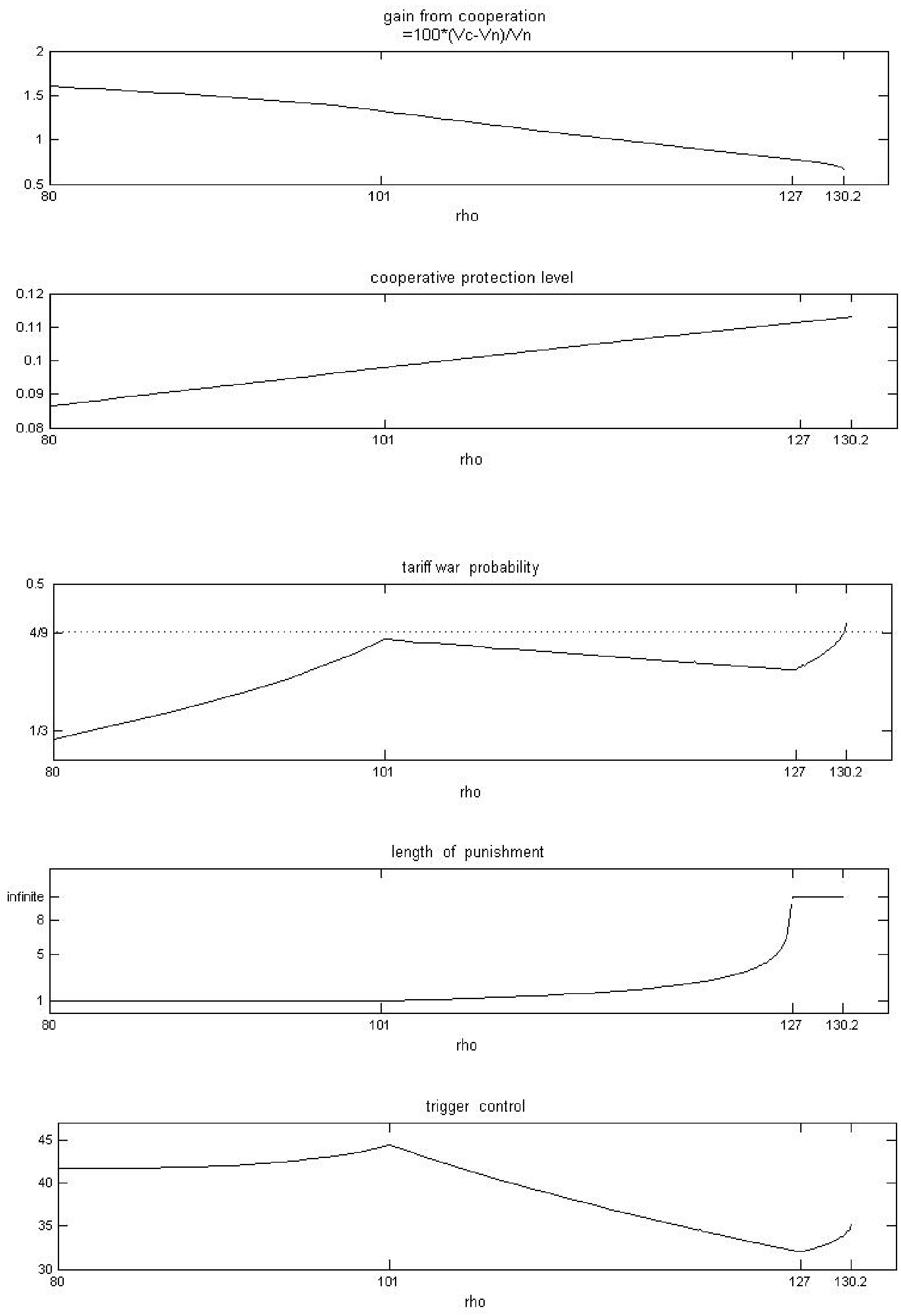

Figure 4. A numerical analysis of the optimal *TTS* for different values of ρ (rho) with $\chi = 1$, $\alpha_1 - \alpha_1^* = 3$, $\beta + B = 1$ (so, $h = 1$), and $\delta^C = 0.5$

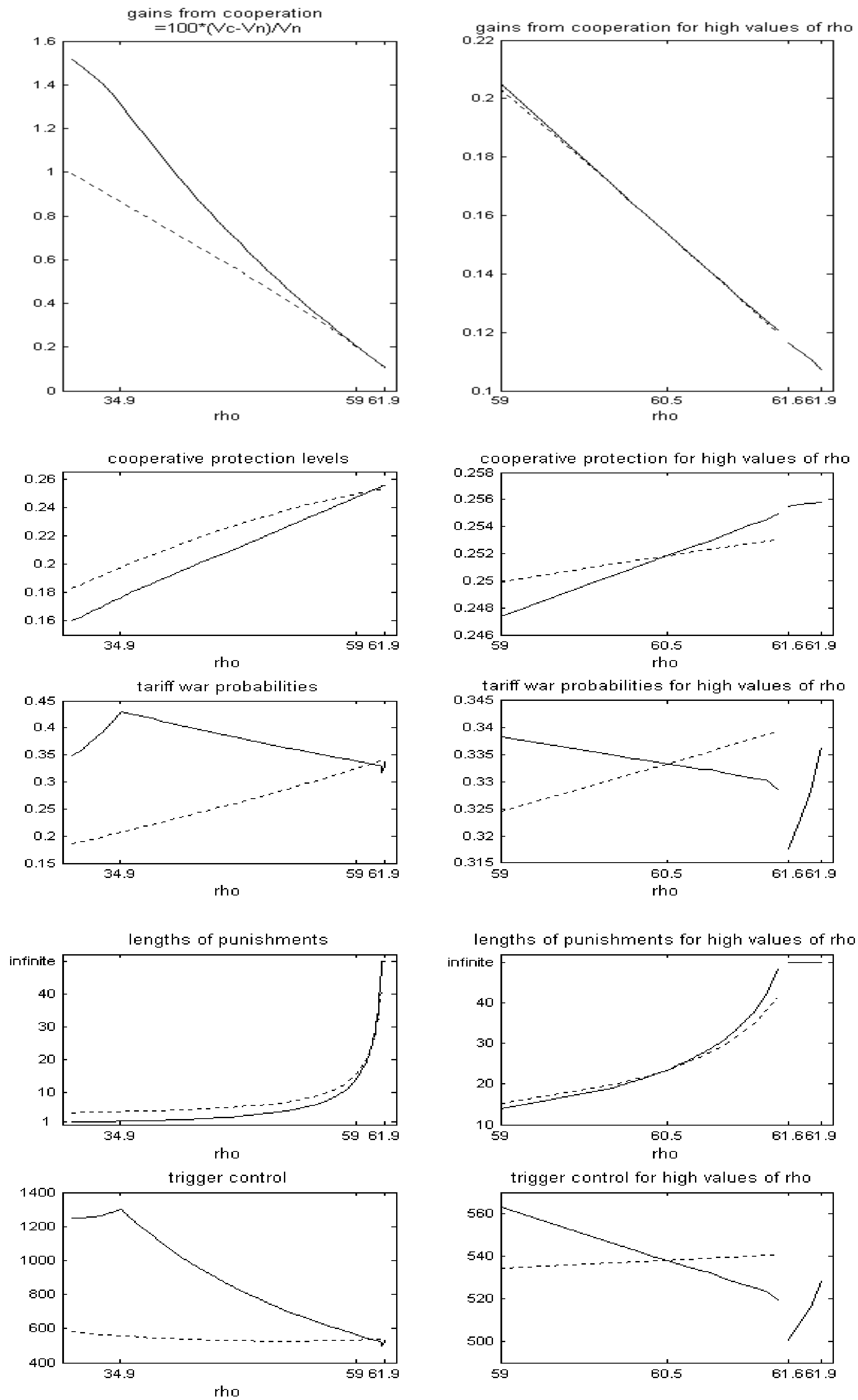

Figure 5. A numerical analysis of the optimal *TTS* and optimal *PTS* for different values of ρ (rho) with $\chi = 100$, $\alpha_1 - \alpha_1^* = 3$, $\beta + B = 1$ (so, $h = 1$), and $\delta^C = 0.5$