

# Reexamination of Dynamic Beta International CAPM: a SUR with GARCH Approach

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## Abstracts

Considering the heteroscedasticity and cross-correlation in the error terms of international stock market returns, International Capital Asset Pricing Model (CAPM) is reinvestigated under Seemingly Unrelated Regression (SUR) and SUR with GARCH (SUR-GARCH) framework. We modified Feasible Generalized Least Square (FGLS) estimator to take into account multivariate GARCH error structure in estimating the model. World market portfolio was constructed to ensure that the market portfolio is mean-variance efficient under no restriction on short selling and borrowing at riskless rate. CAPM fits well only on *ex-post* SUR test, but it is rejected on SUR-GARCH for both *ex-ante* and *ex-post* test. However, this paper found that CAPM could be applied for most stock market indexes when each equation in SUR system was analyzed individually.

*JEL Classification: G11, G12, G15*

*Keywords: CAPM, Time Varying Beta, Seemingly Unrelated Regression (SUR), multivariate GARCH*

## 1. Background

The basic objective of forming a portfolio is to diversify the assets' risks and return so that the portfolio attains a particular target of expected return with minimum variance or attains maximum expected return within specified variance. The diversification may take place within a market by mixing assets that have non-perfect correlation to each other. In a broader setting, the assets included in the diversification process may come from different markets and even from across the borders of economies or countries. In this setting, when the markets become more integrated, the comovements of assets' returns tend to be higher.

The main motivation of this paper is to investigate the presence of benefit of forming internationally well-diversified portfolio. Capital Asset Pricing Model (CAPM) was applied to examine whether investors in different stock markets require different international market risk premium for each offshore asset (proxied by national stock market indexes).

The results are expectedly able to show the effects of recent developments in various stock markets and economies, especially to see the impact of policies that foster freer flow of capital across the borders. By constructing internationally diversified portfolio (world market portfolio) that is mean-variance efficient and at tangent of security market line, International CAPM was tested. Assuming that stock markets in the sample are perfectly integrated, prices of market (beta) risk across the stock markets were tested with the null hypothesis that the risk premiums are homogenous. The world market portfolio was constructed by estimating expected returns and conditional variance-covariance matrix at every observation using Vector Error Correction Model with GARCH (VEC-GARCH) model. The portfolio is derived from an efficient frontier such that it always be mean-variance efficient. Market risk, beta, is assumed to vary over time and measured based on the conditional covariance between the asset and the world market portfolio. The international assets pricing model was examined under seemingly unrelated regression (SUR) and SUR-GARCH framework. To take into account the multivariate GARCH error structure, standard FGLS needs to be adjusted, we call it modified FGLS estimator (mFGLS).

## 2. Literature Review

Debate over the usefulness of the most popular asset pricing model, the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), took place since the model itself was introduced. Fama and French (2004) summarized some important empirical tests on CAPM, concluding that tests based on cross-section regression (i.e. Fama and MacBeth, 1973) and time series regression (i.e. Gibbons, Ross and Shanken, 1989) do not test the CAPM. Those tests only examine whether a specific proxy for the market portfolio is efficient. Furthermore, they concluded that CAPM has never been tested, and the prospects for testing it are not satisfactory. Previous literatures supporting the findings, among others are Fama and French (1992) and Chan, Hamao and Lakanishok (1991) that argued market risk beta is not the only factor that explains the expected return of an asset, and Kothari, Shanken and Sloan (1995) for finding the weak relationship between beta and expected return. Other supporting arguments for rejecting the CAPM are unrealistic assumption (homogeneous expectation and investors' sole attention to the mean-variance of one-period portfolio returns) and presence of market anomalies (i.e. overreaction hypothesis of deBondt and Thaler (1987)). The alternative models to CAPM that partly tackles such issues are Arbitrage Pricing Theory (APT) and Intertemporal CAPM.

However, apart from those critics, the standard CAPM is still the most practically applied and widely accepted by financial practitioners. Following are some recent literatures supporting the CAPM. Levy (2010) shows that CAPM works under both expected utility theory framework and prospect theory framework. Test based on *ex-post* data by Levy (2008) also show that the weak relationship between beta and expected return takes place on the sub-

period selected for such testing, it implies that CAPM may work well in other particular periods.

There is also important caveat for testing CAPM that is the appropriateness of the proxy of market portfolio. The CAPM test based on cross-section regression as in Fama and MacBeth (1973), assumes that market is efficient, the homogeneous expectation is prevail and market is in an equilibrium state, such that according to Black (1972) any weighted average market portfolio is always efficient and it is a good proxy for the market portfolio. However, the assumptions are the main critic of the model, so that rejection of the test model might be interpreted as the violation of the assumption (that the proxy for market portfolio is inefficient) and not the rejection of the CAPM itself. Haugen and Baker (1991) show the evidence that such market weighted index is not efficient and thus market-matching strategy is not a good investment strategy. This issue becomes relevant when CAPM is applied for pricing international assets (such as Exchange Traded Fund (ETF) that directly tracks stock market index in different country), such that using synthetic world market index (e.g. MSCI World Index) does not guarantee that the index represent the true market portfolio in which all assumptions behind the CAPM are put upon it. This issue already discussed in Black and Litterman (1992) that analyzed global portfolio optimization methods. More on this issue will be discussed in section of research method. Previous works on international asset pricing that utilize such world index can be found in the work of French and Poterba (1991), Fama and French (1998), Das and Uppal (2004), Fernandes (2005), and Wu (2008) among other abundant literatures in this area.

The development in econometric area such as non-stationary model and heteroscedasticity model has contributed in testing the CAPM and its extension. Bollerslev, Engle and Wooldridge (1988) applied GARCH model to test conditional (time varying beta) CAPM. Engel and Rodrigues (1989) tested International CAPM using time varying covariance, and found that the estimation method performed is much better than that if constant variances were used. Tsuji (2009) tested conditional CAPM and Conditional Consumption-CAPM using Japan datasets and found that conditional CAPM works better than the competing model. On the contrary, Kumar, Sorescu, Boehme and Danielsen (2008) found that multi factors of risk help explain the expected return in the US markets instead of the single market risk in the conditional CAPM. The result confirmed findings in Lewellen and Nagel (2006).

### **3. Methods**

The proposed test model is aimed to examine the relationship between expected returns of national stock market indexes and the world market portfolio returns. The national stock market indexes are weighted average of the constituent stocks prices based on either market capitalization (e.g. S&P500 Index) or liquidity (e.g. Nikkei 225). The riskless asset is proxied by government securities; 3-month T-Bill.

In previous international CAPM literatures, MSCI world index or other Exchange Traded Fund (ETF) that consists of national market indexes were used as proxy of the world market portfolio. It should be noted that such index weighs the composing asset based on market capitalization or liquidity where the weight is always nonnegative. It means that the world market portfolio consists of assets in long position. Meanwhile CAPM assumes that unrestricted short selling of those assets is allowed. One may argue that we can short sell the index instead of short selling its composing assets. However, the strategy of short selling the world market index does not ensure us that the portfolio is efficient and at tangent of capital market line. To overcome these problems, world market portfolio is constructed following Merton (1972) procedure and Tobin's separation theory (Tobin, 1958) to guarantee that the

portfolio is not only mean-variance efficient, but also located at a point which is at tangent of the capital market line.

### 3.1. Expected return and Conditional Variance-Covariance Matrix of Each Asset

Considering that the stock markets has long-run equilibrium with the other markets and disturbance errors of the estimation model are correlated and heteroscedastic, vector error correction model with GARCH (VEC-GARCH Model) is applied to estimate the expected returns of each national market index and their conditional variance-covariance matrix. The VEC-GARCH model consists of mean equations and variance equations as follows.

The mean equations (the unrestricted VECM) is

$$\mathbf{R}_t^d = \widehat{\mathbf{C}} + (\widehat{\mathbf{\Pi}})\mathbf{M}_{t-1}^d + \widehat{\mathbf{\Phi}}\mathbf{R}_{t-1}^d + \widehat{\boldsymbol{\varepsilon}}_t \quad (1)$$

where,

$\mathbf{R}_t^d = [\Delta m_{1,t} \Delta m_{2,t} \dots \Delta m_{i,t} \dots \Delta m_{N,t}]'$  is vector of first order difference of log national market indexes at time t, where  $\Delta m_{i,t} = r_{i,t} = \log\left(\frac{m_{i,t}}{m_{i,t-1}}\right)$  is also national market return at time t.

$\mathbf{M}_{t-1}^d = [m_{1,t-1} \dots m_{i,t-1} \dots m_{N,t}]'$  is vector of first order lagged of log national market indexes

$\widehat{\mathbf{C}} = [\widehat{c}_1 \widehat{c}_2 \dots \widehat{c}_i \dots \widehat{c}_N]'$  is vector of constant terms

$\widehat{\mathbf{\Pi}}$  =  $N \times N$  matrix of error correction coefficients. When  $\text{rank}(\widehat{\mathbf{\Pi}}) < N$ ,  $\widehat{\mathbf{\Pi}}$  can be decomposed into  $\mathbf{AB}$  by Granger representation theorem, where  $\mathbf{A}$  is vector of coefficient of cointegrating equation (adjustment parameters) and,  $\mathbf{B}$  is vector of cointegrating coefficient.

$\widehat{\mathbf{\Phi}} = \begin{bmatrix} \varphi_{11} & \dots & \varphi_{1N} \\ \vdots & \ddots & \vdots \\ \varphi_{P1} & \dots & \varphi_{NN} \end{bmatrix}$  is a  $N \times N$  matrix of VAR parameters

$\widehat{\boldsymbol{\varepsilon}}_t = [\varepsilon_1 \varepsilon_2 \dots \varepsilon_i \dots \varepsilon_N]$  is the vector of disturbance errors, where  $\boldsymbol{\varepsilon}_t \sim (0, \widehat{\mathbf{H}}_t)$

and the variance equations (Diagonal BEKK Model, Engle and Kroner (1995)) is

$$\widehat{\mathbf{H}}_t = \widehat{\boldsymbol{\Psi}}\widehat{\boldsymbol{\Psi}}' + [\widehat{\mathbf{A}}_1\widehat{\mathbf{A}}_1'] \odot [\widehat{\boldsymbol{\varepsilon}}_{t-1}\widehat{\boldsymbol{\varepsilon}}_{t-1}'] + [\widehat{\mathbf{A}}_2\widehat{\mathbf{A}}_2'] \odot \widehat{\mathbf{H}}_{t-1}, \quad (2)$$

where,  $\widehat{\mathbf{H}}_t$  is  $N \times N$  conditional variance-covariance matrix (its diagonal elements are conditional variances,  $[\widehat{\sigma}_t^2(r_i)]_{ii}$ , and the off-diagonal elements are conditional covariances,  $[\widehat{\sigma}_t(r_i r_j)]_{ij}$ , where  $i \neq j$ , for  $i$  and  $j = [1 N]$ ),  $\widehat{\boldsymbol{\Psi}}\widehat{\boldsymbol{\Psi}}'$ ,  $\widehat{\mathbf{A}}_1\widehat{\mathbf{A}}_1'$ , and  $\widehat{\mathbf{A}}_2\widehat{\mathbf{A}}_2'$  are diagonal matrix of constants, coefficients of ARCH terms, and coefficients of GARCH terms respectively, and  $\odot$  is element by element (Hadamard) product operator.

The parameters in the mean equations and the variance equations theoretically can be estimated by maximum likelihood estimator (MLE). However, when the system is large as in our case, MLE often produces inaccurate results because too many parameters need to be estimated such that the optimization of the log likelihood function failed. To overcome this problem, the mean equation (VECM) parameters were estimated as those in Seemingly Unrelated Regression (SUR) system using modified feasible generalized least square (mFGLS) estimator that taking into account the GARCH error structure. This estimation strategy was also used in testing the CAPM and shall be explained later.

For estimating conditional variance of realized return, the mean equation in equation (1) was replaced by  $\tilde{\mathbf{R}}_t = \hat{\mathbf{C}} + \hat{\boldsymbol{\varepsilon}}_t$  and the conditional variance-covariance matrix was estimated by Diagonal BEKK. Henceforth, accent “ $\tilde{\phantom{x}}$ ” and “ $\hat{\phantom{x}}$ ” are used for indicating variable based on the realized return and the estimated expected return respectively.

### 3.2. World Market Portfolio ( $M$ ) Formation

World market portfolio was constructed by assuming that unrestricted short selling and borrowing at riskless rate in domestic or national market are allowed. The assumptions were made to follow the underlying assumptions in CAPM.

The proportion of each asset in an efficient portfolio was obtained by minimizing objective function of portfolio variance with respect to following constraints: [1] a set of target portfolio expected return and, [2] the sum of proportion of each asset (including riskless asset) is equal to one. When short selling is prohibited, constraint [2] is modified by adding restriction on proportion of each risky asset to vary between 0 to 1, yet in this paper the proportion is unrestricted to indicate that the short selling can be done without any restriction.

Suppose that country  $i$  is our focus of analysis and call it home country. Portfolio  $P$  consists of riskless asset available at domestic market  $i$  and  $N$  international risky portfolios ( $m_1^d, \dots, m_i^d, \dots, m_N^d$ ). The rate of return of  $P$  is the weighted average of rate of return of its composing assets. Our objective is to construct world market portfolio denoted by  $M$  that consists of risky portfolios only (proxied by market indexes). Let us define  $r_{f,t}$ ,  $\boldsymbol{\omega}_t = (\omega_{1,t} \dots \omega_{i,t} \dots \omega_{N,t})'$ , and  $\mathbf{e}$  as riskless rate of return, vector of proportion of risky assets in portfolio  $M$  and vector of ones respectively. Constraint [2] implies that  $(1 - \boldsymbol{\omega}'\mathbf{e})$  is the proportion of riskless asset in portfolio  $P$ . Applying constraint [2] to the expected return of risk-free asset and risky assets definition, the expected return of  $P$  may be stated as:

$$\check{r}_t^P = [r_{f,t} + \boldsymbol{\omega}'_t(\tilde{\mathbf{R}}_t^d - r_{f,t}\mathbf{e})] \quad (3)$$

Having conditional variance-covariance matrix  $\hat{\mathbf{H}}_t$  from (2), variance of portfolio  $P$  at time  $t$  is computed by,

$$\hat{\sigma}_t^2(\check{r}^P) = \boldsymbol{\omega}'_t \hat{\mathbf{H}}_t \boldsymbol{\omega}_t. \quad (4)$$

The optimal weight of the  $N$  risky assets and risk-free asset was obtained by solving following optimization problem:

$$\min_{\boldsymbol{\omega}_t} \frac{1}{2} \boldsymbol{\omega}'_t \hat{\mathbf{H}}_t \boldsymbol{\omega}_t + \lambda \{ \check{r}_t^P - [r_{f,t} + \boldsymbol{\omega}'_t(\tilde{\mathbf{R}}_t^d - r_{f,t}\mathbf{e})] \}. \quad (5)$$

The first-order condition of (5) leads to following solution:

$$\boldsymbol{\omega}_t^* = \lambda \hat{\mathbf{H}}_t^{-1} (\tilde{\mathbf{R}}_t^d - r_{f,t}\mathbf{e}). \quad (6)$$

Taking  $\boldsymbol{\omega}_t^*$  from (6) and apply the  $\mathbf{e}'\boldsymbol{\omega}_t^* = 1$  restriction, we may obtain  $\lambda$ :

$$\begin{aligned} \mathbf{e}'\boldsymbol{\omega}_t^* &= \mathbf{e}'[\lambda \hat{\mathbf{H}}_t^{-1} (\tilde{\mathbf{R}}_t^d - r_{f,t}\mathbf{e})] = 1 \\ \lambda &= \alpha = [\alpha - \delta r_{f,t}]^{-1} \end{aligned} \quad (7)$$

where  $\alpha = \tilde{\mathbf{R}}_t^d \hat{\mathbf{H}}_t^{-1} \mathbf{e}$  and  $\delta = \mathbf{e}' \hat{\mathbf{H}}_t^{-1} \mathbf{e}$ .

From (6), the expected return of risky portfolio  $M$  is  $\check{r}_t^M = \boldsymbol{\omega}_t^* \check{\mathbf{R}}_t^d$  and the variance of portfolio  $P$  will be equal to the variance of portfolio  $M$  defined as  $\hat{\sigma}_t^2(\check{r}^P) = \hat{\sigma}_t^2(\check{r}^M) = \boldsymbol{\omega}_t^* \hat{\mathbf{H}}_t \boldsymbol{\omega}_t^*$ . Define  $\hat{\boldsymbol{\sigma}}_t(\check{r}^M)$  as  $nx1$  vector of covariance of the tangency portfolio  $M$  with each of the risky asset. Then using (6) and (7), we have

$$\hat{\boldsymbol{\sigma}}_t(\check{r}^M) = \hat{\mathbf{H}}_t \boldsymbol{\omega}_t^* = m(\check{\mathbf{R}}_t^d - r_{f,t} \mathbf{e}) \quad (8)$$

Pre-multiply (8) by  $\boldsymbol{\omega}_t^{*'}$  we have  $\hat{\sigma}_t^2(\check{r}^M)$  restated as

$$\hat{\sigma}_t^2(\check{r}^M) = \boldsymbol{\omega}_t^{*'} \hat{\boldsymbol{\sigma}}_t(\check{r}^M) = m \boldsymbol{\omega}_t^{*'} (\check{\mathbf{R}}_t^d - r_{f,t} \mathbf{e}) = m(\check{r}_t^M - r_{f,t}) \quad (9)$$

Rearranging (8) and substituting in for  $m$  from (9) we have the CAPM:

$$(\check{\mathbf{R}}_t^d - r_{f,t} \mathbf{e}) = \frac{1}{m} \hat{\boldsymbol{\sigma}}_t(\check{r}^M) = \frac{\hat{\boldsymbol{\sigma}}_t(\check{r}^M)}{\hat{\sigma}_t^2(\check{r}^M)} (\check{r}_t^M - r_{f,t}). \quad (10)$$

The LHS of (10) is the expected excess return from each asset, while on the RHS,  $\frac{\hat{\boldsymbol{\sigma}}_t(\check{r}^M)}{\hat{\sigma}_t^2(\check{r}^M)} = \hat{\boldsymbol{\beta}}_t$  is vector of time varying betas of each risky asset, and  $(\check{r}_t^M - r_{f,t})$  is the expected market risk premium that prevails for all risky assets. Note that because we are assuming that short selling is unrestricted,  $\check{r}_t^M$  is always nonnegative, and the portfolio  $M$  is always in the efficient frontier of portfolio  $P$  (the risk-free and risky assets portfolio). However, elements of  $\check{\mathbf{R}}_t^d$ , the estimated expected return of each asset could be positive or negative. When the expected return of an asset is negative, it will be more likely to be short sold. Thus,  $\hat{\boldsymbol{\sigma}}_t(\check{r}^M)$  is not always positive. As a result we may find that an asset's beta and the beta risk premium is negative<sup>1</sup>.

### 3.3. Testing Conditional CAPM

The capital asset pricing model in equation (10) will serve as our test model. In addition, because we consider international assets, we must put additional risk factor other than the world market risk (represented by the betas) that indicates the required adjustment for the excess return. In this paper we include exchange rate returns in the model. We can consider the international CAPM being tested in this paper is involving Exchange Traded Funds (ETFs) that track directly the respective stock market indexes. Therefore, like the CAPM test for assets traded in one market, we can ignore the transaction cost of acquiring the cross-border assets. The test model is defined in a system equation as follows:

$$\mathbf{ER}_t = \hat{\boldsymbol{\alpha}} + \hat{\boldsymbol{\beta}}_t' \hat{\boldsymbol{\theta}} + \boldsymbol{\Xi}_t' \hat{\boldsymbol{\xi}} + \boldsymbol{\eta}_t \quad (11)$$

where  $\mathbf{ER}_t$  and  $\hat{\boldsymbol{\beta}}_t$  are vector of excess returns and market betas as defined in (10),  $\boldsymbol{\Xi}_t$  is vector of exchange rate returns for the respective markets. The vectors of estimated coefficients are  $\hat{\boldsymbol{\alpha}}$ ,  $\hat{\boldsymbol{\theta}}$ , and  $\hat{\boldsymbol{\xi}}$ .

CAPM is said works well when all elements in  $\hat{\boldsymbol{\alpha}}$  are statistically not different from zero (the test does not reject  $H_0^1: \hat{\alpha}_i = 0, \forall i$ ). However, evaluating  $\hat{\alpha}_i$  individually shall show the applicability of CAPM for pricing that asset. In addition, because of the fully integrated market assumption, we expect that the (beta) market risk premium for every markets are homogenous. However, since short selling is allowed, the negative betas and risk premiums

<sup>1</sup> See Pennacchi (2008) pp. 37-60 for more detailed derivation of the market portfolio.

are possible. Thus, the homogeneity test was carried out by taking the absolute values of the premium (the test does not reject  $H_0^2: |\hat{\theta}_1| = |\hat{\theta}_2| = \dots = |\hat{\theta}_N|$ ). Rejection of the null hypothesis indicates that markets are not fully integrated, in other words, the risk is priced differently for different assets; a violation of the law of one price. The elements in  $\hat{\xi}$  show additional risk price required with respect to the exchange rate changes. As exchange rate policies are different across the countries, we expect that the estimated coefficient in  $\hat{\xi}$  will be higher for countries that adopt free float regime than those that adopt fixed exchange rate or dollar pegged regime. Moreover, exchange rates against US Dollar in emerging markets are also tend to be more volatile than those in developed countries, thus it is also expected that the estimated coefficient is significantly different from zero for countries with non-fixed exchange rate regime.

Under fully integrated market assumption the unexpected returns or shocks in one stock market may affect or spill over to the others. Moreover, we also found common cyclicality of business cycles in the stock markets. Therefore, we are assuming that the error terms  $\eta_t$  has multivariate GARCH error structure. In order to estimate the parameters, we apply SUR with GARCH (SUR-GARCH) estimation. Estimation from the standard SUR was also presented to see the effect of ignoring the GARCH error structure.

#### 4. Estimation Strategy

Equation (1) and (11) can be restated as SUR model. For simplicity, we will use system equation (11) as a sample to explain the estimation strategy.

Let us define  $ER_i$  as  $T$ -vector of excess return of asset- $i$ , matrix  $X_i = [e, \hat{\beta}_i, \Xi_i]$  is vector of independent variables, where its respective elements are  $T$ -vector of ones,  $T$ -vector of time varying beta for asset- $i$ , and  $T$ -vector of exchange rate changes for market- $i$ , and  $\Gamma_i = [\alpha_i, \theta_i, \xi_i]$  is vector of coefficients for equation- $i$ . Then, equation- $i$  in the system equation (11) can be restated as follows:

$$ER_i = X_i \Gamma_i + \eta_i, \quad i = 1, \dots, N \quad (12)$$

where  $\eta_i$  is  $T$ -vector of the disturbance errors for the equation. In stacked model, the system equation (11) can be restated as follows:

$$\begin{bmatrix} ER_1 \\ ER_2 \\ \vdots \\ ER_N \end{bmatrix} = \begin{bmatrix} X_1 & 0 & \dots & 0 \\ 0 & X_2 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & X_N \end{bmatrix} \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \\ \vdots \\ \Gamma_N \end{bmatrix} + \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_P \end{bmatrix},$$

In general, the corresponding matrices define the following system,

$$Y = X\Gamma + \eta. \quad (13)$$

VECM in system equation (1) also can be stated similar to the above system equation by redefining the  $X$  and  $\Gamma$  accordingly. To reduce the number of parameters needs to be estimated in the VEC-GARCH, we first estimate the VECM (without GARCH), create series of cointegrating equation (we assume that there is only one cointegrating equation), and use it as new variable in a VAR system. Thus, the  $X_i$  for system equation (1) defined as  $X_i = [e, CE, R_{i,t-1}^d]$ , where  $CE$  is  $T$ -vector of cointegrating series that applied for every  $i$ .

#### 4.1. Feasible Generalized Least Square (FGLS) SUR Estimation

FGLS or also known as Zellner's estimator (Zelner, 1962), assumes that  $E[\boldsymbol{\eta}_i | \mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_N] = 0$  (strict exogeneity of  $\mathbf{X}_i$ ), and  $E[\boldsymbol{\eta}_i \boldsymbol{\eta}_i' | \mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_N] = \sigma_{ii} \mathbf{I}_T$  (homoscedasticity). As stock markets are assumed to be fully integrated, the disturbances might be correlated across equations. Therefore,  $E[\eta_{it} \eta_{js}' | \mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_N] = \sigma_{ij}$  for  $t = s$  and 0 for  $t \neq s$ . The  $\sigma_{ij}$  is covariance between disturbances  $i$  and  $j$ ; it is  $ij$ th element of variance-covariance matrix  $\boldsymbol{\Sigma}$ . Let us also define  $\boldsymbol{\Omega} = \boldsymbol{\Sigma} \otimes \mathbf{I}$ . The generalized least square estimator under the covariance structures assumption is

$$\hat{\boldsymbol{\Gamma}} = [\mathbf{X}'(\hat{\boldsymbol{\Sigma}} \otimes \mathbf{I})^{-1} \mathbf{X}]^{-1} [\mathbf{X}'(\hat{\boldsymbol{\Sigma}} \otimes \mathbf{I})^{-1} \mathbf{Y}]. \quad (14)$$

Because  $\sigma_{ij}$  is generally unknown, it is estimated by  $\hat{\sigma}_{ij} = s_{ij} = \frac{\hat{\boldsymbol{\eta}}_i' \hat{\boldsymbol{\eta}}_j}{T}$  where  $\hat{\boldsymbol{\eta}}_i$  is vector of residuals in equation  $i$ . By doing so, the estimated variance-covariance matrix  $\hat{\boldsymbol{\Sigma}}$  can be computed. The FGLS estimator requires inversion of matrix  $\hat{\boldsymbol{\Sigma}}$ , so that the matrix must have a non-zero discriminant.

The standard errors of the parameters were estimated by taking the square root of elements in sampling variances:

$$\text{Var}[\hat{\boldsymbol{\Gamma}} | \mathbf{X}] = \hat{\sigma}^2 (\mathbf{X}' \hat{\boldsymbol{\Omega}}^{-1} \mathbf{X})^{-1} \quad (15)$$

$$\text{where, } \hat{\sigma}^2 = \frac{\hat{\boldsymbol{\eta}}_i' \hat{\boldsymbol{\eta}}_j}{T-N} = \frac{(\mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\Gamma}})' \hat{\boldsymbol{\Omega}}^{-1} (\mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\Gamma}})}{T-N}.$$

The joint hypotheses of  $H_0^1$  and  $H_0^2$ , were tested by Wald coefficient test with  $J$  degree of freedom, where  $J$  is  $N$  and  $N-1$  respectively. The restriction is defined by  $\mathbf{R}\hat{\boldsymbol{\Gamma}} = \mathbf{q}$ , where  $\mathbf{R}$  is  $(J \times K)$  matrix of restriction with  $K$  is number of the parameters in  $\hat{\boldsymbol{\Gamma}}$ , and  $\mathbf{q}$  is  $J$ -vector of the true values. The Wald statistic is  $\chi^2[J]$  distributed and computed by

$$W[J] = (\mathbf{R}\hat{\boldsymbol{\Gamma}} - \mathbf{q})' [\mathbf{R} \hat{\sigma}^2 (\mathbf{X}' \hat{\boldsymbol{\Omega}}^{-1} \mathbf{X})^{-1} \mathbf{R}'] (\mathbf{R}\hat{\boldsymbol{\Gamma}} - \mathbf{q}). \quad (16)$$

#### 4.2. Modified Feasible Generalized Least Square (mFGLS) SUR-GARCH Estimation

Considering that system equation (1) and (11) are estimated in fully integrated markets and there were shocks and crises during the observation periods that spilled over among the samples, the multivariate GARCH error structure should be considered. To do so, following is steps to include the error structure for estimating the parameters in the models:

1. Estimate the mean equations by first ignoring the GARCH error structure and obtain the residuals.
2. Use the residuals to estimates conditional variance-covariance matrix by using Diagonal BEKK model. At every observation  $t$ , we have  $\hat{\mathbf{H}}_t$  with elements of  $\hat{h}_{ij,t}$ .
3. Use the variance-covariance matrix from step 2 to construct  $\hat{\boldsymbol{\Omega}}$ . Note that  $\hat{\boldsymbol{\Omega}}$  in (14) is defined as  $\hat{\boldsymbol{\Sigma}} \otimes \mathbf{I}$  where the diagonal elements are the vector of variances of each equation (which is a constant variance) and the off-diagonal elements are all zeros (there is no covariance across the equations). The modified  $\hat{\boldsymbol{\Omega}}$  at this step is considering the heteroscedasticity and covariance across the equations. To illustrate it in a simple example, for  $N=3$ ,  $\hat{\boldsymbol{\Omega}}$  is:



Replacing  $\hat{h}_{ij,t}$  with  $\hat{h}_t^{ij}$  in  $\hat{\Omega}$  we have easily obtain  $\hat{\Omega}^{-1}$  without inverting a large matrix  $\hat{\Omega}$ .

Having the modified  $\hat{\Omega}$  and  $\hat{\Gamma}$  the hypotheses can be tested using the Wald Test as described previously. The performance of the modified FGLS estimator has been examined by carrying out Monte Carlo simulation. The modified FGLS estimator is still an unbiased estimator and it is more efficient and consistent than the standard FGLS estimator when multivariate GARCH error structure does exist (Maekawa and Setiawan, 2012).

## 5. Data

Stock market index from 12 economies were collected with its respective currency. The indexes represent 6 developed stock markets: United States S&P500 (US), Germany DAX (GE), Hong Kong Hang Seng (HK), Japan Nikkei225 (JP), Singapore Strait Times (SI) and FTSE100 (UK), and 6 emerging markets: Argentina MerVal (AR), Brazil BOVESPA (BR), China SSEC (CH), Indonesia IDX composite (ID), Malaysia KLSE composite (MA), and Mexico IPC (ME). The market indexes are exchange rate adjusted, with US Dollar as the home currency.

The dataset starts from July 1997 to July 2012 and in weekly basis for avoiding non-synchronous trading time effect. Data were collected from Yahoo Finance service through its website. Because weekly data is used, and it is assumed that portfolios rebalancing are done weekly, the returns are not including dividends. Most of the stock market indexes are value-weighted indexes and the remaining are equally weighted index and top performers' index. However, the indexes used in this paper are assumed sufficient in representing the market portfolio in the respective markets because the indexes used to be regarded as the market references. As the US is regarded as home country, US 3-month T-Bills is used as the risk-free rate.

## 6. Findings

### 6.1. Data Properties

Based on Augmented Dickey-Fuller (ADF) Tests and Common Unit Root Tests performed for data in level (log market index) and its first difference (return), the results indicate that all series in level are non-stationary (except for Indonesia and Malaysia when intercept and trend are included), but all series in its first difference are stationary.

Granger causality test for returns of the US Dollar adjusted market indexes were performed, and the results are shown in Table 1. It indicates that US stock market Granger-caused the other markets (except for China, Malaysia and Mexico). The result implies that US stock market is still very dominant and has greater influence to other markets in the world.

**Table 1** Granger Causality Test for Stock Markets Returns

	US	GE	HK	JP	SI	UK	AR	BR	CH	ID	MA	ME
US		<b>0.031</b>	<b>0.000</b>	<b>0.014</b>	<b>0.001</b>	<b>0.006</b>	<b>0.078</b>	<b>0.000</b>	0.072	<b>0.000</b>	0.091	0.139
GE	0.579		<b>0.010</b>	0.696	<b>0.024</b>	0.942	0.107	<b>0.008</b>	<b>0.007</b>	<b>0.001</b>	<b>0.014</b>	<b>0.744</b>
HK	0.293	0.638		0.432	<b>0.042</b>	0.338	0.067	<b>0.006</b>	0.091	<b>0.000</b>	<b>0.027</b>	0.355
JP	0.243	0.500	0.157		<b>0.049</b>	0.522	0.509	0.277	0.120	<b>0.004</b>	<b>0.016</b>	0.583
SI	0.548	0.892	0.753	0.807		0.501	0.110	0.055	0.082	<b>0.000</b>	0.092	0.133
UK	0.253	0.784	0.052	0.411	<b>0.018</b>		0.156	<b>0.003</b>	<b>0.017</b>	<b>0.012</b>	<b>0.039</b>	0.894
AR	0.279	0.178	0.088	0.462	0.071	0.644		0.194	<b>0.017</b>	0.076	0.349	0.614
BR	0.976	0.107	0.180	0.368	0.201	0.229	0.743		0.083	<b>0.001</b>	0.423	0.413
CH	0.126	0.495	0.085	0.703	0.286	0.246	0.663	0.589		0.389	0.954	0.923
ID	0.717	0.423	0.196	0.687	0.115	0.913	0.421	0.474	0.853		0.615	0.819
MA	0.925	0.995	0.879	0.897	<b>0.006</b>	0.339	0.561	0.098	<b>0.044</b>	0.106		0.672
ME	0.928	0.275	0.112	0.454	0.081	0.159	0.184	0.318	0.438	<b>0.000</b>	0.262	

The numbers represent  $p$ -value on Granger Causality  $F$ -Statistic with lag-1 of the stock markets returns. The table is read 'row' Granger-cause 'column'.

**Table 2** Johansen's Cointegration Test

Trace Test				
Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.110	464.063	374.908	0.000
At most 1 *	0.103	372.930	322.069	0.000
At most 2 *	0.087	288.022	273.189	0.010
At most 3	0.060	216.213	228.298	0.157
Maximum Eigenvalue Test				
Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.1096	91.1335	80.8703	0.0048
At most 1 *	0.1025	84.9072	74.8375	0.0047
At most 2 *	0.0874	71.8092	68.8121	0.0256
At most 3	0.0596	48.2450	62.7521	0.5701

Trace test and Max-eigenvalue test indicates 3 cointegrating equations at the 0.05 level.

\* denotes rejection of the hypothesis at the 0.05 level, \*\*MacKinnon-Haug-Michelis (1999) p-values

The cointegration test in Table 2 shows that there are three cointegrating equations. It shows that there is long-term equilibrium relationship among the market indices. However, for simplicity and reducing computational burden in VECM estimation, only one cointegrating equation was applied. The cointegrating equation in the VECM is shown in Table 3. Using the cointegration equation, VECM and VEC-GARCH Model parameters are shown in Table 4 and 5.

**Table 3** Cointegrating Equation in Vector Error Correction Model (VECM)

	log(US)	log(GE)	log(HK)	log(JP)	log(SI)	log(UK)	log(AR)	log(BR)	log(CH)	log(ID)	log(MA)	log(ME)	Const.
Coeff.	1.000	0.094	0.526	-0.371	0.021	-0.528	-0.138	-0.008	-0.022	0.364	-0.996	-0.023	2.027
S.E.		-0.170	-0.182	-0.127	-0.234	-0.167	-0.061	-0.091	-0.055	-0.071	-0.140	-0.082	-1.002
t-stat.		0.554	<b>2.884</b>	<b>-2.916</b>	0.090	<b>-3.153</b>	<b>-2.282</b>	-0.083	-0.397	<b>5.117</b>	<b>-7.131</b>	-0.282	<b>2.024</b>

Number in bold face indicates the coefficient is significant at 5% level.

**Table 4** Vector Error Correction Model (VECM)

	$\tilde{R}_{US,t}$	$\tilde{R}_{GE,t}$	$\tilde{R}_{HK,t}$	$\tilde{R}_{JP,t}$	$\tilde{R}_{SI,t}$	$\tilde{R}_{UK,t}$	$\tilde{R}_{AR,t}$	$\tilde{R}_{BR,t}$	$\tilde{R}_{CH,t}$	$\tilde{R}_{ID,t}$	$\tilde{R}_{MA,t}$	$\tilde{R}_{ME,t}$
Coint.Eq.	<b>-0.027</b>	-0.022	0.011	<b>0.027</b>	0.021	-0.017	-0.002	-0.019	<b>-0.025</b>	<b>0.043</b>	<b>0.095</b>	-0.011
S.E.	<b>-0.009</b>	-0.014	-0.013	<b>-0.012</b>	-0.013	-0.010	-0.019	-0.021	<b>-0.012</b>	<b>-0.022</b>	<b>-0.014</b>	-0.015
$\tilde{R}_{US,t-1}$	-0.034	0.159	<b>0.218</b>	<b>0.203</b>	<b>0.185</b>	<b>0.171</b>	0.071	<b>0.289</b>	-0.023	0.107	-0.049	0.182
S.E.	-0.062	-0.092	<b>-0.088</b>	<b>-0.078</b>	<b>-0.088</b>	<b>-0.069</b>	-0.128	<b>-0.138</b>	-0.083	-0.148	-0.093	-0.103
$\tilde{R}_{GE,t-1}$	0.006	<b>-0.178</b>	0.043	-0.064	-0.004	-0.066	0.046	0.020	0.080	0.090	0.091	-0.002
S.E.	-0.045	<b>-0.067</b>	-0.065	-0.057	-0.064	-0.050	-0.094	-0.101	-0.060	-0.108	-0.068	-0.076
$\tilde{R}_{HK,t-1}$	-0.004	0.021	-0.050	0.012	0.041	0.028	0.088	0.137	0.018	0.080	0.017	0.036
S.E.	-0.042	-0.063	-0.060	-0.053	-0.060	-0.047	-0.087	-0.094	-0.056	-0.101	-0.063	-0.070
$\tilde{R}_{JP,t-1}$	-0.033	0.018	0.021	-0.064	0.029	0.002	-0.037	-0.052	0.005	0.022	0.064	-0.072
S.E.	-0.036	-0.053	-0.051	-0.045	-0.051	-0.040	-0.074	-0.080	-0.048	-0.086	-0.054	-0.060
$\tilde{R}_{SI,t-1}$	0.007	-0.005	-0.002	-0.015	<b>-0.128</b>	-0.004	0.066	-0.040	-0.007	<b>0.199</b>	<b>0.012</b>	0.105
S.E.	-0.042	-0.062	-0.060	-0.053	<b>-0.059</b>	-0.046	-0.086	-0.093	-0.056	<b>-0.100</b>	<b>-0.063</b>	-0.070
$\tilde{R}_{UK,t-1}$	-0.049	-0.062	-0.057	-0.007	0.025	<b>-0.177</b>	-0.039	0.038	0.021	-0.193	0.034	-0.110
S.E.	-0.062	-0.092	-0.088	-0.078	-0.088	<b>-0.069</b>	-0.128	-0.138	-0.082	-0.147	-0.093	-0.103
$\tilde{R}_{AR,t-1}$	-0.027	0.024	0.033	0.011	0.028	-0.009	-0.001	0.027	0.051	-0.027	0.017	0.022
S.E.	-0.023	-0.034	-0.032	-0.029	-0.032	-0.025	-0.047	-0.050	-0.030	-0.054	-0.034	-0.038
$\tilde{R}_{BR,t-1}$	0.012	0.034	-0.004	0.009	-0.017	0.012	-0.037	<b>-0.130</b>	0.022	0.042	-0.020	-0.050
S.E.	-0.024	-0.036	-0.034	-0.030	-0.034	-0.027	-0.050	<b>-0.054</b>	-0.032	-0.057	-0.036	-0.040
$\tilde{R}_{CH,t-1}$	-0.044	-0.035	-0.059	-0.009	-0.050	-0.043	-0.045	-0.006	0.002	-0.105	0.006	-0.012
S.E.	-0.028	-0.042	-0.040	-0.035	-0.040	-0.031	-0.058	-0.062	-0.037	-0.067	-0.042	-0.046
$\tilde{R}_{ID,t-1}$	0.009	-0.020	-0.033	-0.017	0.019	0.001	-0.052	0.007	-0.020	-0.047	<b>-0.060</b>	-0.013
S.E.	-0.018	-0.027	-0.026	-0.023	-0.026	-0.020	-0.037	-0.040	-0.024	-0.043	<b>-0.027</b>	-0.030
$\tilde{R}_{MA,t-1}$	0.003	0.003	0.007	0.012	0.105	0.019	0.005	0.043	0.057	-0.010	0.025	-0.017
S.E.	-0.029	-0.043	-0.041	-0.037	-0.041	-0.032	-0.060	-0.065	-0.039	-0.069	-0.044	-0.048
$\tilde{R}_{ME,t-1}$	0.023	-0.019	-0.025	-0.029	-0.021	0.008	0.033	-0.084	-0.085	0.058	-0.016	-0.027
S.E.	-0.036	-0.053	-0.051	-0.045	-0.051	-0.040	-0.074	-0.079	-0.048	-0.085	-0.054	-0.059

Number in bold face indicates the coefficient is significant at 5% level.

**Table 5** VEC-GARCH Model By Modified FGLS Estimator

	$\check{R}_{US,t}$	$\check{R}_{GE,t}$	$\check{R}_{HK,t}$	$\check{R}_{JP,t}$	$\check{R}_{SI,t}$	$\check{R}_{UK,t}$	$\check{R}_{AR,t}$	$\check{R}_{BR,t}$	$\check{R}_{CH,t}$	$\check{R}_{ID,t}$	$\check{R}_{MA,t}$	$\check{R}_{ME,t}$
Coint.Eq.	<b>-0.021</b>	<b>-0.025</b>	0.019	<b>0.030</b>	<b>0.025</b>	<b>-0.014</b>	0.001	<b>-0.041</b>	-0.017	<b>0.035</b>	<b>0.095</b>	-0.011
S.E.	<b>0.006</b>	<b>0.008</b>	0.010	<b>0.008</b>	<b>0.009</b>	<b>0.007</b>	0.010	<b>0.016</b>	0.011	<b>0.017</b>	<b>0.012</b>	0.011
$\check{R}_{US,t-1}$	<b>-0.106</b>	0.044	<b>0.149</b>	0.131	<b>0.165</b>	0.083	0.005	-0.016	-0.056	<b>0.217</b>	-0.012	0.000
S.E.	<b>0.050</b>	0.071	<b>0.066</b>	0.068	<b>0.061</b>	0.053	0.100	0.111	0.078	<b>0.100</b>	0.060	0.082
$\check{R}_{GE,t-1}$	0.032	<b>-0.117</b>	0.094	-0.043	0.034	-0.059	0.076	0.033	0.080	0.055	<b>0.094</b>	0.044
S.E.	0.037	<b>0.056</b>	0.048	0.050	0.045	0.040	0.074	0.081	0.057	0.070	<b>0.044</b>	0.058
$\check{R}_{HK,t-1}$	0.018	0.076	-0.051	0.034	0.017	<b>0.102</b>	0.109	<b>0.279</b>	0.018	0.037	0.016	0.059
S.E.	0.031	0.044	0.046	0.043	0.041	<b>0.033</b>	0.060	<b>0.074</b>	0.051	0.073	0.045	0.053
$\check{R}_{JP,t-1}$	-0.029	0.025	-0.031	-0.060	0.023	0.007	-0.010	-0.078	-0.013	0.018	0.004	-0.089
S.E.	0.028	0.040	0.038	0.042	0.035	0.029	0.058	0.065	0.044	0.060	0.036	0.047
$\check{R}_{SI,t-1}$	-0.004	-0.042	-0.027	-0.009	<b>-0.107</b>	-0.049	0.009	-0.085	-0.019	0.064	0.016	0.039
S.E.	0.032	0.046	0.049	0.046	<b>0.046</b>	0.034	0.066	0.079	0.051	0.078	0.048	0.057
$\check{R}_{UK,t-1}$	0.028	0.061	-0.018	0.051	-0.038	-0.048	0.017	0.199	0.057	-0.172	-0.054	0.042
S.E.	0.050	0.073	0.064	0.068	0.059	0.055	0.101	0.112	0.078	0.095	0.059	0.080
$\check{R}_{AR,t-1}$	-0.015	<b>0.021</b>	0.029	0.016	<b>0.048</b>	-0.003	0.035	0.032	0.052	-0.027	0.025	0.048
S.E.	0.018	<b>0.028</b>	0.023	0.027	<b>0.022</b>	0.019	0.044	0.041	0.028	0.034	0.021	0.029
$\check{R}_{BR,t-1}$	-0.002	0.012	-0.006	0.001	-0.013	-0.011	0.009	-0.096	0.005	<b>0.065</b>	0.010	-0.020
S.E.	0.019	0.028	0.025	0.027	0.023	0.020	0.040	0.045	0.030	<b>0.039</b>	0.024	0.031
$\check{R}_{CH,t-1}$	<b>-0.068</b>	<b>-0.071</b>	<b>-0.074</b>	<b>-0.069</b>	<b>-0.073</b>	<b>-0.083</b>	<b>-0.145</b>	<b>-0.129</b>	0.005	<b>-0.118</b>	0.002	<b>-0.097</b>
S.E.	<b>0.020</b>	<b>0.029</b>	<b>0.027</b>	<b>0.029</b>	<b>0.025</b>	<b>0.022</b>	<b>0.041</b>	<b>0.046</b>	0.036	<b>0.043</b>	0.026	<b>0.034</b>
$\check{R}_{ID,t-1}$	0.014	-0.008	-0.043	-0.009	0.027	0.002	0.003	0.043	-0.031	0.030	<b>-0.063</b>	0.013
S.E.	0.014	0.019	0.023	0.021	0.021	0.015	0.030	0.037	0.022	0.041	<b>0.024</b>	0.026
$\check{R}_{MA,t-1}$	0.028	0.051	<b>0.085</b>	0.015	<b>0.132</b>	<b>0.056</b>	0.044	0.071	0.069	0.069	<b>0.089</b>	0.028
S.E.	0.023	0.031	<b>0.035</b>	0.030	<b>0.032</b>	<b>0.023</b>	0.044	0.056	0.036	0.058	<b>0.040</b>	0.040
$\check{R}_{ME,t-1}$	0.031	-0.024	-0.002	-0.025	0.014	0.014	-0.035	-0.099	-0.056	-0.002	0.010	-0.048
S.E.	0.028	0.040	0.038	0.040	0.035	0.029	0.058	0.066	0.045	0.061	0.035	0.049

Number in bold face indicates the coefficient is significant at 5% level.

### 6.2. Expected Return of Risky Asset

The estimated parameters and their standard errors in the VECM and VEC-GARCH model are different. Because MGARCH error structure is assumed, the estimated expected returns are based on the VEC-GARCH model.

The statistics of the estimated expected return and realized return are presented in Table 6. In general, emerging stock markets such as Brazil, China, and Mexico had higher expected return, yet they were also more volatile than those in the developed markets. Several economic crises and recessions took place during the observation period, such that the averages of expected returns in most countries were negative. The long period of recession in Japan was causing both its realized and expected returns are negative. In emerging markets, only stock market in Argentina that consistently has negative realized and expected return.

**Table 6**  
Annualized Weekly Statistics

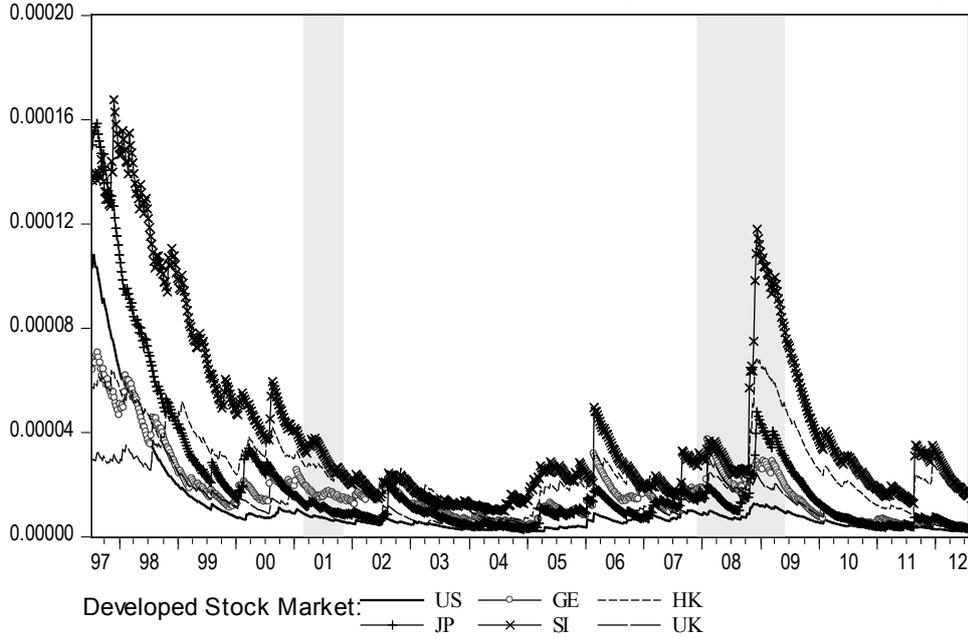
Realized Return	US	GE	HK	JP	SI	UK	AR	BR	CH	ID	MA	ME
Mean	0.024	0.034	0.015	-0.030	0.033	0.004	-0.029	0.044	0.056	0.020	0.014	0.096
Std.Dev.	0.178	0.265	0.255	0.225	0.255	0.199	0.367	0.397	0.238	0.430	0.275	0.295
Expected Return*	US	GE	HK	JP	SI	UK	AR	BR	CH	ID	MA	ME
Mean	0.008	-0.001	-0.005	-0.014	-0.010	0.002	-0.006	0.000	0.002	-0.005	-0.031	-0.001
Std.Dev.	0.026	0.033	0.048	0.035	0.061	0.031	0.057	0.081	0.037	0.081	0.076	0.039

\*The estimated expected return was estimated by VEC-GARCH using modified FGLS estimator

### 6.3. Conditional Variance of Risky Asset

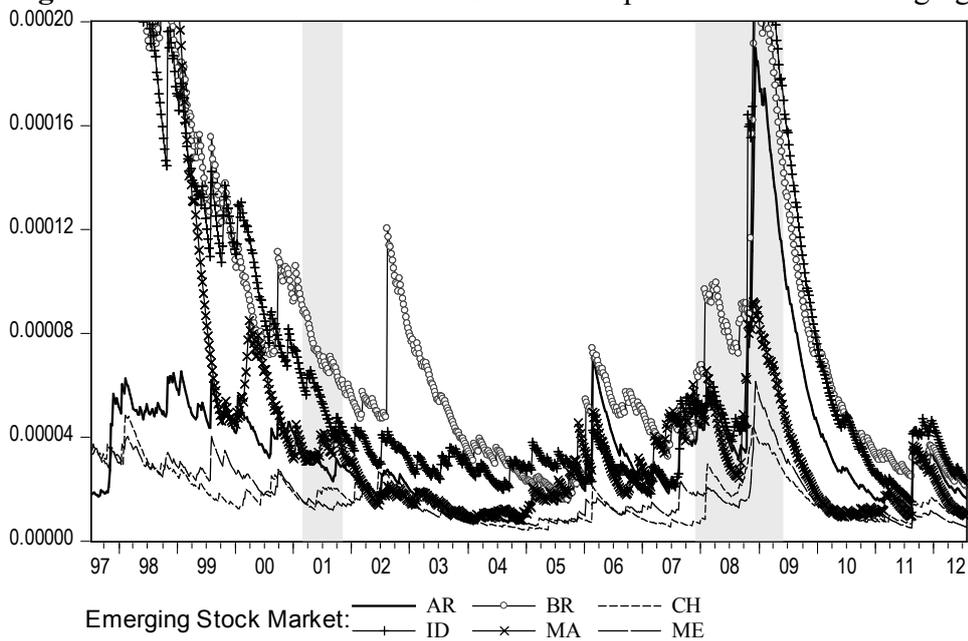
Conditional variances for each market expected return were estimated using Diagonal BEKK model of Engle and Kroner (1995) as specified in equation (2) and the results are shown in Figure 1 and 2 for the developed and emerging markets respectively.

**Figure 1** Conditional Variance of Estimated Expected Return in Developed Markets



Note: Shaded area is the US recession period (based on NBER Business Cycle Dating Committee report, last update was on September 20, 2010).

**Figure 2** Conditional Variance of Estimated Expected Return in Emerging Markets



Note: The scale was trimmed to conform to the Figure 1. Shaded area is the US recession period (based on NBER Business Cycle Dating Committee report, last update was on September 20, 2010).

The figures show that the conditional variances were increasing during period of crises, yet the magnitudes varied across the samples. For example, non-Asian developed and emerging stock markets were less affected by the Asian financial crisis in 1997-1998. However, the US financial crisis in 2008-2009 seems spilled over to other markets and the Asian markets were becoming more volatile in that period. Emerging markets apparently show higher volatility than that for the developed markets. The different magnitudes of volatility indicate that there is opportunity to obtain lower diversifiable risk by investing in those markets; this could be the driving factor of stronger price comovements and stock market integration.

#### 6.4. Test of International CAPM

The *ex-ante* and *ex-post* test for the CAPM were carried out under both SUR (without GARCH) and SUR-GARCH. The results are shown in Table 7 and 8 respectively. In *ex-ante* test (Table 7), the null hypotheses  $H_0^1$  and  $H_0^2$  are all rejected, they indicate that CAPM does not work well for the international assets and the market risk premiums are heterogeneous across the markets. However, individual test of the hypothesis that  $\hat{\alpha}_i = 0$  (*t*-test) show that CAPM can be applied for pricing of all market indexes (except for the Malaysian market), even under the SUR-GARCH test, all alphas are not statistically different from zero. It is susceptible that the rejection of  $H_0^1$  were caused by large differences in the standard errors<sup>2</sup>. This is an indication that the market risk premium adjustments across the markets were so vary during the observation period. The results suggest that removing some markets from the sample might alter the verdict that the CAPM does not fit well for international asset pricing.

**Table 7** *Ex-Ante* International Dynamic Beta CAPM Test

	SUR				SUR-GARCH			
	Coef.	S.E.	t-Stat.	Prob.	Coef.	S.E.	t-Stat.	Prob.
$\hat{\alpha}_{US}$	0.000	0.000	1.885	0.059	0.000	0.000	0.279	0.781
$\hat{\gamma}_{US}$	<b>0.021</b>	<b>0.000</b>	<b>51.835</b>	<b>0.000</b>	<b>0.021</b>	<b>0.000</b>	<b>74.689</b>	<b>0.000</b>
$\hat{\alpha}_{GE}$	0.000	0.000	1.144	0.253	0.000	0.000	-1.152	0.252
$\hat{\gamma}_{GE}$	<b>0.022</b>	<b>0.000</b>	<b>65.176</b>	<b>0.000</b>	<b>0.022</b>	<b>0.000</b>	<b>95.359</b>	<b>0.000</b>
$\hat{\phi}_{GE}$	-0.002	0.003	-0.596	0.552	-0.001	0.002	-0.564	0.574
$\hat{\alpha}_{HK}$	0.000	0.000	0.180	0.857	0.000	0.000	1.747	0.084
$\hat{\gamma}_{HK}$	<b>0.022</b>	<b>0.000</b>	<b>70.061</b>	<b>0.000</b>	<b>0.022</b>	<b>0.000</b>	<b>104.130</b>	<b>0.000</b>
$\hat{\phi}_{HK}$	0.066	0.056	1.175	0.240	<b>0.063</b>	<b>0.031</b>	<b>2.032</b>	<b>0.045</b>
$\hat{\alpha}_{JP}$	<b>0.000</b>	<b>0.000</b>	<b>-2.233</b>	<b>0.026</b>	0.000	0.000	0.165	0.869
$\hat{\gamma}_{JP}$	<b>0.022</b>	<b>0.000</b>	<b>71.136</b>	<b>0.000</b>	<b>0.021</b>	<b>0.000</b>	<b>105.469</b>	<b>0.000</b>
$\hat{\phi}_{JP}$	0.002	0.003	0.829	0.407	0.002	0.002	0.951	0.344
$\hat{\alpha}_{SI}$	0.000	0.000	-0.349	0.727	0.000	0.000	0.293	0.770
$\hat{\gamma}_{SI}$	<b>0.024</b>	<b>0.000</b>	<b>67.676</b>	<b>0.000</b>	<b>0.023</b>	<b>0.000</b>	<b>101.945</b>	<b>0.000</b>
$\hat{\phi}_{SI}$	0.018	0.010	1.765	0.078	<b>0.012</b>	<b>0.006</b>	<b>2.037</b>	<b>0.044</b>
$\hat{\alpha}_{UK}$	0.000	0.000	1.751	0.080	0.000	0.000	0.043	0.966
$\hat{\gamma}_{UK}$	<b>0.022</b>	<b>0.000</b>	<b>71.038</b>	<b>0.000</b>	<b>0.022</b>	<b>0.000</b>	<b>103.266</b>	<b>0.000</b>
$\hat{\phi}_{UK}$	0.001	0.003	0.465	0.642	0.002	0.002	0.822	0.413
$\hat{\alpha}_{AR}$	0.000	0.000	-0.122	0.903	0.000	0.000	0.053	0.958
$\hat{\gamma}_{AR}$	<b>0.023</b>	<b>0.000</b>	<b>81.410</b>	<b>0.000</b>	<b>0.022</b>	<b>0.000</b>	<b>121.855</b>	<b>0.000</b>
$\hat{\phi}_{AR}$	0.000	0.003	-0.098	0.922	0.000	0.002	-0.143	0.886
$\hat{\alpha}_{BR}$	0.000	0.000	0.820	0.412	0.000	0.000	-1.226	0.223
$\hat{\gamma}_{BR}$	<b>0.023</b>	<b>0.000</b>	<b>62.376</b>	<b>0.000</b>	<b>0.022</b>	<b>0.000</b>	<b>92.981</b>	<b>0.000</b>
$\hat{\phi}_{BR}$	0.000	0.005	-0.007	0.994	0.001	0.003	0.253	0.801
$\hat{\alpha}_{CH}$	0.000	0.000	-0.298	0.766	0.000	0.000	-1.898	0.061
$\hat{\gamma}_{CH}$	<b>0.022</b>	<b>0.000</b>	<b>75.680</b>	<b>0.000</b>	<b>0.021</b>	<b>0.000</b>	<b>102.483</b>	<b>0.000</b>
$\hat{\phi}_{CH}$	-0.040	0.023	-1.710	0.087	-0.011	0.015	-0.749	0.455
$\hat{\alpha}_{ID}$	0.000	0.000	-0.721	0.471	0.000	0.000	0.426	0.671
$\hat{\gamma}_{ID}$	<b>0.024</b>	<b>0.000</b>	<b>67.258</b>	<b>0.000</b>	<b>0.023</b>	<b>0.000</b>	<b>103.701</b>	<b>0.000</b>
$\hat{\phi}_{ID}$	0.003	0.002	1.471	0.141	0.003	0.001	1.943	0.055
$\hat{\alpha}_{MA}$	<b>-0.001</b>	<b>0.000</b>	<b>-2.937</b>	<b>0.003</b>	0.000	0.000	0.765	0.446
$\hat{\gamma}_{MA}$	<b>0.024</b>	<b>0.000</b>	<b>53.605</b>	<b>0.000</b>	<b>0.023</b>	<b>0.000</b>	<b>91.641</b>	<b>0.000</b>
$\hat{\phi}_{MA}$	0.008	0.008	0.929	0.353	0.004	0.005	0.774	0.441
$\hat{\alpha}_{ME}$	0.000	0.000	-0.348	0.728	0.000	0.000	-1.380	0.171
$\hat{\gamma}_{ME}$	<b>0.022</b>	<b>0.000</b>	<b>69.672</b>	<b>0.000</b>	<b>0.022</b>	<b>0.000</b>	<b>99.530</b>	<b>0.000</b>
$\hat{\phi}_{ME}$	0.000	0.003	-0.126	0.900	0.000	0.002	-0.177	0.860
Coefficients Wald Test								
Null Hypothesis	d.f.	Chi-sq	Prob.	d.f.	Chi-sq	Prob.		
$\hat{\alpha}_i = 0, \forall i$	12	<b>30.843</b>	<b>0.002</b>	12	<b>26.463</b>	<b>0.009</b>		
$ \hat{\gamma}_1  =  \hat{\gamma}_2  = \dots =  \hat{\gamma}_N $	11	<b>69.525</b>	<b>0.000</b>	11	<b>174.795</b>	<b>0.000</b>		

Number in bold face indicates the coefficient is significant at 5% level.

<sup>2</sup> The differences in the standard errors cannot be seen in the tables because the numbers were rounded to only three decimals.

The homogeneity test of the market risk premiums are also rejected in both tests. It suggests that the stock markets were not fully integrated yet. Market risk is priced higher in Asian stock markets such as in Singapore, Indonesia, and Malaysia, than that in other markets. Meanwhile, in the US and the Japan, the market risk premium is lower than that in the other markets. Note that all market risk premiums are nonnegative. This is the expected result. It indicates that the constructed world market portfolio is always in the efficient frontier and is at the tangency of capital market line.

The effect of changes in currency exchange rates seemed to be absorbed by the market risk factor. Under the SUR-GARCH test, the additional required rate of return to compensate the exchange rate risk is only applied for assets from Hong Kong and Singapore stock market. This result indicates that the market risk (beta) is still the only relevant risk factor in the International CAPM (provided that un-integrated stock markets were removed from the sample such that  $H_0^1$  could not be rejected).

**Table 8** *Ex-Post* International Dynamic Beta CAPM Test

	SUR				SUR-GARCH			
	Coef.	S.E.	t-Stat.	Prob.	Coef.	S.E.	t-Stat.	Prob.
$\hat{\alpha}_{US}$	0.000	0.001	0.199	0.842	0.000	0.000	0.670	0.504
$\hat{\gamma}_{US}$	-0.019	0.003	-6.893	<b>0.000</b>	-0.015	0.002	-8.040	<b>0.000</b>
$\hat{\alpha}_{GE}$	0.000	0.001	0.219	0.827	0.000	0.001	0.472	0.638
$\hat{\gamma}_{GE}$	-0.017	0.003	-6.124	<b>0.000</b>	-0.011	0.002	-5.722	<b>0.000</b>
$\hat{\phi}_{GE}$	0.644	0.051	12.604	<b>0.000</b>	0.652	0.036	18.327	<b>0.000</b>
$\hat{\alpha}_{HK}$	0.000	0.001	-0.346	0.729	0.000	0.001	-0.222	0.825
$\hat{\gamma}_{HK}$	-0.006	0.003	-1.792	0.073	-0.002	0.002	-0.793	0.430
$\hat{\phi}_{HK}$	1.300	0.830	1.566	0.117	1.782	0.485	3.672	<b>0.000</b>
$\hat{\alpha}_{JP}$	-0.001	0.001	-1.316	0.188	-0.002	0.001	-2.260	<b>0.026</b>
$\hat{\gamma}_{JP}$	-0.011	0.003	-3.260	<b>0.001</b>	-0.007	0.002	-3.027	<b>0.003</b>
$\hat{\phi}_{JP}$	0.658	0.062	10.635	<b>0.000</b>	0.623	0.045	13.819	<b>0.000</b>
$\hat{\alpha}_{SI}$	-0.001	0.001	-0.737	0.461	0.000	0.001	-0.007	0.994
$\hat{\gamma}_{SI}$	-0.003	0.003	-0.977	0.329	0.000	0.002	0.023	0.981
$\hat{\phi}_{SI}$	1.227	0.102	11.996	<b>0.000</b>	1.112	0.061	18.143	<b>0.000</b>
$\hat{\alpha}_{UK}$	0.000	0.001	0.236	0.813	0.000	0.000	0.150	0.881
$\hat{\gamma}_{UK}$	-0.018	0.002	-7.196	<b>0.000</b>	-0.012	0.002	-7.459	<b>0.000</b>
$\hat{\phi}_{UK}$	0.653	0.040	16.178	<b>0.000</b>	0.661	0.027	24.050	<b>0.000</b>
$\hat{\alpha}_{AR}$	-0.002	0.002	-0.928	0.353	-0.002	0.001	-1.898	0.061
$\hat{\gamma}_{AR}$	0.008	0.004	2.189	<b>0.029</b>	0.008	0.003	3.051	<b>0.003</b>
$\hat{\phi}_{AR}$	0.345	0.072	4.759	<b>0.000</b>	0.462	0.077	6.027	<b>0.000</b>
$\hat{\alpha}_{BR}$	-0.001	0.002	-0.408	0.683	0.001	0.001	0.591	0.556
$\hat{\gamma}_{BR}$	-0.005	0.003	-1.717	0.086	-0.005	0.002	-2.877	<b>0.005</b>
$\hat{\phi}_{BR}$	1.037	0.054	19.222	<b>0.000</b>	1.050	0.035	30.399	<b>0.000</b>
$\hat{\alpha}_{CH}$	0.000	0.001	-0.350	0.726	-0.001	0.001	-0.647	0.519
$\hat{\gamma}_{CH}$	0.002	0.004	0.568	0.570	0.005	0.003	1.558	0.122
$\hat{\phi}_{CH}$	0.853	0.736	1.159	0.247	1.389	0.524	2.652	<b>0.009</b>
$\hat{\alpha}_{ID}$	0.000	0.002	-0.078	0.938	0.003	0.001	2.919	<b>0.004</b>
$\hat{\gamma}_{ID}$	-0.001	0.003	-0.424	0.671	0.001	0.002	0.476	0.635
$\hat{\phi}_{ID}$	0.978	0.035	28.243	<b>0.000</b>	1.064	0.032	33.572	<b>0.000</b>
$\hat{\alpha}_{MA}$	0.000	0.001	-0.276	0.783	0.001	0.001	1.404	0.163
$\hat{\gamma}_{MA}$	-0.004	0.004	-1.014	0.310	0.002	0.002	0.674	0.502
$\hat{\phi}_{MA}$	1.093	0.081	13.481	<b>0.000</b>	1.090	0.061	17.976	<b>0.000</b>
$\hat{\alpha}_{ME}$	0.002	0.001	1.391	0.164	0.003	0.001	3.718	<b>0.000</b>
$\hat{\gamma}_{ME}$	-0.007	0.003	-2.307	<b>0.021</b>	-0.007	0.002	-3.845	<b>0.000</b>
$\hat{\phi}_{ME}$	1.128	0.071	15.891	<b>0.000</b>	1.067	0.042	25.520	<b>0.000</b>
Coefficients Wald Test								
Null Hypothesis	d.f.	Chi-sq	Prob.	d.f.	Chi-sq	Prob.		
$\hat{\alpha}_i = 0, \forall i$	12	9.442	0.665	12	<b>45.876</b>	<b>0.000</b>		
$ \hat{\gamma}_1  =  \hat{\gamma}_2  = \dots =  \hat{\gamma}_N $	11	<b>36.816</b>	<b>0.000</b>	11	<b>148.947</b>	<b>0.000</b>		

Number in bold face indicates the coefficient is significant at 5% level.

The *ex-post* test presented in Table 8 examined whether CAPM was applied in the market for pricing the international assets. The test was using realized or actual returns. From the table, CAPM fits well in pricing the assets when multivariate GARCH error structure was ignored (the test under SUR). Testing the alphas individually shall support the finding. However, from the data properties of the sample, the multivariate GARCH error structure does exist. Ignoring the error structure proved that the latter conclusion was inaccurate; when GARCH error structure was considered, the CAPM does not fit well! This result is inline with previous findings, for example in Lewellen and Nagel (2006) and Wu and Chiou (2007). The latter paper used Kalman filter method in testing the CAPM. Thus, this paper shows the applicability of SUR-GARCH model using the modified FGLS estimator that is simpler than other method such as the Kalman filter.

As the previous analysis in the *ex-ante* test, removing some stock market indexes might make CAPM works. Under the SUR-GARCH test, CAPM only failed to work for pricing the Japanese and the Indonesian stock market returns. This research shows that constructing world market portfolio from the twelve indexes leads to inapplicable CAPM. Finding the stock market indexes that make CAPM works is subject to future research.

The realized market risk premiums are also heterogeneous. Some of the market risk premiums are negative. They indicate that those indexes returns have negative covariance with the world market portfolio. This is as a result of allowing the short selling. Moreover, this result also suggests that the stock markets were not fully integrated yet. The price of the market risk for the assets varies. In addition, exchange rate risk is also priced differently for the market indexes. It indicates that beta risk is not the only risk factor considered by investors. Multi factors CAPM may be applied in the markets.

## 7. Conclusion

Conditional International CAPM was tested under assumptions of unrestricted short selling and borrowing at riskless rate such that the constructed world market portfolio is a mean-variance efficient portfolio at the tangency of the capital market line. Previous researches that attempt to test the CAPM for international assets were using readily available index that was subject to the critique that the market portfolio was not mean-variance efficient. By constructing the world market portfolio that meets with the assumptions, we could estimates not only the expected return and conditional variance of that portfolio, but also the conditional covariance between the world market portfolio and its composing portfolios (national stock market indices), such that time varying betas for each international asset can be estimated. Thus, all assumptions used in building the CAPM were all met so that the test of the model can be emphasized on the estimated parameters of the model.

The test of the international asset-pricing model was carried out by assuming that the markets were fully integrated so that covariance among the disturbance terms is not zero. Furthermore, because crises, recession, and other kind of market shocks happened during the observation period in 1997:7 until 2012:7, heteroscedasticity of the errors is expected, and the sample data properties said so. To consider the covariance and the heteroscedasticity, the time varying beta CAPM was tested under SUR-GARCH model. In *ex-ante* test CAPM is rejected, but individual analysis of the assets showed that CAPM may be applied for all of the assets. It indicates that the risk premium adjustments were done during the period of analysis, and the levels of adjustments varied across the assets. The market risk is also priced differently for each assets, it indicates that the markets were not fully integrated yet. However, exchange rate risk is not significantly affecting the expected excess return. The latter shows that theoretically CAPM is correct, that the only risk factor worth to consider is the market risk (beta). However, in the real world, the *ex-post* tests show that CAPM had failed in pricing the international assets and it suggests that other risk factors might exist.

The findings are subject to the stock market index selection for constructing the world market portfolio. In addition, CAPM might also work under sub-sample periods, for example under non-recession period. Finding the structural break point under SUR-GARCH also a challenge in the econometric side. These issues should be addressed in the future research.

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