Volatility and Quantile Forecasts of Financial Returns Using Realized Stochastic Volatility Models with Generalized Hyperbolic Distribution

Makoto Takahashi (Northwestern University)
Yasuhiro Omori (University of Tokyo)
Toshiaki Watanabe (Hitotsubashi University)

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Introduction

Joint modelling of daily returns and RV

- Realized SV
  - Takahashi, Omori and Watanabe (2009)
  - Dobrev and Szerszen (2010)
  - Koopman and Scharth (2012)
- Realized GARCH
  - Hansen, Huang and Shek (2012)
- Why is joint modelling needed?
  - Adjusting the bias of RV caused by microstructure noise and non-trading hours.
  - Estimating the parameters in return equation jointly with the parameters in volatility equation.

Introduction

Purpose of this paper

- We examine whether the realized SV model will improve the performance of volatility and quantile forecasts.

Return distribution

- For quantile forecast, not only volatility but also return distribution are important.
- We use the GH skew Student's *t* distribution.

Realized SV Model

Notation

- r_t= daily financial return
- h_t= log of true volatility
- x_t= log of RV

Realized SV model

$$
\begin{aligned}
r_t &= \exp(h_t/2)\epsilon_t, \\
h_{t+1} &= \mu + \phi(h_t - \mu) + \eta_t, \\
x_t &= \xi + \psi h_t + u_t, \\
\begin{bmatrix} \epsilon_t \\ \eta_t \\ u_t \end{bmatrix} &\sim N(\mathbf{0}, \mathbf{\Sigma}), \quad \mathbf{\Sigma} = \begin{bmatrix} 1 & \rho\sigma_\eta & 0 \\ \rho\sigma_\eta & \sigma_\eta^2 & 0 \\ 0 & 0 & \sigma_u^2 \end{bmatrix}.
\end{aligned}
$$

Realized SV Model

$$x_t = \xi + \psi h_t + u_t$$

- If $\xi = 0$ and $\psi = 1$, x_t is the unbiased estimator of h_t.
- These parameters play a role to adjust the bias of RV caused by microstructure noise and non-trading hours.
- Takahashi, Omori and Watanabe (2009) set $\psi = 1$.
- In what follows, we also set $\psi = 1$ because the performance of quantile forecast is not improved by estimating ψ.

Return Distribution

GH skew Student's *t* distribution

- Suppose
  - $\epsilon_t \sim N(0, 1)$
  - $z_t \sim IG(\nu/2, \nu/2)$
  - ϵ_t and z_t are independent.
  - $\mu_z = E[z_t] = \nu/(\nu - 2)$
- Then, the distribution of $\{\beta(z_t - \mu_z) + \sqrt{z_t}\epsilon_t\}$ is called the GH skew Student's *t* distribution.
- It includes the Student's *t* distribution as a special case of $\beta = 0$.
- It collapses to the standard normal distribution when $\beta = 0, \nu \to \infty$ (i.e., $z_t = 1$ for all t).
- $r_t = \exp(h_t/2)\,\{\beta(z_t - \mu_z) + \sqrt{z_t}\epsilon_t\}$

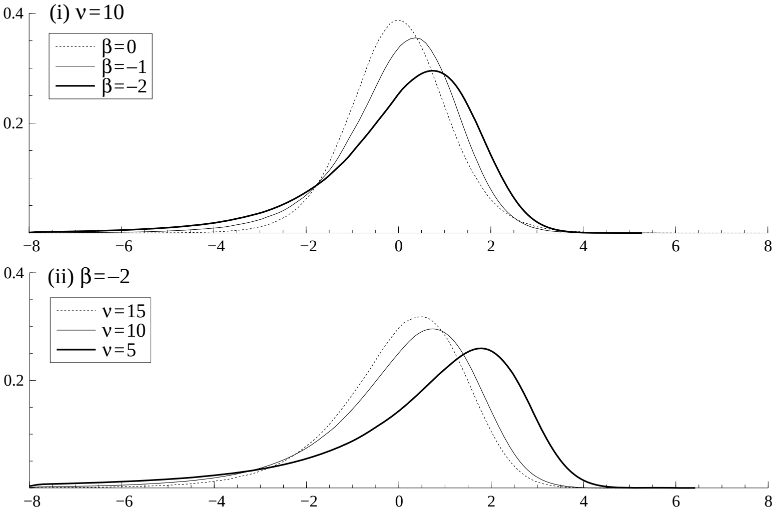
Return Distribution

GH distribution

- There is a more general distribution called the GH distribution, which includes the GH skew Student's t distribution.
- It is difficult to estimate the parameters in the GH distribution (Prause, 1999; Aas and Haff, 2006; Nakajima and Omori, 2012)
- We use the GH skew Student's t distribution in this paper.

Return Distribution

Examples of GH skew Student's *t* distribution



Return Distribution

Realized SV model with the GH skew Student's t return distribution

$$
\begin{aligned}
r_t &= \exp(h_t/2)\left\{\beta(z_t-\mu_z)+\sqrt{z_t}\epsilon_t\right\},\\
h_{t+1} &= \mu+\phi(h_t-\mu)+\eta_t,\\
x_t &= \xi+h_t+u_t,\\
z_t &\sim IG(\nu/2,\nu/2),
\end{aligned}
$$

$$
\begin{bmatrix} \epsilon_t \\ \eta_t \\ u_t \end{bmatrix} \sim N(\mathbf{0},\boldsymbol{\Sigma}), \quad \boldsymbol{\Sigma} = \begin{bmatrix} 1 & \rho\sigma_\eta & 0 \\ \rho\sigma_\eta & \sigma_\eta^2 & 0 \\ 0 & 0 & \sigma_u^2 \end{bmatrix}.
$$

- Let $\theta=(\phi,\sigma_\eta,\rho,\mu,\beta,\nu,\xi,\sigma_u)$, $y=\{r_t,x_t\}_{t=1}^n$, $h=\{h_t\}_{t=1}^n$ and $z=\{z_t\}_{t=1}^n$.

Return Distribution

- This model enables us to estimate the parameters (β, ν) for the GH skew Student's t distribution jointly with the other parameters.

- Giot and Laurent (2004) applies the ARFIMA model for RV to VaR, where they first estimate the parameters in the ARFIMA model and then estimate the parameters in the return distribution.

Bayesian Estimation Using MCMC

- We sample (θ, h, z) from their posterior distribution using the Gibbs sampler:
  0. Initialize (θ, h, z).
  1. Sample from $\phi | \sigma_\eta, \rho, \mu, \beta, \nu, \xi, \sigma_u, h, z, y$.
  2. Sample from $(\sigma_\eta, \rho) | \phi, \mu, \beta, \nu, \xi, \sigma_u, h, z, y$.
  3. Sample from $\mu | \phi, \sigma_\eta, \rho, \beta, \nu, \xi, \sigma_u, h, z, y$.
  4. Sample from $\beta | \phi, \sigma_\eta, \rho, \mu, \nu, \xi, \sigma_u, h, z, y$.
  5. Sample from $\nu | \phi, \sigma_\eta, \rho, \mu, \beta, \xi, \sigma_u, h, z, y$.
  6. Sample from $\xi | \phi, \sigma_\eta, \rho, \mu, \beta, \nu, \sigma_u, h, z, y$.
  7. Sample from $\sigma_u | \phi, \sigma_\eta, \rho, \mu, \beta, \nu, \xi, h, z, y$.
  8. Sample from $z | \theta, h, y$.
  9. Sample from $h | \theta, z, y$.
  10. Go to 1.

Bayesian Estimation Using MCMC

- We can sample from the full conditional distributions in Steps 1–9 by extending the method proposed by Takahashi, Omori and Watanabe (2009).
- We use the method proposed by Nakajima and Omori (2012) in Steps 4, 5 and 8, where the parameters (β, ν) and latent variable z_t for the GH skew Student's t distribution are sampled.
- We sample the latent variable h_t efficiently by applying the block sampler for the asymmetric stochastic volatility model proposed by Omori and Watanabe (2008).

Quantile Forecasts

Sampling one-day-ahead return

- We add the following sampling scheme after Step 9 of the Gibbs sampler to sample one-day-ahead return for quantile forecast.

  i. Generate $h_{n+1}|\theta, h, z, y \sim N(\mu_{n+1}, \sigma^2_{n+1})$, where

$$\begin{aligned}\mu_{n+1} =& \mu + \phi(h_n - \mu) \\ & + z_n^{-1/2} \exp(-h_n/2)\rho\sigma_\eta\{r_n - \beta\bar{z}_n \exp(h_n/2)\}, \\ \sigma^2_{n+1} =& (1 - \rho^2)\sigma^2_\eta.\end{aligned}$$

  ii. Generate $z_{n+1} \sim IG(\nu/2, \nu/2)$.

  iii. Generate $r_{n+1}|\theta, h_{n+1}, z_{n+1} \sim N(\hat{\mu}_{n+1}, \hat{\sigma}^2_{n+1})$, where

$$\begin{aligned}\hat{\mu}_{n+1} &= \beta(z_{n+1} - \mu_z) \exp(h_{n+1}/2), \\ \hat{\sigma}^2_{n+1} &= z_{n+1} \exp(h_{n+1}).\end{aligned}$$

  iv. Generate $x_{n+1}|\theta, h_{n+1} \sim N(\xi + \psi h_{n+1}, \sigma^2_\eta)$.

Quantile Forecasts

VaR

- $\text{VaR}_{n+1}(\alpha)$ denotes the one-day-ahead forecast for the VaR of the daily return r_{n+1} with probability α.
- We concentrate on the long position.
- Then, $\Pr[r_{n+1} < \text{VaR}_{n+1}(\alpha) | \mathcal{I}_n] = \alpha$.

Quantile Forecasts

ES

- Although the VaR has been widely used to evaluate the quantile forecast of financial returns, it only measures a quantile of the distribution and ignores the important information of the tail beyond the quantile.
- To evaluate the quantile forecast with tail information, we compute the expected shortfall (ES), which is defined as the conditional expectation of the return given that it violates the VaR.
- The one-day-ahead forecast of the ES with probability α is defined as

$$\text{ES}_{n+1}(\alpha) = \text{E}[r_{n+1} | r_{n+1} < \text{VaR}_{n+1}(\alpha), I_n].$$

Quantile Forecasts

Computation of VaR and ES

- The one-day-ahead forecasts ($\text{VaR}_{n+1}(\alpha), \ldots,$ $\text{VaR}_{n+T}(\alpha)$) and ($\text{ES}_{n+1}(\alpha), \ldots, \text{ES}_{n+T}(\alpha)$) are computed repeatedly in the following way.
  0. Set $i = 1$.
  1. Sample the model parameters and one-day-ahead return r_{n+i} from their posterior distribution conditional on the data $(y_i, \ldots, y_{n+i-1})$.
  2. Compute $\text{VaR}_{n+i}(\alpha)$ as the α-percentile of the sample of r_{n+i}.
  3. Compute $\text{ES}_{n+i}(\alpha)$ as a sample average of r_{n+i} which satisfies $r_{n+i} < \text{VaR}_{n+i}(\alpha)$.
  4. Set $i = i + 1$ and return to 1 if $i < T$.

Empirical Application

Data

- Spyder, the S&P 500 exchange-traded fund, obtained from NYSE TAQ database.
- Sample period: February 1, 2001–August 29, 2008.
- Sample size: 1,886.
- We compute daily returns as the log difference in the closing prices.
- We compute daily RV using 1-minute returns.
- We also compute daily RK (Barndorff-Nielsen et al., 2008) using 1-minute returns.
- We neglect overnight in computing RV and RK.

Empirical Application

Descriptive statistics for the full sample.

Variable	Mean	SD	Skew	Kurt	JB	LB(10)
r	-0.0033	1.0897	0.0448	5.3255	0.00	0.72
RV	0.9548	1.1375	4.3027	34.3299	0.00	0.00
RK	0.8493	1.0077	4.0660	28.4696	0.00	0.00
$\log RV$	-0.4705	0.8862	0.3627	2.6823	0.00	0.00
$\log RK$	-0.5773	0.8701	0.3849	2.8369	0.00	0.00

Empirical Application

Estimation results of realized SV model with the GH skew Student's *t* distribution and RV during the period of February 1, 2001 to February 10, 2005 (1,000 observations).

	Mean	Stdev.	95%L	95%U	CD	Inef.
ϕ	0.9759	0.0065	0.9629	0.9884	0.238	2.16
σ_η	0.1643	0.0075	0.1504	0.1796	0.812	9.55
ρ	-0.3850	0.0529	-0.4864	-0.2802	0.041	12.26
μ	-0.1652	0.2191	-0.5991	0.2741	0.694	3.19
β	0.5647	0.6460	-0.7901	1.8300	0.375	104.53
ν	24.1302	5.2360	15.4345	35.4438	0.248	125.85
ξ	0.0127	0.0509	-0.0800	0.1184	0.834	59.00
σ_u	0.2661	0.0084	0.2505	0.2834	0.765	4.08

Empirical Application

Estimation results of realized SV model with the GH skew Student's *t* distribution and RK during the period of February 1, 2001 to February 10, 2005 (1,000 observations).

	Mean	Stdev.	95%L	95%U	CD	Inef.
ϕ	0.9716	0.0069	0.9574	0.9848	0.550	2.29
σ_η	0.1761	0.0085	0.1601	0.1939	0.072	8.56
ρ	-0.4350	0.0542	-0.5362	-0.3230	0.551	11.63
μ	0.0056	0.1903	-0.3712	0.3811	0.356	3.45
β	0.1308	0.3126	-0.4947	0.7448	0.730	22.97
ν	27.1193	4.9390	18.2402	37.9927	0.074	51.66
ξ	-0.3134	0.0470	-0.4098	-0.2239	0.378	28.32
σ_u	0.3101	0.0094	0.2929	0.3295	0.434	2.26

Empirical Application

Forecasting periods

- Low volatility period: February 11, 2005–December 29, 2006.
- High volatility period: January 3, 2007–August 29, 2008.

Empirical Application

Failure rates of the VaR forecasts for the low volatility period.

Model	RM	0.5%	1%	5%	10%
SVn		0.0021	0.0021	0.0297	0.0488
SVt		0.0021	0.0042	0.0255	0.0552
SVskt		0.0000	0.0021	0.0276	0.0637
RSVn	RV	0.0064	0.0127	0.0467	0.0913
RSVt	RV	0.0042	0.0064	0.0467	0.0892
RSVskt	RV	0.0085	0.0085	0.0425	0.0807
RSVn	RK	0.0042	0.0085	0.0403	0.0786
RSVt	RK	0.0042	0.0064	0.0403	0.0786
RSVskt	RK	0.0042	0.0064	0.0403	0.0722

Empirical Application

Failure rates of the VaR forecasts for the high volatility period.

Model	RM	1%	5%	10%
SVn		0.0337	0.0771	0.1325
SVt		0.0289	0.0795	0.1446
SVskt		0.0217	0.0771	0.1398
RSVn	RV	0.0241	0.0651	0.1012
RSVt	RV	0.0120	0.0602	0.1012
RSVskt	RV	0.0120	0.0578	0.1012
RSVn	RK	0.0217	0.0699	0.1060
RSVt	RK	0.0169	0.0675	0.1036
RSVskt	RK	0.0145	0.0675	0.1036

Empirical Application

Likelihood ratio tests for VaR

- Kupiec (1995)$\cdots$ violations are independent.
- Christoffersen (1998)$\cdots$ violations follow a Markov process.
- Chritoffersen and Pelletier (2004)$\cdots$ duration follows the Weibull distribution or the exponential autoregressive conditional duration (EACD) model of Engle and Russell (1998).

Empirical Application

P-values of the Markov, Weibull and EACD tests for VaR forecasts for the low volatility period ($\alpha = 1\%$).

Model	RM	Markov	Weibull	EACD
SVn		0.0910	.NaN	.NaN
SVt		0.2355	.NaN	.NaN
SVskt		0.0819	.NaN	.NaN
RSVn	RV	0.6222	0.3211	0.7571
RSVt	RV	0.4966	0.2243	0.6414
RSVskt	RV	0.7881	0.2340	0.8037
RSVn	RK	0.7620	0.2387	0.8093
RSVt	RK	0.3925	0.2211	0.6459
RSVskt	RK	0.4234	0.2239	0.6357

Empirical Application

P-values of the Markov, Weibull and EACD tests for VaR forecasts for the low volatility period ($\alpha = 5\%$).

Model	RM	Markov	Weibull	EACD
SVn		0.0993	0.2111	0.2538
SVt		0.0409	0.4401	0.5239
SVskt		0.0683	0.1385	0.3544
RSVn	RV	0.2693	0.9577	0.1757
RSVt	RV	0.2678	0.9606	0.1727
RSVskt	RV	0.5482	0.8936	0.1441
RSVn	RK	0.3748	0.5349	0.1367
RSVt	RK	0.3700	0.5341	0.1379
RSVskt	RK	0.1093	0.8654	0.1456

Empirical Application

P-values of the Markov, Weibull and EACD tests for VaR forecasts for the low volatility period ($\alpha = 10\%$).

Model	RM	Markov	Weibull	EACD
SVn		0.0005	0.4849	0.2412
SVt		0.0039	0.6030	0.0392
SVskt		0.0349	0.6025	0.2024
RSVn	RV	0.5323	0.8663	0.0051
RSVt	RV	0.4302	0.9630	0.0039
RSVskt	RV	0.1643	0.9871	0.0615
RSVn	RK	0.1134	0.9732	0.0078
RSVt	RK	0.1116	0.9712	0.0103
RSVskt	RK	0.0261	0.9705	0.0196

Empirical Application

P-values of the Markov, Weibull and EACD tests for VaR forecasts for the high volatility period ($\alpha = 1\%$).

Model	RM	Markov	Weibull	EACD
SVn		0.0149	0.2177	0.0159
SVt		0.0182	0.3909	0.6009
SVskt		0.0642	0.5396	0.0709
RSVn	RV	0.0378	0.0535	0.9548
RSVt	RV	0.6678	0.8824	0.9791
RSVskt	RV	0.7169	0.9872	0.7384
RSVn	RK	0.0621	0.7332	0.4002
RSVt	RK	0.2095	0.4914	0.3957
RSVskt	RK	0.4370	0.9241	0.8101

Empirical Application

P-values of the Markov, Weibull and EACD tests for VaR forecasts for the high volatility period ($\alpha = 5\%$).

Model	RM	Markov	Weibull	EACD
SVn		0.0058	0.0032	0.2854
SVt		0.0036	0.0025	0.2644
SVskt		0.0061	0.0032	0.2891
RSVn	RV	0.0957	0.2898	0.2596
RSVt	RV	0.1679	0.1221	0.4072
RSVskt	RV	0.2175	0.1014	0.4697
RSVn	RK	0.0498	0.0383	0.1040
RSVt	RK	0.0683	0.0392	0.1320
RSVskt	RK	0.0687	0.0369	0.1253

Empirical Application

P-values of the Markov, Weibull and EACD tests for VaR forecasts for the high volatility period ($\alpha = 10\%$).

Model	RM	Markov	Weibull	EACD
SVn		0.0080	0.0064	0.2502
SVt		0.0003	0.0006	0.2685
SVskt		0.0016	0.0038	0.3428
RSVn	RV	0.2085	0.0213	0.2427
RSVt	RV	0.2139	0.0174	0.2434
RSVskt	RV	0.2125	0.0166	0.2526
RSVn	RK	0.1435	0.0158	0.1324
RSVt	RK	0.1812	0.0098	0.1780
RSVskt	RK	0.1826	0.0126	0.1963

Empirical Application

Backtesting measure of Embrechts, Kaufman and Patie (2005) for the ES forecasts.

- To backtest the ES forecasts with probability α, we use the measure proposed by Embrechts, Kaufmann and Patie (2005).
- $\delta_t(\alpha) = r_t - \mathrm{ES}_t(\alpha)$.
- $\kappa(\alpha) =$ set of time points for which a violation of the VaR occurs.
- $T_1 =$ number of time points in $\kappa(\alpha)$.
- $V_1(\alpha) = \frac{1}{T_1} \sum_{t \in \kappa(\alpha)} \delta_t(\alpha)$.
- This is a standard backtesting measure for the ES estimates but depends strongly on the VaR estimates without adequately reflecting the correctness of these values.

Empirical Application

- To correct for this, it is combined with the following measure, where the empirical α-quantile of $\delta_t(\alpha)$ is used in place of the VAR estimates.
- $q(\alpha) =$ empirical α-quantile of $\delta_t(\alpha)$.
- $\tau(\alpha) =$ set of time points for which $\delta_t(\alpha) < q(\alpha)$ occurs.
- $T_2 =$ number of time points in $\tau(\alpha)$.
- $V_2(\alpha) = \frac{1}{T_2} \sum_{t \in \tau(\alpha)} \delta_t(\alpha)$.
- The Embrechts, Kaufmann and Patie (2005) measure is given by
$$V(\alpha) = \frac{|V_1(\alpha)| + |V_2(\alpha)|}{2}.$$
- A good estimation of ES will lead to a low value of $V(\alpha)$.

Empirical Application

Backtesting measure of Embrechts, Kaufman and Patie (2005) for the ES forecasts for the low volatility period.

Model	RM	1%	5%	10%
SVn		0.409	0.437	0.256
SVt		0.320	0.412	0.271
SVskt		0.577	0.353	0.298
RSVn	RV	0.058	0.040	0.024
RSVt	RV	0.077	0.043	0.022
RSVskt	RV	0.036	0.027	0.034
RSVn	RK	0.054	0.035	0.037
RSVt	RK	0.072	0.058	0.043
RSVskt	RK	0.084	0.068	0.049

Empirical Application

Backtesting measure of Embrechts, Kaufman and Patie (2005) for the ES forecasts for the high volatility period.

Model	RM	1%	5%	10%
SVn		0.320	0.439	0.329
SVt		0.254	0.376	0.280
SVskt		0.155	0.295	0.230
RSVn	RV	0.275	0.189	0.187
RSVt	RV	0.346	0.142	0.139
RSVskt	RV	0.291	0.132	0.122
RSVn	RK	0.314	0.187	0.185
RSVt	RK	0.214	0.118	0.146
RSVskt	RK	0.217	0.099	0.133

Conclusions

1. The realized SV model performs better than the SV model at least for the low volatility periods.

2. The GH skew Student's *t* distribution performs better than the *t* and normal distributions for the both periods.

3. The realized SV model with RK does not perform better than that with RV, indicating that the realized SV model can adjust the bias caused by microstructure noise well.

Extensions

1. Realized range-based volatility
   - Christensen and Podolskij (2007)
   - Martens and van Dijk (2007)
   - Divide a day into n intervals.
   - m prices are observed in each interval.
   - $p^H_{i,t}$: highest price in the ith interval on day t.
   - $p^L_{i,t}$: lowest price in the ith interval on day t.
   - Realized range-based volatility:

$$RRV_t = \frac{1}{\lambda_m} \sum_{i=1}^{n} \left\{ \log(p^H)_{i,t} - \log(p^L_{i,t}) \right\}^2$$

   - $\lambda_m \to 4\ln(2)$ as $m \to \infty$.
   - λ_m cannot be obtained analytically if m is finite (Christensen. Podolskij and Vetter, 2009).

Extensions

2. Jump

3. Long memory

4. Multiperiod forecasts

5. Comparison with the other models
    - ARFIMA and HAR models for RV
    - Realized GARCH (Watanabe, 2012)

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