Normalizing Returns with Realized Measures

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Abstract

Financial returns are often modeled as the product of a random variable with zero mean and unit variance and a positive random variable, which is called volatility. Once the volatility is given, the former variable can be calculated as the return divided by the volatility, which can be used to test the assumptions of the former variable. This is true for GARCH models because they specify the volatility as a function of past observed variables only and hence the volatility on day t is known on day $t-1$ once the parameters in GARCH models are given. It is, however, not true for stochatic volatility models because the volatility is a latent variable. This paper considers the return devided by the square root of realized measures. Specifically, we analyze how using the realized measures affects the asymptotic and bootstrap analyses for statistics of normalized returns, compared with the use of the true volatility. It is shown that the asymptotic distributions of statistics of normalized returns with the realized measures are the same as those with the true conditional variances and the bootstrap method consistently estimates the asymptotic distribution of the statistics if the number of intra day returns for calculating the realized measures increases at a sufficiently fast rate.

Key Words: Realized Measure; Integrated Variance; Normalization; Bootstrap

References

[1] Beran, R. and G.R. Ducharme (1991), Asymptotic Theory for Bootstrap Methods in Statistics, (Les Publications CRM, Centre de recherches mathematiques, Universite de Montreal, Montreal, Canada).

[2] Lahiri, S.N. (2003), Resampling Methods for Dependent Data. Springer-Varlag, New York.

[3] Mammen, E. (1992), When Does Boostrap Work? Asymptotic Results and Simulations. Springer-Verlag, New York.

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