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Dukpa Kim
Yohei Yamamoto

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Institute of Economic Research
Hitotsubashi University
2-1 Naka, Kunitatchi Tokyo, 186-8601 Japan
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Time Instability of the U.S. Monetary System: Multiple Break Tests and Reduced Rank TVP VAR

Dukpa Kim
Korea University*

Yohei Yamamoto
Hitotsubashi University†

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Abstract

Earlier attempts to find evidence of time varying coefficients in the U.S. monetary vector autoregression have been only partially successful. Structural break tests applied to typical data sets often fail to reject the null hypothesis of no break. Bayesian inferences using time varying parameter vector autoregressions provide posterior median values that capture some important movements over time, but the associated confidence intervals are often very wide and make the entire results less conclusive. We apply recently developed multiple structural break tests and find statistically significant evidence of time varying coefficients. We also develop a reduced rank time varying parameter vector autoregression with multivariate stochastic volatility. Our model has a smaller number of free parameters thereby yielding tighter confidence intervals than previously employed unrestricted time varying parameter models.

Keywords: Time Varying Monetary Policy Rule, Inflation Persistence, Multivariate Stochastic Volatility

JEL Code: C32, E52

*Department of Economics, Korea University, Anam-dong, Seongbuk-gu, Seoul, 136-701, Korea (dukpakim@korea.ac.kr)

†Department of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo, 186-8601, Japan (yohei.yamamoto@econ.hit-u.ac.jp).

1 Introduction

It has been of great interest to many macroeconomists whether or not the parameters in the U.S. monetary vector autoregression (VAR) have been changing over time. The goal of this paper is to provide further evidence of time instability using two different approaches.

The first is to use recently developed multiple structural break tests. There have been similar attempts in the macroeconomic literature. The results from these studies are somewhat mixed and perhaps slightly favor time stability rather than time instability. For example, Bernanke and Mihov (1998) test for time invariance of the U.S. monetary VAR coefficients and do not find any strong evidence against time invariance. Sims and Zha (2006) also find little evidence against time invariance using a Bayesian approach, though they argue that the innovation variances show sizable time variation. Cogley and Sargent (2005) confirm similar results, but argue that the non-rejections result from the low power of structural break tests. Indeed, their simulation results suggest that the structural break tests they consider have very small power when the true data generating process is the same as the posterior mean of their Bayesian time varying parameter (TVP) model. We view these non-rejections of the structural break tests as a consequence of low power, agreeing with Cogley and Sargent (2005). Naturally, we apply recently developed break tests that are supposed to be more powerful than the tests used in the earlier studies. We consider both pure and partial change specifications and reject the null hypothesis of no time change for most cases in all equations of our VAR system. We also test for time stability of innovation covariance matrix using Aue et al. (2009) and find strong evidence of time instability.

The second is to build a reduced rank TVP VAR. Earlier studies typically use unrestricted TVP models where all autoregressive (AR) coefficients are permitted to change freely. However, our initial estimates from a similar unrestricted model suggest that most important time changes in the AR coefficients are highly synchronized across equations and autoregressive lags. Also, the dates of significant structural breaks are clustered around certain periods. These findings imply that the variations of the AR coefficients are likely to be structured in some way and the unrestricted model might be inefficient. This is exactly our motivation to build a reduced rank model. Our reduced rank model also complies with Cogley and Sargent's (2005) finding that a few principal components can explain the majority of the time variations in the AR coefficients. The direct benefit of reduced rank is that our model produces tighter confidence intervals than the unrestricted model thereby providing more definite statistical evidence. Canova and Ciccarelli (2009) use a similar reduced rank model in a panel VAR.

Another methodological contribution is that we adopt Uhlig (1997) type multivariate stochastic volatility process. In other words, the VAR error covariance is an inverse Wishart process with multivariate beta shocks. This stochastic volatility model is less restrictive than the ones used in earlier studies such as Cogley and Sargent (2001, 2005) and Primiceri (2005).

To sum up our empirical findings, the U.S. monetary VAR has statistically significant time variations not only in the covariance of the errors but also in the AR coefficients. We estimate time varying natural rate of unemployment and core inflation. These two variables show similar time patterns that they rise throughout the 1970s, reach their peak at the end of 1970s, sharply drop during the Volcker chairmanship, and remain mostly stable during the Greenspan chairmanship. The historical path of inflation persistence, measured as the normalized spectral density at zero frequency, is greatly correlated with the core inflation. The persistence builds steadily before the Volcker chairmanship, drops sharply and remains at a low level afterwards. The monetary policy rule also shows some variations over time. The policy reaction to inflation seems the strongest during the Volcker chairmanship, but there is no period that can be characterized as violating the Taylor principle.

The rest of the paper is organized as the following. Section 2 shows our results with multiple structural break tests. Section 3 explains the details of the reduced rank TVP vector autoregression with multiple stochastic volatility. Section 4 presents the estimation results from the reduced rank model. Section 5 contains concluding remarks.

2 Structural Break Tests and Time Instability

We first apply recently developed structural break tests to the U.S. monetary VAR. To be comparable with earlier studies, we focus on the second order VAR with three variables, unemployment rate, inflation rate and interest rate. Hence, the equation to which we apply structural break tests is given by

$$\begin{aligned} y_t &= v_t + A_{1,t} y_{t-1} + A_{2,t} y_{t-2} + u_t, \quad t = 1, 2, \ldots \qquad (1) \\ &= A_t x_t + u_t \\ x_t &= (1, y_{t-1}', y_{t-2}')' \\ A_t &= (v_t, A_{1,t}, A_{2,t}) \end{aligned}$$

where y_t is a vector of dependent variables, x_t is a vector of regressors, and A_t is a matrix of coefficients whose time stability is being tested.

We consider the following tests: 1) the SupLM test and 2) the SupWald test proposed by Andrews (1993), 3) the Nyblom (1989) test, 4) Elliott and Müller (2006)'s $\widehat{qLL}$ test, 5) Bai and Perron (1998)'s UDmax test, 6) the $\xi_{\sup}$ test and 7) the ξ_{trace} test proposed by

Eliasz, Stock, and Watson (2004)[1].

The null hypothesis in all of these tests specifies the VAR coefficients to be time invariant, that is,

$$A_t = A \text{ for all } t.$$

On the other hand, the alternative hypotheses in these tests specify different forms of time changes in A_t. The SupLM and SupWald tests use an alternative hypothesis of a single change in A_t at an unknown date. The UDmax test is an extension of the SupWald test and its alternative hypothesis assumes an unknown number of multiple breaks at unknown dates, where the maximum number of breaks considered is usually much smaller than the sample size. The Nyblom test assumes under the alternative that A_t is a martingale with its difference being i.i.d and that the magnitude of changes in A_t is extinct as the sample size increases. The alternative hypothesis in the $\widehat{qLL}$ test is similar to the one in the Nyblom test but it allows A_t to be a persistent mixing process. The alternative hypothesis in the $\xi_{\sup}$ and ξ_{trace} tests is such that

$$\begin{aligned} A_t &= A + B_t G, \\ \Delta B_t &= \theta_B(L)\eta_t, \quad \theta_B(1) = \gamma I_r \end{aligned}$$

where G is an $r \times (pk+1)$ matrix that reduces the rank of the time varying parameters.[2] The $\xi_{\sup}$ and ξ_{trace} tests are of interest because the reduced rank structure specified under the alternative hypothesis exactly corresponds to the model we estimate in the next section.

All of aforementioned tests have some power even if A_t changes differently from the way specified in their alternative hypothesis. The A_t process specified in the alternative hypothesis is only intended to be the data generating process against which the power of each test is optimal or admissible in some asymptotic sense. Hence, there is no one test that is the most powerful against all forms of changes in A_t.

Inflation rate is measured by log difference of the Consumer Price Index (CPI) for all urban consumers. The CPI is point sampled in the last month of every quarter. Unemployment rate is measured by the quarterly average of monthly rates for civilians. Interest rate is the monthly average of the daily rates of the 3 month Treasury Bill rates in the last month of every quarter. The data spans from 1948.Q1 to 2010.Q4, but the periods from 1948.Q1 to 1954.Q4 are used as our pre-sample to work out our priors.

All tests are conducted equation-by-equation. White (1980)'s heteroskedasticity robust variance is used. The results are presented in Table 1, which are obtained using the asymptotic critical values. Both pure and partial change cases are considered. The left-hand side

[1] We are grateful to an anonymous referee who suggests the $\xi_{\sup}$ and ξ_{trace} tests to us.

[2] r does not need to be estimated for these tests to be implemented.

variable is denoted by Y (current value) and the right-hand side variables with possibly unstable coefficients are denoted by X (lagged values). The right hand side variables that are not included in X are assumed to have a time invariant coefficient in the testing procedure.

Table 1. Parameter Instability Tests in the U.S. monetary VAR

Y	X	SupLM	SupWald	Nyblom	$\widehat{qLL}$	UDmax	ξ_{sup}	ξ_{trace}
unemp	all	12.46	39.37***	1.43	-38.50*	41.52***	21.07**	36.44**
	unemp	7.49	9.70	0.41***	-16.73	13.69*	-	-
	inflation	7.44	20.80***	1.25**	-14.73	20.80***	-	-
	interest	8.54	16.45**	0.60	-21.53**	16.85**	-	-
inflation	all	14.98	93.52***	1.30	-48.84***	93.52***	26.32***	46.31***
	unemp	6.39	6.42	0.54	-19.15*	37.78***	-	-
	inflation	14.96**	51.47***	0.86*	-31.08***	51.47***	-	-
	rate	19.71***	38.38***	0.44	-18.08*	45.69***	-	-
interest	all	11.78	24.76**	1.30	-44.03**	57.85***	30.17***	41.71**
	unemp	8.20	8.88	0.82	-11.32	10.39	-	-
	inflation	9.07	14.75**	0.59	-28.28***	14.75*	-	-
	interest	5.87	6.50	0.78	-14.65	14.91**	-	-

1. ***, **, and * are significant at 1%, 5%, and 10% levels respectively.
2. For the $\widehat{qLL}$ test, $c = 10$ as recommended in the original paper.
3. For the UDmax test, the maximum number of breaks is 5 and the truncation level is 10% for the SupLM, SupWald, and UDmax tests.

Our results from single break tests are consistent with earlier studies in the sense that there is little evidence of instability. In particular, the SupLM test rejects in only two cases at 5% significance. The SupWald test rejects more often as noted by Boivin (2002) and Cogley and Sargent (2005). The UDmax test rejects in most cases at 5% significance. This is mainly due to the fact that the UDmax test has more power than single break tests, if the true process has multiple breaks. The UDmax test also rejects more often than the Nyblom and $\widehat{qLL}$ tests. This may be caused by the well known "non-monotonic power" problem of structural break tests, which is first extensively analyzed in Vogelsang (1999) and further explored in Kim and Perron (2009) and Perron and Yamamoto (2011). The non-monotonic power means that the power function decreases and can even reach zero when the alternative hypothesis moves further away from the null hypothesis for a given sample size. This phenomenon appears when one uses LM type test (SupLM) or so called sum type tests (Nyblom and $\widehat{qLL}$). The ξ_{sup} and ξ_{trace} tests reject in all equations at 5% significance.

A natural concern is size distortion. To investigate this issue, we conduct a Monte Carlo experiment to obtain finite sample critical values. In doing so, it is important to use a data generating process that contains important characteristics of the data. As we show later in this section, there is strong evidence of changing volatility of VAR innovations. Hence, we first estimate the null model ($A_t = A$) which incorporates time varying VAR innovation covariance. Then, we simulate the test statistics under pseudo data generated from the estimates of A and the estimated time profile of innovation covariance matrix[3].

In particular, we use Uhlig's (1997) multivariate stochastic volatility model[4] for u_t in (1), which is given by

$$u_t|H_t \sim ind\ N\left(0, H_t^{-1}\right).$$

where the inverse covariance matrix H_t evolves over time by

$$\begin{aligned} H_{t+1} &= \frac{1}{\lambda} T_t' \Theta_t T_t \quad \text{with} \quad \Theta_t \sim B_k\left(\frac{d}{2}, \frac{1}{2}\right) \\ H_1|Y_0 &\sim W_k\left(d, \Sigma_{1|0}\right). \end{aligned} \tag{2}$$

Here, $Y_t = (y_t, y_{t-1}, \ldots, y_{-p})$, T_t is an upper triangular matrix obtained from the Cholesky decomposition of H_t, $1 > \lambda > 0$ and $d \geq k+3$ are scalar parameters, and $\Sigma_{1|0}$ is a $k \times k$ symmetric matrix of parameters. $B_k(\cdot,\cdot)$ and $W_k(\cdot,\cdot)$ stand for multivariate beta and Wishart distribution respectively. Also, the singular Wishart and singular multivariate beta distributions are as defined by Uhlig (1994). Throughout the paper, we assume that d, λ and $\Sigma_{1|0}$ are known, because they usually do not bear economic meanings and can be better understood as tuning parameters. We set $d = 20$ and $\lambda = \exp\left(-\Psi(d+1) + \Psi(d)\right)$ where $\Psi(\cdot)$ denotes the Euler Psi function. $\Sigma_{1|0}$ is set at the inverse of the covariance of u_t estimated from the pre-sample data (1948.Q1$\sim$1954.Q4) scaled by $(d-k+1)^{-1}$.

Instead of the Bayesian estimation method Uhlig (1997) proposes, we obtain the maximum likelihood estimate of the AR coefficients. Rewrite the null model of (1) as $y_t = Z_t'\mu + u_t$ where $Z_t' = I_k \otimes x_t'$ and $\mu = vec\,(A')$. Then, the maximum likelihood estimate for μ is

[3] We divide all AR coefficient estimates by two, because, otherwise, they would imply an unstable system. It is a well known fact that the ordinary least squares estimate is biased toward an unstable system if changes in the intercepts and AR coefficients are ignored (for example, Perron, 1989), and a similar bias is likely to exist in our estimate. This adjustment is somewhat arbitrary, but we find that the critical values are not very sensitive to a change in the adjustment.

[4] Uhlig's (1997) model has important advantages over other approaches. For example, Harvey, Ruiz and Shephard (1994), adopted in Cogley and Sargent (2005), do not allow covariances to evolve. Primiceri (2005) allows covariances to change but assumes the row independence of the covariance matrix and a certain time ordering among variables within the same time period. Uhlig's model is subject to none of these restrictions. Also, Uhlig's model has the simplest estimation procedure.

given by

$$\hat{\mu} = \left(\sum_t \frac{1}{\delta_t^2} Z_t \Omega_t^{-1} Z_t'\right)^{-1} \left(\sum_t \frac{1}{\delta_t^2} Z_t \Omega_t^{-1} y_t\right) \tag{3}$$

where

$$\delta_t^2 = 1 + \frac{1}{(d+1-k)}(y_t - Z_t'\hat{\mu})'\Omega_t^{-1}(y_t - Z_t'\hat{\mu}).$$

and

$$\begin{aligned} \Omega_t &= \frac{1}{(d+1-k)}\Sigma_{t|t-1}^{-1} \\ \Sigma_t^{-1} &= (y_t - Z_t'\hat{\mu})(y_t - Z_t'\hat{\mu})' + \Sigma_{t|t-1}^{-1} \\ \Sigma_{t+1|t} &= \frac{1}{\lambda}\Sigma_t \end{aligned} \tag{4}$$

Given the maximum likelihood estimate[5] of the AR coefficients, the covariance process is simulated from its posterior distribution. Note that conditional on the parameters (μ, d, λ, $\Sigma_{1|0}$),

$$p(H_T, \ldots, H_1|Y_T) = p(H_T|Y_T)p(H_{T-1}|Y_T, H_T)\cdots p(H_1|Y_T, H_T, \ldots, H_2)$$

and

$$p(H_t|Y_T, H_T, \ldots, H_{t+1}) = p(H_t|Y_t, H_{t+1}) \tag{5}$$

Combining these two equations yields that

$$p(H_T, \ldots, H_1|Y_T) = p(H_T|Y_T)p(H_{T-1}|Y_{T-1}, H_T)\cdots p(H_1|Y_1, H_2). \tag{6}$$

which is the same type of decomposition as reported by Carter and Kohn (1994) for the linear Gaussian state space model. In addition, we can see that conditional on Y_t and H_{t+1}

$$\begin{aligned} H_t &= \lambda(H_{t+1} + R_t) \\ R_t &\sim W_k\left(1, \frac{1}{\lambda}\Sigma_t\right). \end{aligned} \tag{7}$$

Therefore, drawing from $p(H_T, \ldots, H_1|Y_T)$ is equivalent to drawing $H_T^{(i)}$ from $p(H_T|Y_T)$ and subsequently drawing $H_t^{(i)} = \lambda(H_{t+1}^{(i)} + R_t)$ for $t = T-1, \ldots, 1$. For a given history of covariance process, a sequence of innovations is drawn to create a set of pseudo data.

[5] The maximum likelihood estimate $\hat{\mu}$ can be obtained via an iterative method. That is to obtain the sequence of Σ_t from (4) for a given value of μ and to obtain μ from (3) for a given sequence of Σ_t. The iteration can be initiated from the least squares estimate of μ. This estimate belongs to the class of iteratively reweighted least squares estimates by Rubin (1983).

All test statistics are computed for each pseudo data set. The number of replications is 1,000. We then evaluate the 99^{th}, 95^{th} and 90^{th} percentiles of the empirical distributions and use them as critical values.

In Table 2, we report the test results using the finite sample critical values. It shows that the SupWald and UDmax tests have less rejections than in Table 1. However, the overall results are qualitatively similar. We have more rejections with multiple break tests than single break tests. The ξ_{sup} and ξ_{trace} tests still reject in all equations.

Table 2. Parameter Instability Tests in the U.S. monetary VAR: Empirical Critical Values from Stochastic Volatility Model

Y	X	SupLM	SupWald	Nyblom	$\widehat{qLL}$	UDmax	ξ_{sup}	ξ_{trace}
unemp	all	12.46	39.37**	1.43**	-38.50**	41.52**	21.07**	36.44*
	unemp	7.49	9.70	0.41	-16.73	13.69	-	-
	inflation	7.44	20.80**	1.25	-14.73	20.80**	-	-
	interest	8.54	16.45**	0.60	-21.53**	16.85*	-	-
inflation	all	14.98	93.52***	1.30*	-48.84***	93.52***	26.32***	46.31***
	unemp	6.39	6.42	0.54*	-19.15*	37.78***	-	-
	inflation	14.96**	51.47***	0.86	-31.08***	51.47***	-	-
	rate	19.71***	38.38***	0.44	-18.08*	45.69***	-	-
interest	all	11.78	24.76*	1.30*	-44.03**	57.85***	30.17***	41.71***
	unemp	8.20	8.88	0.82*	-11.32	10.39	-	-
	inflation	9.07	14.75*	0.59	-28.28***	14.75	-	-
	interest	5.87	6.50	0.78	-14.65	14.91	-	-

Notes: See Table 1.

We also investigate the break dates which minimize the sum of squared residuals using the so-called dynamic programming procedure suggested by Bai and Perron (1998). In order to appropriately determine the number of breaks in each equation, one needs to use a sequential test or information criteria. However, we simplify this process by looking at the number of breaks at which the UDmax test becomes the most significant.

Figure 1 is a histogram of these break dates. We include only the break dates from the partial change models to avoid double counting. We observe that the dates are clustered in the mid 1970s and the early 1980s and the pattern is very close to the illustration by Stock and Watson (1996). This suggests that modeling parameter variations in some systematic manner should result in efficiency gain compared to a TVP model where all regression coefficients vary freely.

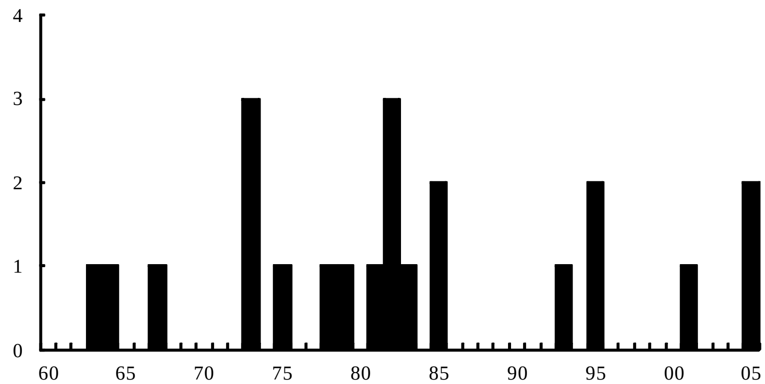

Figure 1. Histogram of Estimated Break Dates

Finally, we investigate time instability of the innovation covariance matrix using the tests recently proposed by Aue et al. (2009). Applying these tests to standard VAR residuals may result in over-rejection if there are neglected changes in the coefficients, because they may translate into additional changes in the covariances. Hence we conduct the covariance stability tests not only with the residuals from the constant coefficient VAR but also from the VAR with multiple structural breaks in the coefficients[6]. The results are reported in Table 3. It is strongly suggested that the VAR system has an unstable covariance structure in the full sample. Even after taking the coefficient breaks into account, the Ω_n test and the Λ_n test are significant at 1% and 10% respectively.

Table 3. Results of the Covariance Stability Tests

sample	Ω_n (CUSUM type)	Λ_n (Sup type)
constant coefficients	6.47^{***}	4.46^{***}
multiple breaks in coefficients	5.05^{***}	3.14^{*}

1. ***, **, and * are significant at 1%, 5%, and 10% levels respectively.

2. Asymptotic critical values for $\delta = 6$ are not reported in the original paper hence they are simulated via approximating a Brownian bridge by 10,000 steps with 10,000 replications.

[6] The break dates are obtained from the pure structural change models. They are 1963.$Q1$ and 2005.$Q2$ in the unemployment equation, 1981.$Q3$ in the inflation equation, and 1969.$Q3$, 1975.$Q1$, 1980.$Q3$, and 1986.$Q3$ in the interest equation.

3 Reduced Rank TVP VAR

The standard TVP VAR where all regression coefficients are freely changing has great flexibility in fitting data. However, previous studies applying this method to the U.S. monetary VAR, for example Cogley and Sargent (2005), find that most important time variations in the AR coefficients can be captured by only a few components. This is exactly the motivation for us to build a reduced rank TVP VAR. We decompose the AR coefficients into time invariant and varying portions and impose the reduced rank condition on the time varying portion. Using unnecessarily many free parameters often results in extremely wide confidence intervals, which makes the inference much less conclusive. One benefit of our approach is having a reduced number of time varying parameters, thereby rendering tighter confidence intervals. For comparison, we also consider unrestricted model. In the following, we lay out the details of our models and estimation procedures.

Unrestricted TVP VAR We start with a simple unrestricted model which will serve as benchmark for the reduced rank model. Our unrestricted TVP VAR is given by (1) where the autoregressive coefficients follow a multivariate random walk

$$\begin{aligned} vec(A_t') &= vec(A_{t-1}') + e_t \\ e_t &\sim N(0, Q) \end{aligned} \tag{8}$$

and

$$u_t | H_t \sim ind\ N\left(0, H_t^{-1}\right).$$

with the inverse covariance matrix H_t as given in (2). Our unrestricted model is a variant of Triantafyllopoulos (2011).

For the estimation of the unrestricted model, we use the Gibbs sampler combing the following two conditional models:

Step 1: Conditional on $(Q, H_t, t = 1, \ldots, T)$, the equations in (1) and (8) constitute a linear Gaussian state space model:

$$y_t = A_t x_t + u_t = (I_k \otimes x_t') vec(A_t') + u_t$$

This can be routinely estimated by the Kalman filter. Define $W_t' = I_k \otimes x_t'$ and $b_t = vec(A_t')$.

The Kalman filter updating equations are:

$$\begin{aligned}
b_{t|t-1} &= b_{t-1|t-1} \\
V_{t|t-1} &= V_{t-1|t-1} + Q \\
K_t &= V_{t|t-1} W_t (W_t' V_{t|t-1} W_t + H_t^{-1})^{-1} \\
b_{t|t} &= b_{t|t-1} + K_t (y_t - W_t' b_{t|t-1}) \\
V_{t|t} &= V_{t|t-1} - K_t W_t' V_{t|t-1}
\end{aligned} \tag{9}$$

Once $b_{T|T}$ and $V_{T|T}$ are obtained, b_T is drawn from its posterior distribution $N(b_{T|T}, V_{T|T})$. Then, b_t for $t = T-1, \ldots, 1$ are subsequently drawn from their posterior distribution $N(b_{t|t+1}, V_{t|t+1})$ where the mean and variance are obtained from the backward recursion formula:

$$\begin{aligned}
b_{t|t+1} &= b_{t|t} + V_{t|t} V_{t+1|t}^{-1} (b_{t+1} - b_{t|t}) \\
V_{t|t+1} &= V_{t|t} - V_{t|t} V_{t+1|t}^{-1} V_{t|t}
\end{aligned} \tag{10}$$

To initiate the Kalman filter, we obtained $b_{0|0}$ and $V_{0|0}$ from the pre-sample data. $b_{0|0}$ is the least squares estimate of the AR coefficients and $V_{0|0}$ is a diagonal matrix taking the diagonal entries of the covariance matrix of the least squares estimate.

We specify that Q has an inverse Wishart prior.

$$Q \sim IW(\bar{Q}^{-1}, df)$$

where

$$\begin{aligned}
df &= k(2k+1) + 1 = 22 \\
\bar{Q} &= I_3 \otimes diag\left\{ sI_4, \frac{s}{2} I_3 \right\} \quad \text{with } s = 3^{-1} \times 10^{-5}.
\end{aligned}$$

Given the sequence of b_t, we can recover the sequence of e_t. Then, the posterior of Q is given by

$$Q \sim IW\left([\bar{Q} + \sum e_t e_t']^{-1}, df + T \right)$$

Step 2: Conditional on $(A_t, t = 1, \ldots, T)$, the model becomes the multivariate stochastic volatility model given in (2). For $t = 1, 2, \ldots,$

$$p(H_t | Y_t) \sim W_k(d+1, \Sigma_t) \tag{11}$$

$$p(H_{t+1} | Y_t) \sim W_k(d, \Sigma_{t+1|t}) \tag{12}$$

where Σ_t and $\Sigma_{t+1|t}$ are obtained through a filter so that

$$\Sigma_t^{-1} = u_t u_t' + \Sigma_{t|t-1}^{-1} \quad \text{and} \quad \Sigma_{t+1|t} = \frac{1}{\lambda}\Sigma_t$$

Upon obtaining Σ_ts, drawing from $p(H_1, \ldots, H_T|Y_T)$ is the same as explained in (7).

Reduced Rank TVP-VAR Now, we impose reduced rank structure on the time varying portion of the autoregressive coefficients. That is to specify $A_t = A + B_t G$ so that

$$y_t = Ax_t + B_t G x_t + u_t \tag{13}$$

where B_t and G are $k \times r$ and $r \times (kp+1)$ respectively. For the model to be properly identified, we assume that

$$B_1 = 0 \text{ and } G = \begin{pmatrix} I_r & G_1 \end{pmatrix}.$$

The time varying parameters follow a multivariate random walk

$$\begin{aligned} vec(B_t') &= vec(B_{t-1}') + e_t \\ e_t &\sim N(0, Q) \end{aligned} \tag{14}$$

We use the Gibbs sampler combing the following four conditional models:

Step 1: Conditional on $(A, Q, G, H_t, t = 1, \ldots, T)$, the equations in (13) and (14) constitute a linear Gaussian state space model:

$$\begin{aligned} y_t - Ax_t &= B_t G x_t + u_t \\ &= vec\left(x_t' G' B_t'\right) + u_t = (I_k \otimes x_t' G') vec(B_t') + u_t \end{aligned}$$

This is again estimated by the Kalman filter. Define $W_t' = I_k \otimes x_t' G'$ and $b_t = vec(B_t')$. The Kalman filter updating equations given in (9) and (10) are directly applied. One difference is that since we assume $B_1 = 0$, the recursion starts from $b_{1|1} = 0$ and $V_{1|1} = 0$. We set $r = 3$ and then our assumption that $G = [I_r, G_1]$ implies $B_t G = [B_t, B_t G_1]$. This means that B_t should have similar time variations to the elements in the first r columns of A_t matrix of the unrestricted model. Hence, the prior for Q is the same as in the unrestricted model. That is,

$$
\begin{aligned}
Q &\sim IW(\bar{Q}^{-1}, df) \\
df &= 22 \\
\bar{Q} &= sI_9 \quad \text{with } s = 3^{-1} \times 10^{-5}.
\end{aligned}
$$

The Q matrix is updated in the same way as before given the sequence of b_ts.

Step 2: Conditional on $(B_t, G, H_t, t = 1, \ldots, T)$, the model in (13) yields a multivariate linear regression with heteroskedastic Gaussian errors.

$$
\begin{aligned}
y_t^* &= T_t(y_t - B_t G x_t) \\
&= T_t A x_t + T_t u_t = (T_t \otimes x_t')vec(A') + T_t u_t = Z_t' \mu_A + T_t u_t
\end{aligned}
$$

where

$$y_t^* = T_t(y_t - B_t G x_t),\ Z_t' = T_t \otimes x_t', \text{ and } \mu_A = vec(A').$$

We specify the prior for μ_A as $N(\bar{\mu}_A, \Omega_A)$. Then, the posterior is given by

$$N\left(\hat{\mu}_A, \left[\Omega_A^{-1} + \sum_{t=1}^{T} Z_t Z_t'\right]^{-1}\right)$$

with

$$\hat{\mu}_A = \left[\Omega_A^{-1} + \sum_{t=1}^{T} Z_t Z_t'\right]^{-1} \left[\Omega_A^{-1}\bar{\mu}_A + \sum_{t=1}^{T} Z_t y_t^*\right].$$

Since A stands for the value of AR coefficients at time 1, $\bar{\mu}_A$ is set at the average of the posterior median values of A_t in the first 10 quarters obtained from the unrestricted model. Ω_A is set at the covariance matrix of the least squares estimate of the AR coefficients from the entire sample scaled by 10^{-1}.

Step 3: Conditional on $(A, B_t, H_t, t = 1, \ldots, T)$, the model in (13) yields another multivariate linear regression with heteroskedastic Gaussian errors. Let $x_t = (x_{1t}', x_{2t}')'$ where x_{1t} collects the first r elements of x_t and x_{2t} the rest.

$$
\begin{aligned}
y_t^{**} &= T_t(y_t - A x_t - B_t x_{1t}) \\
&= T_t B_t G_1 x_{2t} + T_t u_t \\
&= (T_t B_t \otimes x_{2t}')vec(G_1') + T_t u_t = S_t' \gamma_1 + T_t u_t
\end{aligned}
$$

where

$$y_t^{**} = T_t(y_t - Ax_t - B_t x_{1t}),\ S_t' = (T_t B_t \otimes x_{2t}'),\ \text{and}\ \gamma_1 = vec(G_1').$$

We specify the prior for γ_1 as $N(\bar{\gamma}_1, \Omega_\gamma)$. Then, the posterior is given by

$$N\left(\hat{\gamma}_1, \left[\Omega_\gamma^{-1} + \sum_{t=1}^{T} S_t S_t'\right]^{-1}\right)$$

with

$$\hat{\gamma}_1 = \left[\Omega_\gamma^{-1} + \sum_{t=1}^{T} S_t S_t'\right]^{-1} \left[\Omega_\gamma^{-1}\bar{\gamma}_1 + \sum_{t=1}^{T} S_t y_t^{**}\right].$$

Because $B_t G = [B_t, B_t G_1]$, the elements in the first column of B_t correspond to changes in the intercepts, those in the second column to changes in the unemployment coefficients in the first lag and those in the third column to changes in the inflation coefficients in the first lag. Changes in the rest coefficients are some linear combinations of the elements in B_t and G_1 specifies the weights. We specify

$$\begin{aligned}
\bar{\gamma}_1 &= E(vec(G_1')) = vec\left[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.5 & 0.1 & 0 & 0.5 \\ -0.5 & 0 & 0.1 & -0.5 \end{pmatrix}'\right] \\
\Omega_\gamma &= diag\left\{ vec\left[\begin{pmatrix} 10^{-6} & 10^{-6} & 10^{-6} & 10^{-6} \\ 1 & 4^{-1} & 10^{-6} & 10^{-2} \\ 1 & 10^{-6} & 4^{-1} & 10^{-2} \end{pmatrix}'\right]\right\}.
\end{aligned}$$

The zero mean in the first row of G_1 implies that the intercept changes are not coupled with changes in the other coefficients. This prior is imposed somewhat strongly as we put small variances for these elements. The weight $(0.5, -0.5)$ is for the interest coefficients changes, meaning that the unemployment and inflation coefficients changes have the same importance but with opposite signs. This prior is put only very loosely especially for the first lag term. The weight $(0.1, 0)$ is for the unemployment coefficient change in the second lag. We put a relatively large variance for the positive weight and a small variance for the zero weight. This means that the unemployment coefficient change in the first lag would mostly affect the unemployment coefficient change in the second lag with some positive bias while the inflation coefficient change in the first lag would not affect much. A similar interpretation goes for the weight $(0, 0.1)$.

Step 4: Conditional on $(A, G, B_t, t = 1, \ldots, T)$, the model becomes a multivariate stochastic volatility model which is the same as Step 2 of the unrestricted model.

For both the unrestricted and reduced rank models, we executed 40,000 replications and the first 30,000 replications are discarded to ensure convergence of the chain. We checked convergence by looking at various moments of the estimates both within each chain and across multiple parallel chains.

4 Estimation Results

We report the time evolutions of the VAR error covariance matrix, core inflation, natural rate of unemployment and interest rate's longrun response to inflation. In all graphs, the thick line corresponds to the posterior median and the thin lines to 5^{th} and 95^{th} percentiles of the posterior.

VAR error covariance matrix We construct historical paths of the Cholesky decomposition of H_t^{-1} from the posterior draws. Assuming that the structural errors are identified by the usual triangular system, we first report the standard errors of the structural shocks in Figure 2. We report only the results from the reduced rank model, since the two models produce very similar results.

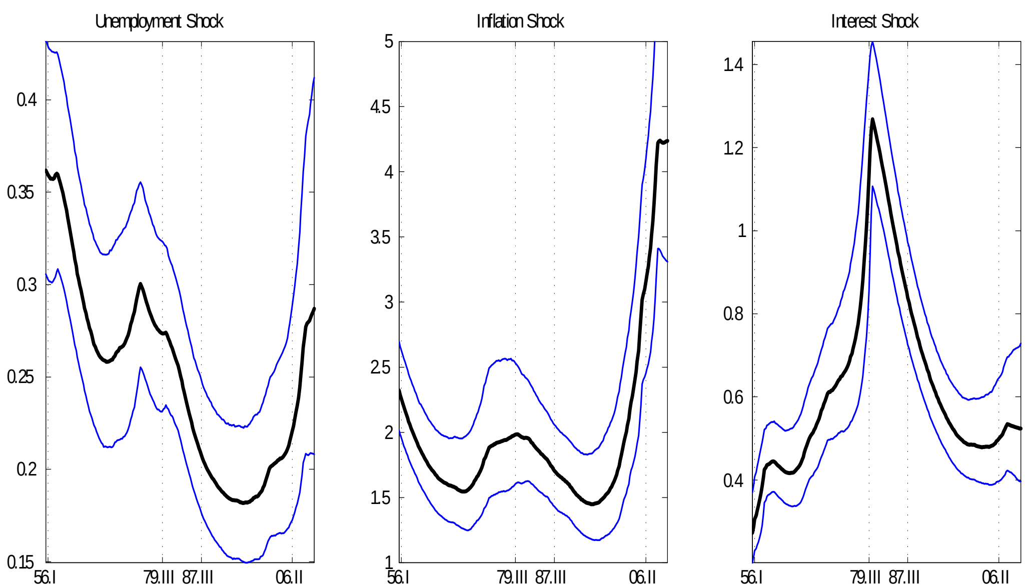

Figure 2. Standard Errors of Structural Shocks

Our estimates share many important features with previous findings in the literature, for example, those in McConnell and Perez Quiros (2000), Primiceri (2005) and Cogley and Sargent (2005) among many others. First, the unemployment shock shows the "Great

Moderation" throughout the 1980s. The drop during the Volcker chairmanship is roughly 40%. However, the unemployment volatility has been on the rise especially during the Bernanke chairmanship. The inflation shock shows a similar pattern, though the moderation captured in the 1980s seems milder. The interest shock shows a huge hike during the Volcker chairmanship, reflecting the non-borrowed reserve targeting.

Figure 3 is obtained from the off diagonal elements of the Cholesky decomposition of H_t^{-1}. Under the triangular identification, these estimates can be interpreted as contemporaneous responses to one standard deviation change in structural shocks. The inflation response to unemployment shock seems negative mostly, but the confidence interval does not rule out zero response. The interest rate responses are more definite statistically. A positive unemployment shock decreases interest rate. The response is the strongest during the Volcker chairmanship, but there is no evidence to differentiate pre and post Volcker periods. A positive inflation shock increases interest rate. Again, the response is the strongest during the Volcker chairmanship, but the Bernanke chairmanship seems almost equally strong.

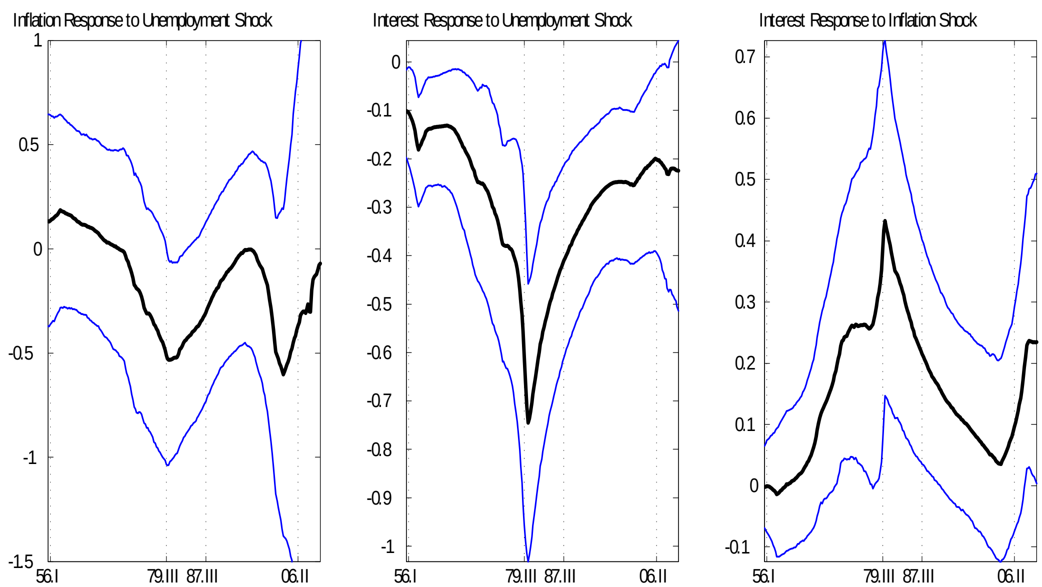

Figure 3. Contemporaneous Responses to Structural Shocks

Natural rate of unemployment and core inflation Figures 4 and 5 show the natural rate of unemployment and core inflation. We obtain them from local linear approximations to mean inflation and unemployment respectively evaluated at the posterior distribution, following Cogley and Sargent (2005). Let the companion form of (1) be $z_t = \mu_t + \bar{A}_t z_{t-1} + u_t$. Then, the natural rate of unemployment and core inflation are given by $s(I - \bar{A}_t)^{-1}\mu_t$, where

s is an appropriate selection matrix. We present the results from the unrestricted model and reduced rank model side by side.

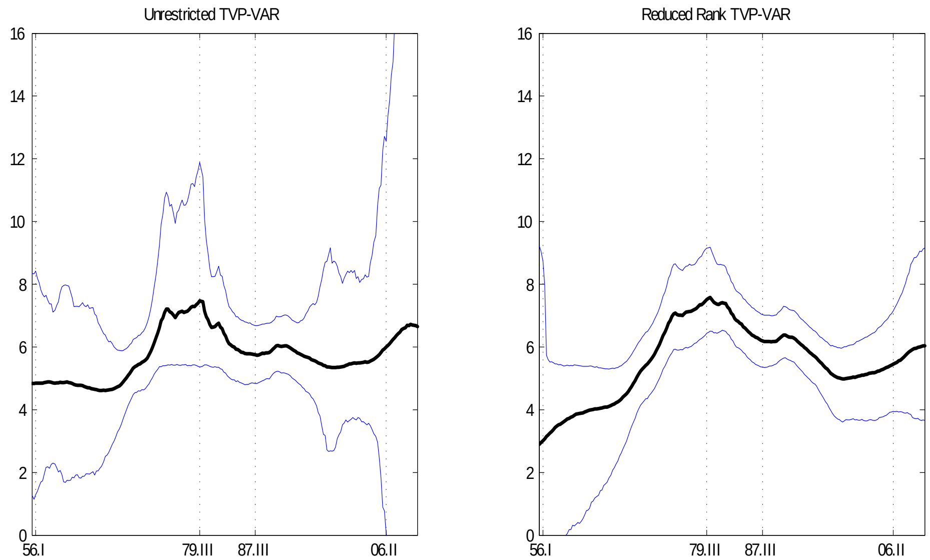

Figure 4. Natural Rate of Unemployment

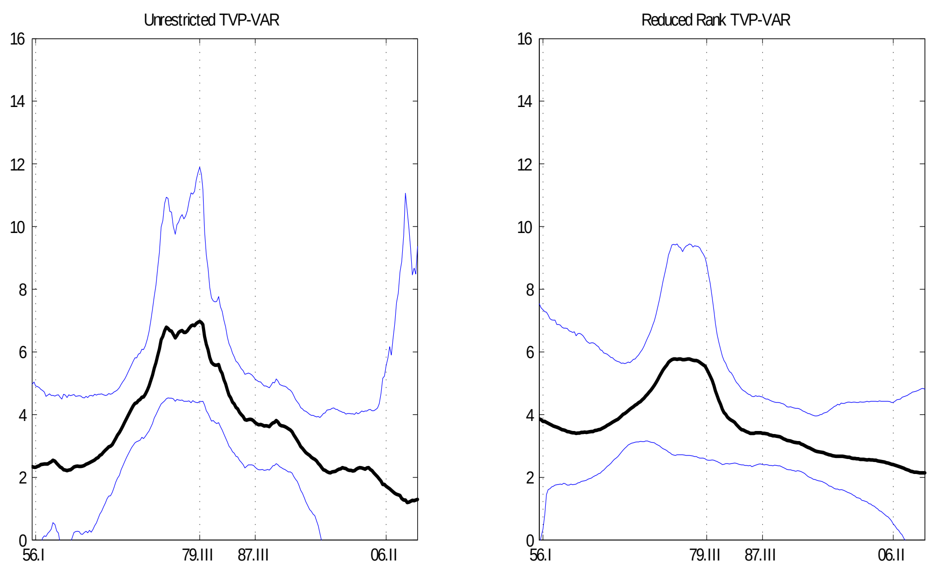

Figure 5. Core Inflation

Overall, the reduced rank model produces much tighter confidence intervals than the unrestricted model, while the two models produce similar trends in the median. In terms of the median values, both the natural rate of unemployment and core inflation rise to the peak of 6%-8% in the late 1970s and fall afterwards throughout the Greenspan chairmanship. These estimates are in accordance with the previous results reported in Cogley and Sargent (2005). However, the results from the unrestricted model are less conclusive about the recent movements of these two variables due to the wider confidence intervals. In Figure 4, for example, the estimates from the unrestricted model somewhat suggest that the natural rate of unemployment declines during the Volcker chairmanship but the following time path during the Greenspan and Bernanke chairmanship is unclear because of the wide confidence intervals. On the other hand, the reduced rank model yields confidence intervals that are tight enough to suggest declining natural rate of unemployment up to the mid 1990s. In Figure 5, the 95^{th} percentile for the core inflation during the Bernanke chairmanship is much higher in the unrestricted model than in the reduced rank model. As a result, if we make a hypothesis that the core inflation during the Bernanke chairmanship is as high as the median estimates for the late 1970s, then the unrestricted model does not reject it while the reduced rank model does and thus provides more evidence of time changes.

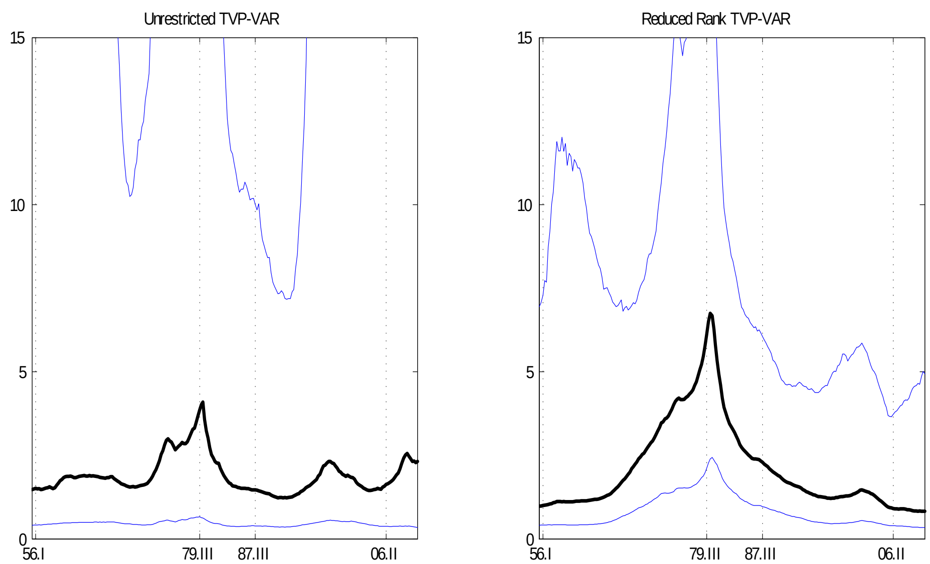

Figure 6. Inflation Persistence

Inflation persistence Figure 6 displays measured inflation persistence. It is measured as the normalized spectral density at zero frequency, where the normalization is taken with respect to the innovation variance. In terms of the posterior median values, the two models produce similar time patterns. The median of the persistence gradually rises throughout the late 1960s and the 1970s, sharply falls in the early 1980s, and shows much less fluctuation afterwards. Our median estimates are comparable to those in Cogley and Sargent (2005), and they bear great resemblance to the historical path of the core inflation. When the confidence intervals are considered, however, the results from the unrestricted model is not informative at all. For example, the 5^{th} percentile from the unrestricted model is almost flat over time and the 95^{th} percentile is much higher than the median. Thus if we make a hypothesis that the inflation persistence around 1979 is as low as the median estimates for other time periods, then such a hypothesis cannot be rejected. On the other hand, the 5^{th} percentile around 1979 obtained from the reduced rank model is higher than the median estimates in other time periods, and provides some evidence of time varying inflation persistence.

Monetary policy rule Figure 7 shows monetary policy rule, which is obtained as interest rate's longrun response to a unit inflation shock. As explained in Primiceri (2005), the Taylor principle requires that the longrun response be greater than one. Our median estimates from the reduced rank model suggest that this requirement is met most periods except for the late 1950s and the early 2000s, which is in general agreement with the claim in Primiceri (2005). In Figure 3, the immediate response of interest to inflation in the Bernanke chairmanship is almost as strong as in the Volcker chairmanship. However, the longrun response seems weaker in the Bernanke chairmanship than in the Volcker chairmanship.

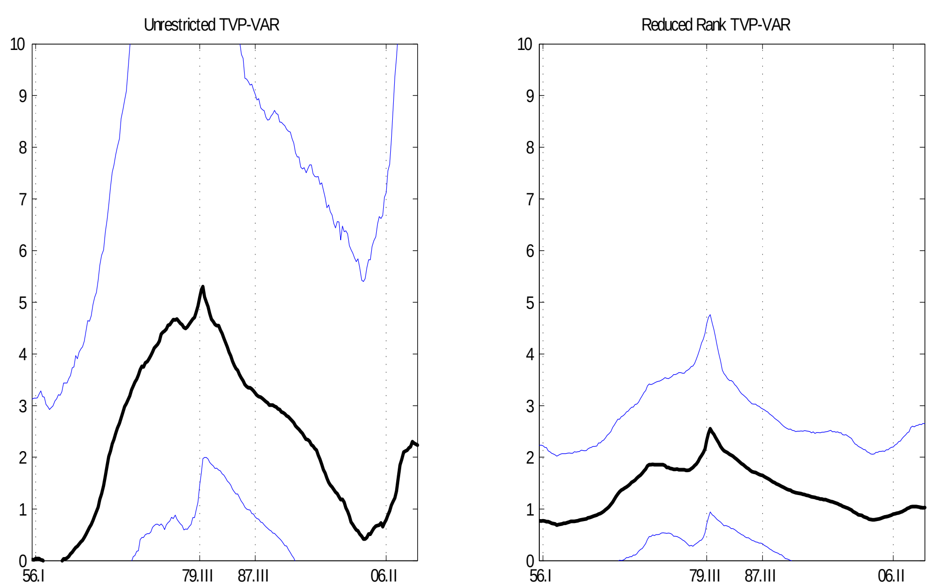

Figure 7. Monetary Policy Rule, Longrun Response of Interest Rate to Inflation Shock

5 Conclusion

We apply more powerful structural break tests and find statistically significant evidence that the coefficients in the U.S. monetary VAR are time varying. We also apply a reduced rank TVP VAR and show that there are important changes in natural rate of unemployment, core inflation, inflation persistence, and monetary policy rule. Our results differ from the previous studies in the literature in that our reduced rank restrictions reduce the number of parameters and yield tighter confidence intervals.

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