Empirical stochastic time change variable of equity returns with high frequency data

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Introduction

- Non-normality of property of return distribution is well-known.
- But it is possible to recover normality considering stochastic time change process.

- Clark (1973)
  - Provide linkage between physical time interval returns and subordinated stochastic time change process.
  - Time change process assumed to be log-normally distributed
  - Observed cumulative trading volumes is a proxy of time change variable

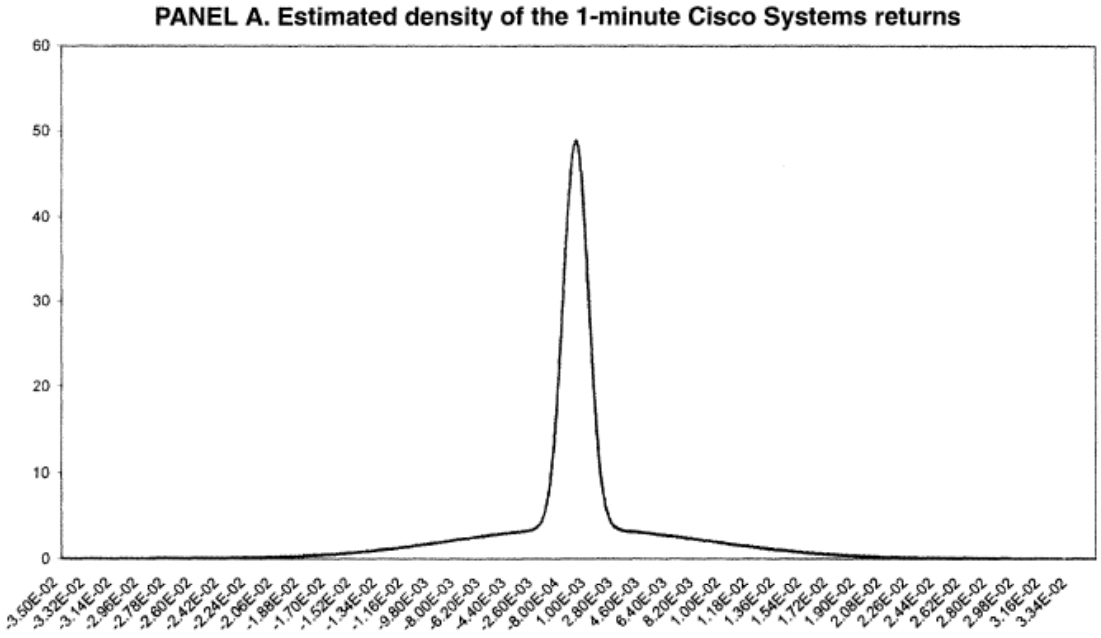
Introduction

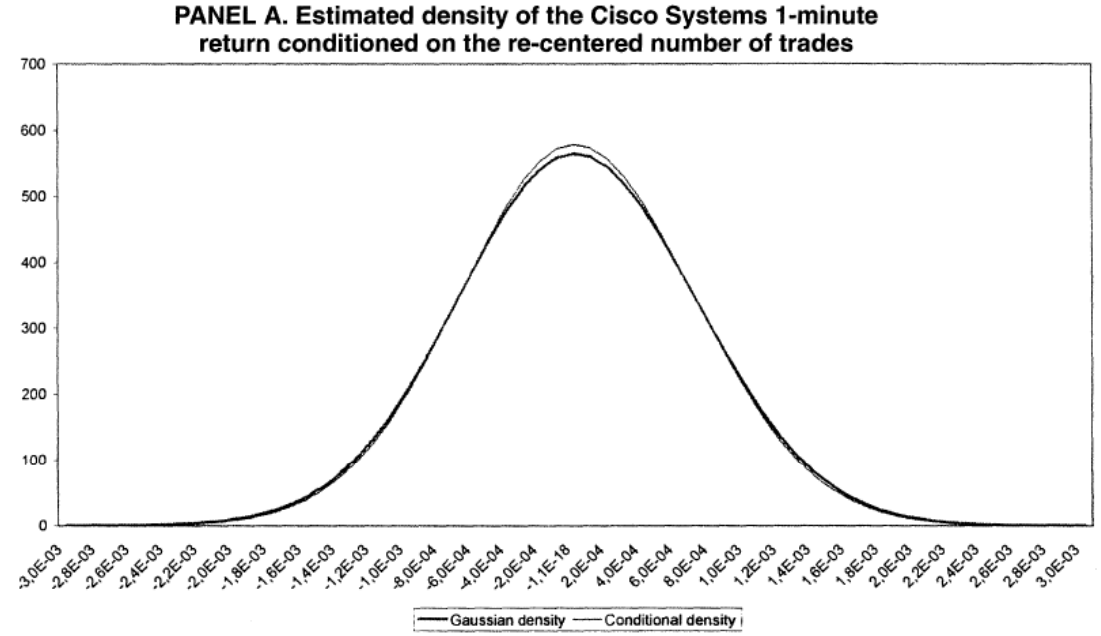
- Ane and Geman (JF 2000)
  - Do not assume distribution of time change process.
  - Assume return process to be a subordinated process of *Brownian process* and *stochastic time change process*.
  - Estimate moments of stochastic time change variable through moments-matching between *observed returns* and *unobserved subordinated process*.
  - Find the best proxy of time change variable through comparing *estimated moments* with *moments of observable economic variables* .
  - Calculate conditional returns base on the selected proxy to recover normality.

Ane and Geman (JF 2000)

- Order Flow, Transaction Clock, and Normality of Asset Returns, Journal of Finance, Vol. 55, No. 5. Oct., 2000
- Data
  - Intraday data of *CISCO* and *Intel* from January to December 1997.
  - Four physical time intervals; 1, 5, 10, and 15 minutes.
  - Candidate economic variables:
    - Cumulative number of trades (cNTD)
    - Cumulative volumes (cVLM)
- Results
  - Moments of cNTD resembles estimated moments of time change variable.
  - Normality of conditional returns on cNTD for both securities can be recovered.

Ane and Geman (JF 2000)





Objectives & Approaches

- Objectives:
  - One year after the implementation of automated trading system “Arrow Head” in TSE
    - Search for proxy of time change variable.
    - Test if the normality of returns conditioned on time change process can be recovered.
- Approaches
  - Use high-frequency intraday data, 6 physical time intervals both seconds and minutes intervals.
  - Select seven securities with different characteristics from various industries.
  - Compare standardized moments instead of raw moments.
  - Consider variety of candidate economic variables.

Estimation Method of time change variable moments

- Return process is a subordinated process

$$Y(t) = X(\tau(t)) \tag{1}$$

  - $Y(t)$: observed return @t
  - $X(s)$: subordinated process @ t, given that $X(s) \sim N(\mu s, \sigma^2 s)$
  - μ & σ^2: parameters of Brownian process
  - $\tau(t)$: time change variable @t

- Moments matching through Moment Generating Function

$$\min U = [MGF[Y(t)]_{empirical} - MGF[X(\tau(t))]_{theoretical}]^2 \tag{2}$$

  - Conditions on least square minimization

$$m_i[Y(t)] = m_i[X(\tau(t))] \tag{3}$$

  - m_i: i^{th} central moment with i = 2,…,6

Zero Percentage of Observations

*All statistics are averaged across 7 securities

		5sec			15sec			30sec		
		N	Zeros	% Zero	N	Zeros	% Zero	N	Zeros	% Zero
1	'RET_T';	123,068	103,975	84.5%	41,023	28,893	70.4%	20,512	12,525	61.1%
2	'cVLM';	123,144	75,822	61.6%	41,099	12,777	31.1%	20,588	2,916	14.2%
3	'cNTD';	123,144	75,822	61.6%	41,099	12,777	31.1%	20,588	2,916	14.2%
6	'RV_T';	123,144	99,976	81.2%	41,099	24,004	58.4%	20,588	8,063	39.2%
7	'PCH_T';	123,144	99,976	81.2%	41,099	24,004	58.4%	20,588	8,063	39.2%
14	'pVLM';	123,144	75,822	61.6%	41,099	12,777	31.1%	20,588	2,916	14.2%
15	'pNTD';	123,144	75,822	61.6%	41,099	12,777	31.1%	20,588	2,916	14.2%
22	'RV';	123,144	116,028	94.2%	41,099	35,704	86.9%	20,588	16,307	79.2%
24	'SPC';	123,144	122,017	99.1%	41,099	39,929	97.2%	20,588	19,499	94.7%
25	'QCH';	123,144	119,159	96.8%	41,099	37,205	90.5%	20,588	17,055	82.8%

Zero Percentage of Observations

*All statistics are averaged across 7 securities

		1min			5min			10min		
		N	Zeros	% Zero	N	Zeros	% Zero	N	Zeros	% Zero
1	'RET_T';	10,256	5,503	53.7%	2,051	775	37.8%	1,026	320	31.2%
2	'cVLM';	10,332	495	4.8%	2,127	76	3.6%	1,102	76	6.9%
3	'cNTD';	10,332	495	4.8%	2,127	76	3.6%	1,102	76	6.9%
6	'RV_T';	10,332	2,171	21.0%	2,127	95	4.5%	1,102	77	7.0%
7	'PCH_T';	10,332	2,171	21.0%	2,127	95	4.5%	1,102	77	7.0%
14	'pVLM';	10,332	495	4.8%	2,127	76	3.6%	1,102	76	6.9%
15	'pNTD';	10,332	495	4.8%	2,127	76	3.6%	1,102	76	6.9%
22	'RV';	10,332	7,115	68.9%	2,127	803	37.8%	1,102	277	25.1%
24	'SPC';	10,332	9,401	91.0%	2,127	1,663	78.2%	1,102	789	71.6%
25	'QCH';	10,332	7,405	71.7%	2,127	834	39.2%	1,102	301	27.3%

Robust Comparison of Moments

- Observed variable has zero observations less than or equal to 25%
  - Use sample moments

- Observed variable has zero observations more than 25%
  - Zero-inflated Negative Binomial moments
  - Density of economic variable X

$$p^*(x|w,\theta) = \begin{cases} w + (1-w)p(0|\theta), & x = 0 \\ (1-w)p(x|\theta), & x > 0 \end{cases} \quad (4)$$

- Compare moments using standardized moments

$$\frac{E[X-\mu]^i}{V[X]^{i/2}} \quad (5)$$

  - X: candidate economic variable,μ: sample mean of X
  - i –order of moments、i = 3,…,6

Time Change Variable Selection

- Sum of Squared Errors (SSE)
  - SSE of each candidate economic variable is:

$$SSE = \sum_{i=3}^{6} \left[\frac{m_i(X) - m_i(\tau(t))}{m_i(\tau(t))} \right]^2 \quad (6)$$

  - $m_i(X\ \ )$: sample standardized i^{th} order moment of variable X
  - $m_i(\tau(t))$: standardized i^{th} order moment of time change variable
  - The closer SSE of X gets near to zero, the better X is a proxy of time change variable $\tau(t)$.

Data

- High-frequency data from Jan – Feb , 2011, 38 trading days.
- Cover seven securities listed on TSE 1st section

Sec. Code	Sec.Name	Ave. Closing Price (Yen)	Min. Closing Price (Yen)	Max. Closing Price (Yen)	Ave. VLM/day (Shares)	Ave.NTD /day (Times)	Tick Size	HPR	Ave. VLM/Trade (Shares)	Ave. Trade Value/day (mil.Yen)
4502	Takeda	4,011	3,950	4,105	2,081,082	1,859	5	1.6%	1,119.21	4.49
6502	Toshiba	503	449	549	44,300,000	3,546	1	17.5%	12,493.41	6.29
7203	Toyota	3,569	3,265	3,910	9,399,018	5,230	5	15.7%	1,797.19	6.41
8802	Mitsubishi Estate	1,611	1,505	1,719	6,456,711	2,032	1	9.4%	3,177.43	5.12
9020	JR East	5,442	5,240	5,730	1,102,537	1,263	10	7.1%	873.22	4.75
9433	KDDI	495,026	461,000	547,000	17,162	1,628	500-1000	10.2%	10.54	5.22
9437	Docomo	149,139	142,400	158,900	53,713	3,117	100	7.1%	17.23	2.57
	Nikkei	10,536	10,238	10,858				2.2%		

5sec

Sec.code	N	mean	var	Skewness	Kurtosis	std m5	std m6
4502	225	1.02E-02	8.41E+02	(0.32)	7.94	(8.65)	76.68
6502	364	1.02E-02	3.40E+02	(0.20)	5.21	(3.62)	33.26
7203	462	4.83E-02	6.97E+03	(0.82)	5.65	(13.40)	73.31
8802	264	1.02E-02	2.63E+01	(0.21)	16.72	(22.58)	845.02
9020	160	1.02E-02	2.08E+02	(0.15)	13.36	(8.98)	265.83
9433	183	3.28E-02	1.74E+02	(0.24)	16.48	(10.46)	468.35
9437	340	1.02E-02	2.35E+01	(0.06)	8.35	(2.29)	209.85

15sec

Sec.code	N	mean	var	Skewness	Kurtosis	std m5	std m6
4502	201	3.05E-02	1.77E+03	(0.23)	3.89	(3.02)	19.10
6502	283	3.05E-02	6.00E+02	(0.16)	3.21	(1.83)	14.94
7203	327	1.45E-01	1.12E+04	(0.62)	4.10	(5.17)	53.19
8802	224	3.05E-02	7.39E+01	(0.26)	10.45	(16.86)	390.91
9020	138	3.05E-02	4.97E+02	(0.12)	6.41	(6.60)	107.13
9433	168	9.83E-02	4.16E+02	(0.17)	9.06	(5.99)	262.97
9437	270	3.05E-02	5.00E+01	(0.03)	6.25	(3.95)	159.41

30sec

Sec.code	N	mean	var	Skewness	Kurtosis	std m5	std m6
4502	163	6.09E-02	2.36E+03	(0.19)	3.04	(1.92)	12.49
6502	20,751	6.09E-02	7.45E+02	(0.14)	2.96	(1.67)	15.50
7203	24,350	2.90E-01	1.36E+04	(0.50)	4.44	0.12	93.97
8802	185	6.09E-02	1.39E+02	(0.23)	9.22	(12.16)	306.51
9020	124	6.09E-02	7.85E+02	(0.09)	4.45	(2.55)	42.75
9433	142	1.97E-01	6.55E+02	(0.10)	7.72	(2.35)	233.62
9437	203	6.09E-02	7.30E+01	0.09	6.16	8.45	156.25

Estimated Moments of Time Change Process $\tau(t)$ based on Transaction Returns

1min

Sec.code	N	mean	var	Skewness	Kurtosis	std m5	std m6
4502	11,077	1.22E-01	2.68E+03	(0.20)	3.01	(2.24)	14.95
6502	11,166	1.22E-01	8.70E+02	(0.15)	3.32	(2.00)	24.68
7203	11,390	5.80E-01	1.60E+04	(0.44)	5.79	6.76	209.11
8802	11,248	1.22E-01	2.61E+02	(0.14)	7.91	(6.00)	190.27
9020	104	1.22E-01	1.11E+03	(0.06)	3.62	(1.49)	28.19
9433	119	3.93E-01	1.01E+03	(0.11)	7.06	(4.27)	173.47
9437	11,564	1.22E-01	1.07E+02	0.17	5.87	7.48	111.00

5min

Sec.code	N	mean	var	Skewness	Kurtosis	std m5	std m6
4502	2,268	6.09E-01	3.87E+03	(0.15)	3.41	(2.32)	23.67
6502	2,381	6.09E-01	1.84E+03	(0.10)	4.76	2.17	70.04
7203	2,447	2.90E+00	4.29E+04	0.03	9.14	26.78	366.72
8802	2,548	6.09E-01	1.11E+03	(0.17)	5.04	(4.92)	60.13
9020	2,473	6.09E-01	2.19E+03	0.26	4.29	4.91	43.78
9433	2,486	1.97E+00	2.97E+03	0.27	7.85	10.21	160.49
9437	2,391	6.09E-01	3.37E+02	0.54	7.96	24.53	214.34

10min

Sec.code	N	mean	var	Skewness	Kurtosis	std m5	std m6
4502	1,257	1.22E+00	5.36E+03	(0.09)	3.65	(0.75)	22.77
6502	1,299	1.22E+00	2.95E+03	0.03	4.63	1.45	54.81
7203	1,323	5.80E+00	7.25E+04	0.48	9.67	40.31	383.84
8802	1,376	1.22E+00	2.15E+03	(0.27)	5.24	(5.12)	61.53
9020	1,302	1.22E+00	3.32E+03	(0.01)	4.20	(1.29)	33.96
9433	1,393	3.93E+00	5.36E+03	0.75	10.15	41.02	368.69
9437	1,293	1.22E+00	5.96E+02	1.12	12.05	72.40	628.34

Estimated Moments of Time Change Process $\tau(t)$ based on Transaction Returns

Transaction Return (Y_t)- based SSE

5sec

	4502	6502	7203	8802	9020	9433	9437	Ave
1 'cVLM';	#######	#######	#######	#######	#######	#######	#######	#######
2 'cNTD';	300.5	561.7	52.2	376.2	118.3	612.3	3,111.6	733.3
5 'RV_T';	6.1	8.0	4.0	17.8	25.2	17.9	85.7	23.5
6 'PCH_T';	12.7	37.1	8.4	32.9	30.1	19.4	264.5	57.9
8 'aVLM';	#######	#######	#######	#######	#######	9,359.2	#######	#######
10 'cVNt';	#######	#######	#######	#######	#######	#######	#######	#######
14 'pNTD';	37.0	132.0	18.8	58.7	121.4	60.7	776.2	172.1
15 'apVLM';	1,245.5	#######	1,245.6	91.0	539.1	376.9	#######	#######
20 'RV';	10.5	22.0	5.2	29.5	19.8	22.9	186.6	42.4
21 'BAS';	9.1	70.7	4.9	14.1	13.2	18.9	28.7	22.8
22 'SPC';	4.8	5.7	3.5	39.1	7.5	14.2	116.4	27.3
23 'QCH';	10.9	19.4	5.3	41.4	25.8	14.8	392.2	72.8

15sec

	4502	6502	7203	8802	9020	9433	9437	Ave
1 'cVLM';	#######	#######	#######	#######	#######	5,334.8	#######	#######
2 'cNTD';	400.9	#######	#######	136.5	810.1	431.9	#######	#######
5 'RV_T';	8.2	27.0	7.6	17.5	21.8	34.4	479.2	85.1
6 'PCH_T';	407.4	609.2	85.1	161.5	70.8	52.0	5,136.2	931.7
8 'aVLM';	#######	#######	#######	#######	#######	1,817.5	#######	#######
10 'cVNt';	#######	#######	#######	93,565.4	#######	1,957.2	#######	#######
14 'pNTD';	185.8	#######	#######	54.5	239.3	154.6	#######	#######
15 'apVLM';	6,812.7	#######	#######	55.5	868.1	775.2	#######	#######
20 'RV';	21.2	52.8	9.8	23.6	39.7	40.4	706.8	127.8
21 'BAS';	15.3	135.3	28.0	30.2	16.9	31.3	1,068.5	189.4
22 'SPC';	5.2	26.2	6.8	38.0	31.7	20.1	1,739.3	266.8
23 'QCH';	19.4	307.2	37.7	518.3	146.1	70.8	2,016.0	445.1

*SSE values over 1e+4 were hidden.

Transaction Return (Y_t)- based SSE

	30sec							
	4502	6502	7203	8802	9020	9433	9437	Ave
1 'cVLM';	#######	#######	#######	#######	#######	#######	#######	#######
2 'cNTD';	#######	#######	#######	978.5	1,051.8	#######	#######	#######
5 'RV_T';	16.6	15.3	3,288.5	17.8	29.8	85.6	17.5	495.9
6 'PCH_T';	343.2	#######	#######	133.5	948.5	1,048.8	268.2	#######
8 'aVLM';	#######	#######	#######	#######	221.1	#######	#######	#######
10 'cVNt';	#######	#######	#######	#######	220.7	#######	#######	#######
14 'pNTD';	#######	#######	#######	518.3	561.9	#######	5,639.1	#######
15 'apVLM';	#######	#######	#######	#######	2,026.8	#######	#######	#######
20 'RV';	37.8	72.3	4,426.6	28.6	75.6	122.0	46.9	687.1
21 'BAS';	181.8	7,351.7	#######	3.0	25.7	2.9	410.8	#######
22 'SPC';	26.0	27.7	1,316.3	276.8	49.1	45.7	145.2	269.5
23 'QCH';	208.5	326.1	#######	305.8	321.7	277.2	3,099.1	5,340.6

	1min							
	4502	6502	7203	8802	9020	9433	9437	Ave
1 'cVLM';	#######	19,638.3	#######	#######	#######	#######	98,937.9	#######
2 'cNTD';	#######	12,046.4	2,152.7	5,359.4	#######	#######	3,155.9	#######
5 'RV_T';	2.6	16.3	5.1	231.5	103.7	77.1	14.8	64.4
6 'PCH_T';	1,114.4	4,668.1	823.1	35,071.3	1,439.0	392.8	408.6	6,273.9
8 'aVLM';	#######	64,642.1	2,349.9	14,391.9	#######	25,177.5	16,904.6	#######
10 'cVNt';	#######	#######	26,902.7	563.0	#######	73,709.9	5,903.3	#######
14 'pNTD';	86,698.3	21,330.0	1,268.2	990.6	#######	10,085.6	992.9	31,887.3
15 'apVLM';	#######	63,154.7	7,483.4	#######	#######	92,674.3	15,895.1	#######
20 'RV';	35.7	57.9	8.1	217.1	187.3	94.0	13.6	87.7
21 'BAS';	22,590.9	#######	#######	21.1	1,569.7	1.6	#######	#######
22 'SPC';	20.9	123.7	13.6	540.6	95.2	168.3	38.3	143.0
23 'QCH';	197.1	#######	7,081.4	434.6	828.0	4,890.0	906.6	21,242.7

*SSE values over 1e+4 were hidden.

Transaction Return (Y_t)- based SSE

	5min							
	4502	6502	7203	8802	9020	9433	9437	Ave
1 'cVLM';	#######	848.7	#######	3,159.1	683.0	1,093.5	86.3	7,265.0
2 'cNTD';	556.8	1,281.8	#######	#######	631.7	#######	56.7	#######
5 'RV_T';	2.3	11.4	408.9	305.7	0.5	19.4	0.3	106.9
6 'PCH_T';	766.9	1,566.5	#######	#######	276.7	191.6	9.2	8,216.5
8 'aVLM';	454.9	228.2	1,905.3	35.8	49.9	24.8	13.2	387.4
10 'cVNt';	1,826.2	1,531.4	6,382.0	51.5	386.9	182.2	5.8	1,480.8
14 'pNTD';	757.1	891.8	7,433.2	640.5	878.0	967.5	5.3	1,653.3
15 'apVLM';	1,582.7	824.5	3,453.9	159.8	105.1	62.1	3.9	884.6
20 'RV';	49.3	110.2	771.0	115.5	4.3	6.4	1.6	151.2
21 'BAS';	#######	#######	#######	#######	#######	3.7	#######	#######
22 'SPC';	77.2	159.8	1,149.5	#######	12.3	43.8	13.3	1,862.0
23 'QCH';	#######	4,337.3	#######	6,219.5	2,432.1	117.9	32.0	6,490.0

	10min							
	4502	6502	7203	8802	9020	9433	9437	Ave
1 'cVLM';	#######	3,931.0	40.9	511.9	25,494.0	17.8	2.1	29,737.0
2 'cNTD';	764.3	5,751.4	56.9	1,476.6	15,031.1	578.1	1.0	3,379.9
5 'RV_T';	64.4	85.3	2.0	70.4	36.4	1.3	3.0	37.5
6 'PCH_T';	1,355.3	6,248.9	49.2	3,968.5	20,221.2	3.8	1.1	4,549.7
8 'aVLM';	673.3	751.6	2.1	0.8	3,935.4	1.5	1.2	766.6
10 'cVNt';	4,741.9	2,066.1	8.5	3.3	20,298.4	1.5	0.2	3,874.3
14 'pNTD';	1,051.0	3,059.5	10.7	99.3	16,663.0	3.3	1.6	2,984.1
15 'apVLM';	3,081.4	1,668.3	4.4	23.3	8,212.1	1.3	1.9	1,856.1
20 'RV';	284.1	911.9	2.6	32.1	3,790.2	1.3	2.1	717.8
21 'BAS';	#######	#######	#######	#######	#######	2.2	#######	#######
22 'SPC';	508.5	6,279.7	6.4	2,260.1	6,537.8	4.2	2.2	2,228.4
23 'QCH';	76,490.3	10,258.4	30.5	1,100.9	50,164.5	14.7	3.4	19,723.2

*SSE values over 1e+4 were hidden.

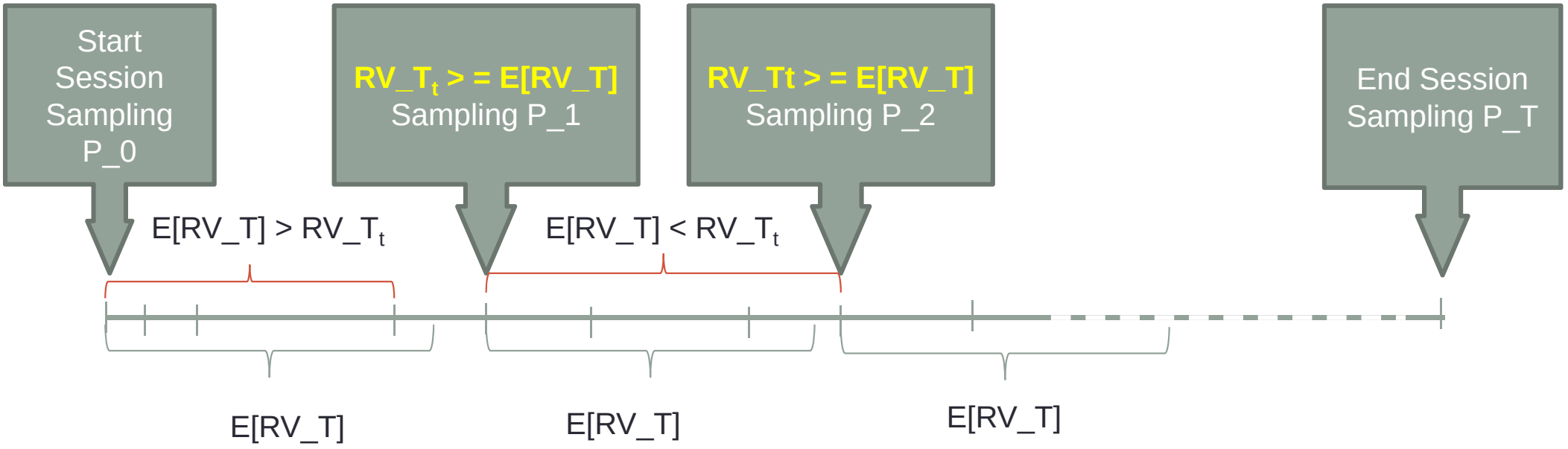
Transaction Return (Y_t)- based SSE

	Sec Average	Min Average	Total Average
1 'cVLM';	#########	529,692.77	#########
2 'cNTD';	#########	116,884.06	697,325.69
5 'RV_T';	201.51	69.64	135.58
6 'PCH_T';	166,903.03	6,346.70	86,624.87
8 'aVLM';	#########	487,958.62	#########
10 'cVNt';	#########	40,531.90	#########
14 'pNTD';	#########	12,174.90	#########
15 'apVLM';	#########	74,745.94	#########
20 'RV';	285.74	318.88	302.31
21 'BAS';	500,601.62	#########	#########
22 'SPC';	187.86	1,411.14	799.50
23 'QCH';	1,952.83	15,818.65	8,885.74

* SSE values over 1e+4 were hidden.

Time Change Conditional Returns

- SSEs of realized volatility estimated from transaction prices (RV_T) were the smallest regardless of physical time intervals and individual security.
- Next step is estimate return for each physical time interval *conditioning on mean value of RV_T*, then test if normality of return distribution can be recovered.



Normality Test – Transaction Returns

5sec

Sec.	Transaction RET				RV_T Cond.Time Change RET			
Code	H0 or H1	pval	JB stat	crit Val	H0 or H1	pval	JB stat	crit Val
4502	1	0.001	116,855.13	5.99	0	0.072	4.80	5.71
6502	1	0.001	23,550.77	5.99	0	0.185	3.07	5.81
7203	1	0.001	11,509.31	5.99	1	0.001	63.06	5.85
8802	1	0.001	962,253.43	5.99	0	0.061	5.25	5.75
9020	1	0.001	546,027.88	5.99	1	0.006	15.00	5.61
9433	1	0.001	914,945.94	5.99	1	0.001	155.97	5.65
9437	1	0.001	146,402.88	5.99	1	0.020	8.48	5.80

15sec

Sec.	Transaction RET				RV_T Cond.Time Change RET			
Code	H0 or H1	pval	JB stat	crit Val	H0 or H1	pval	JB stat	crit Val
4502	1	0.001	1,097.75	5.99	1	0.015	10.09	5.68
6502	1	0.001	49.21	5.99	1	0.001	23.62	5.76
7203	1	0.001	325.33	5.99	1	0.001	73.19	5.79
8802	1	0.001	94,642.55	5.99	0	0.233	2.53	5.71
9020	1	0.001	19,695.58	5.99	1	0.016	10.02	5.56
9433	1	0.001	61,674.43	5.99	1	0.001	65.08	5.63
9437	1	0.001	18,060.28	5.99	1	0.001	34.23	5.75

30sec

Sec.	Transaction RET				RV_T Cond.Time Change RET			
Code	H0 or H1	pval	JB stat	crit Val	H0 or H1	pval	JB stat	crit Val
4502	0	0.500	0.22	5.99	1	0.035	6.70	5.62
6502	0	0.125	4.16	5.99	1	0.001	260,461.55	5.99
7203	1	0.001	822.29	5.99	0	0.500	0.57	5.99
8802	1	0.001	33,086.60	5.99	1	0.010	11.88	5.66
9020	1	0.001	1,754.46	5.99	0	0.427	1.39	5.52
9433	1	0.001	18,852.74	5.99	1	0.001	97.70	5.57
9437	1	0.001	8,640.26	5.99	1	0.030	7.28	5.68

Seconds interval returns

- '0' indicates that null hypothesis is not rejected at 5% significant level or returns series is considered normal
- p-values that are more than 0.5 is shown as 0.5 while that are less than 0.001 is shown as 0.001.

Normality Test – Transaction Returns

1min

Sec. Code	Transaction RET H0 or H1	pval	JB stat	crit Val	RV_T Cond.Time Change RET H0 or H1	pval	JB stat	crit Val
4502	0	0.4035	1.77	5.99	1	0.001	211,829.17	5.99
6502	1	0.001	36.59	5.99	1	0.001	697.70	5.99
7203	1	0.001	2,470.25	5.99	1	0.001	274.23	5.99
8802	1	0.001	10,301.23	5.99	1	0.001	122.85	5.99
9020	1	0.001	159.12	5.99	0	0.168	2.77	5.45
9433	1	0.001	6,950.53	5.99	1	0.001	89.50	5.51
9437	1	0.001	3,596.05	5.99	1	0.001	61.49	5.99

5min

Sec. Code	Transaction RET H0 or H1	pval	JB stat	crit Val	RV_T Cond.Time Change RET H0 or H1	pval	JB stat	crit Val
4502	1	0.004	12.20	5.96	1	0.001	885.73	5.97
6502	1	0.001	260.25	5.96	1	0.001	47.70	5.97
7203	1	0.001	3,317.78	5.96	1	0.001	361.77	5.97
8802	1	0.001	366.42	5.96	1	0.001	14.99	5.97
9020	1	0.001	182.84	5.96	0	0.129	4.03	5.97
9433	1	0.001	2,088.02	5.96	1	0.001	366.53	5.97
9437	1	0.001	2,231.34	5.96	1	0.001	194.14	5.97

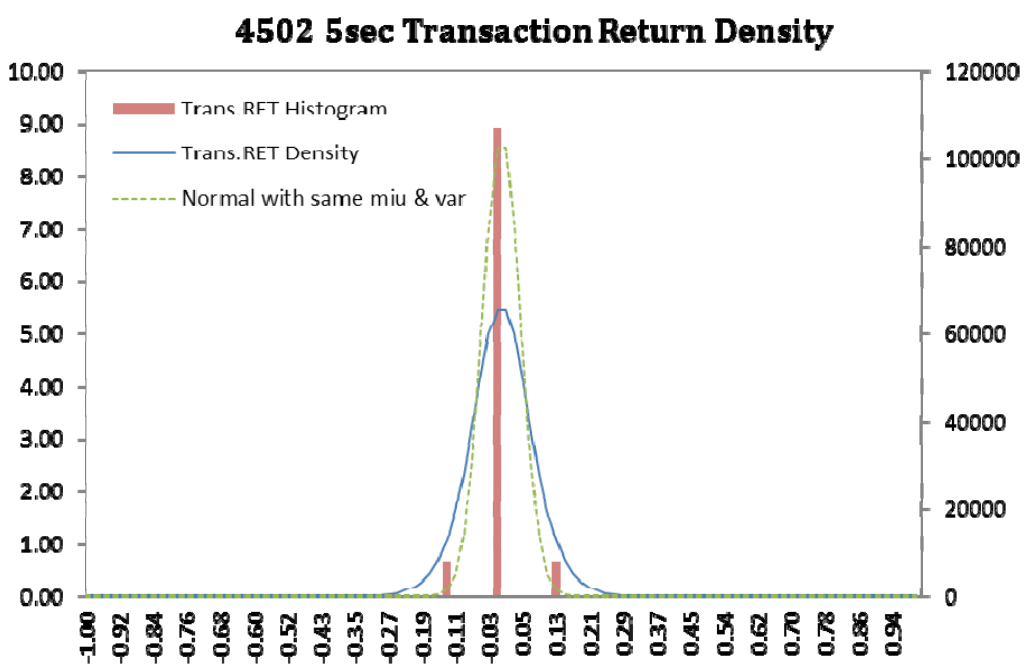
10min

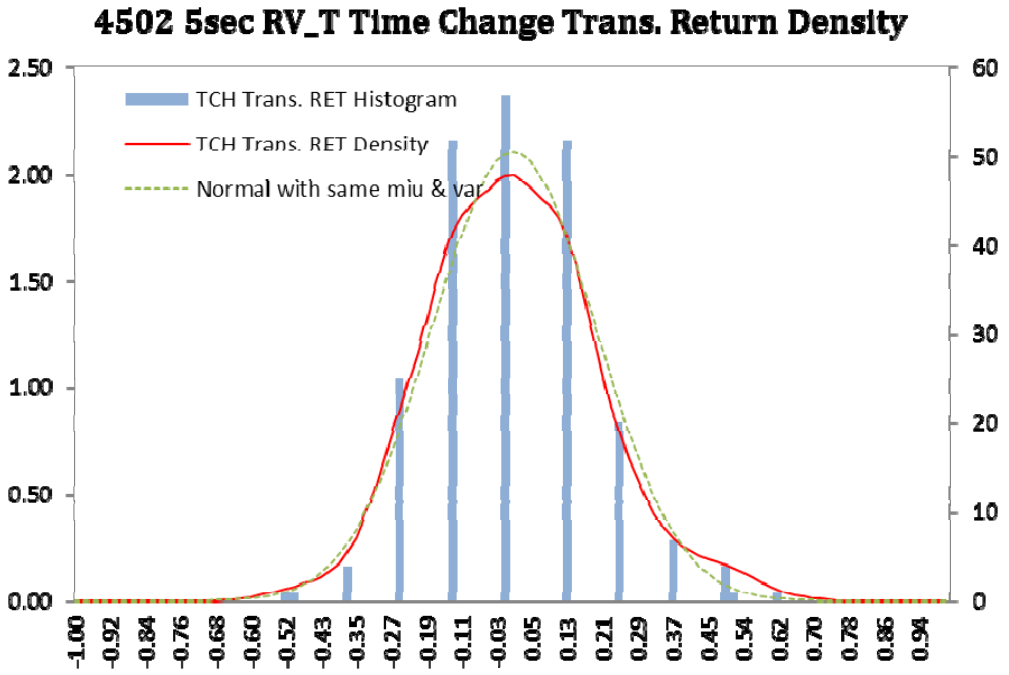
Sec. Code	Transaction RET H0 or H1	pval	JB stat	crit Val	RV_T Cond.Time Change RET H0 or H1	pval	JB stat	crit Val
4502	1	0.0013	17.38	5.93	1	0.001	337.22	5.94
6502	1	0.001	117.45	5.93	1	0.001	25.69	5.94
7203	1	0.001	2,151.56	5.93	1	0.001	196.25	5.95
8802	1	0.001	226.44	5.93	0	0.294	2.39	5.95
9020	1	0.001	62.63	5.93	1	0.001	200.70	5.94
9433	1	0.001	2,354.29	5.93	1	0.001	99.11	5.95
9437	1	0.001	3,750.48	5.93	1	0.001	83.61	5.94

Minutes interval returns

- '0' indicates that null hypothesis is not rejected at 5% significant level or returns series is considered normal
- p-values that are more than 0.5 is shown as 0.5 while that are less than 0.001 is shown as 0.001.

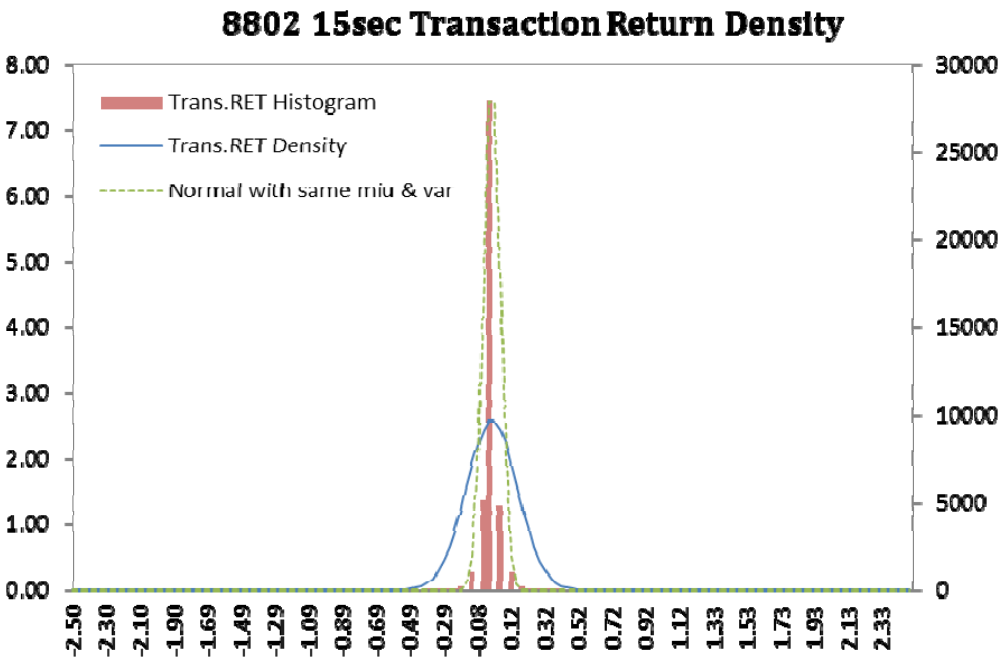
Return Density Comparison

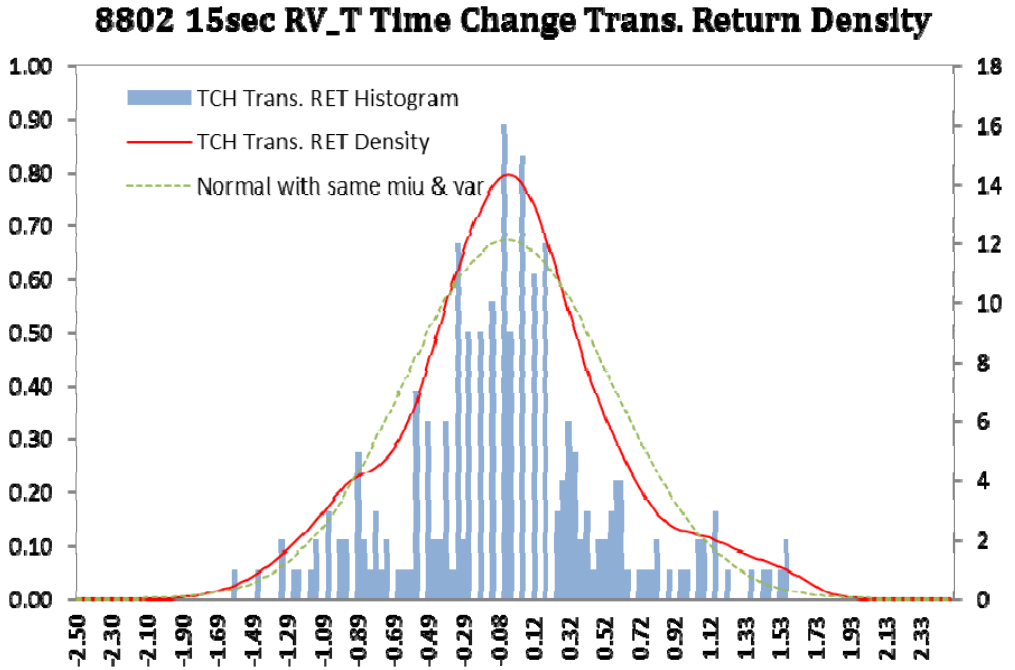




	H0 or H1	pval	JB stat	crit Val
Original Trans.RET	1	0.001	116,855.13	5.99
RV_T Cond.Trans.RET	0	0.072	4.80	5.71

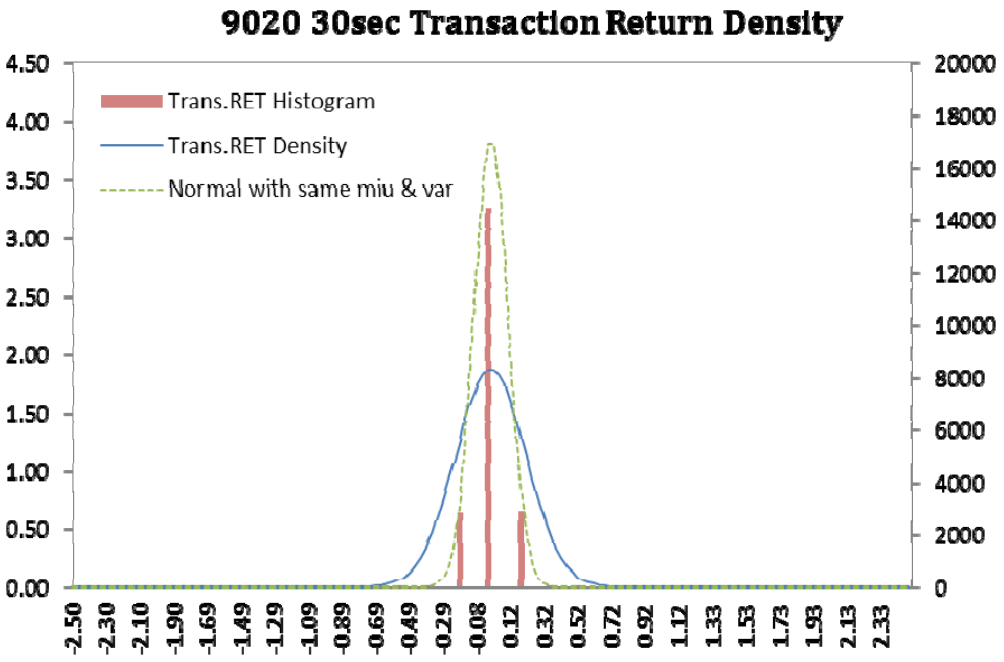
Return Density Comparison

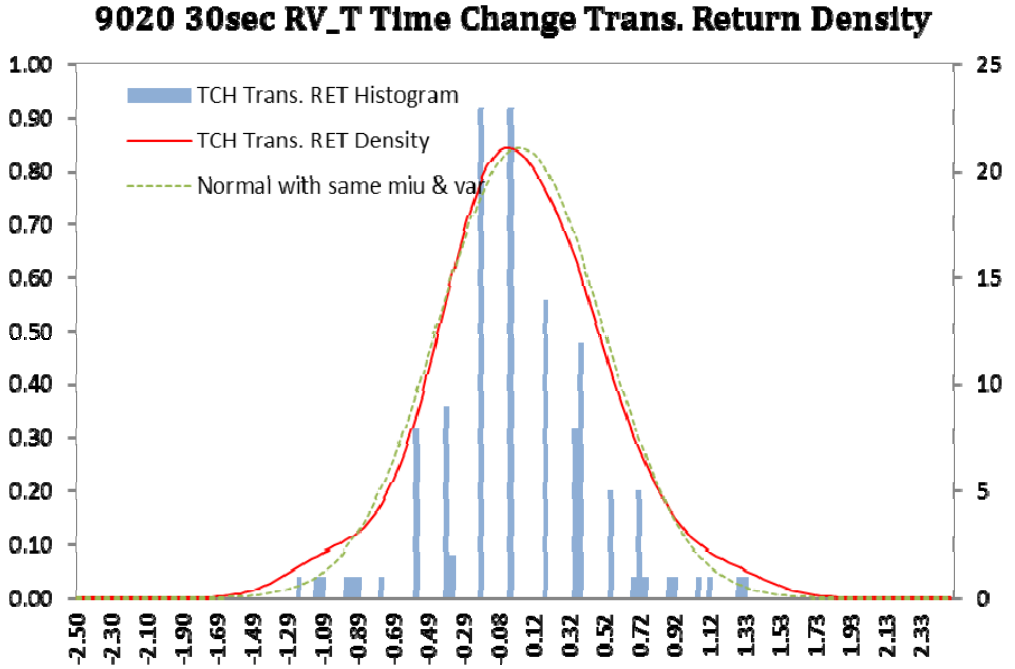




	H0 or H1	pval	JB stat	crit Val
Original Trans.RET	1	0.001	94,642.55	5.99
RV_T Cond.Trans.RET	0	0.233	2.53	5.71

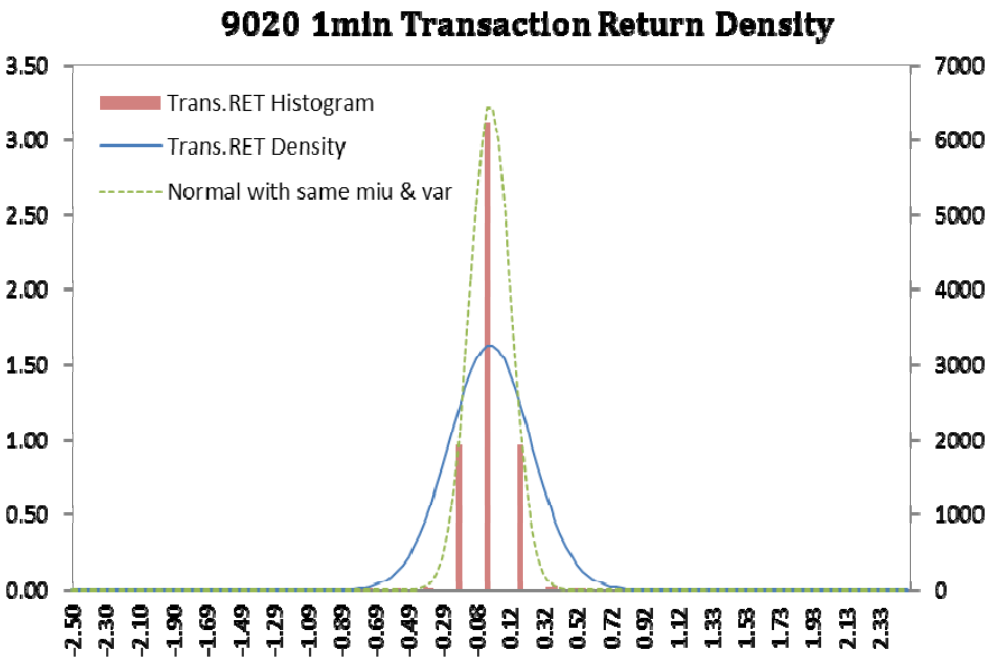
Return Density Comparison

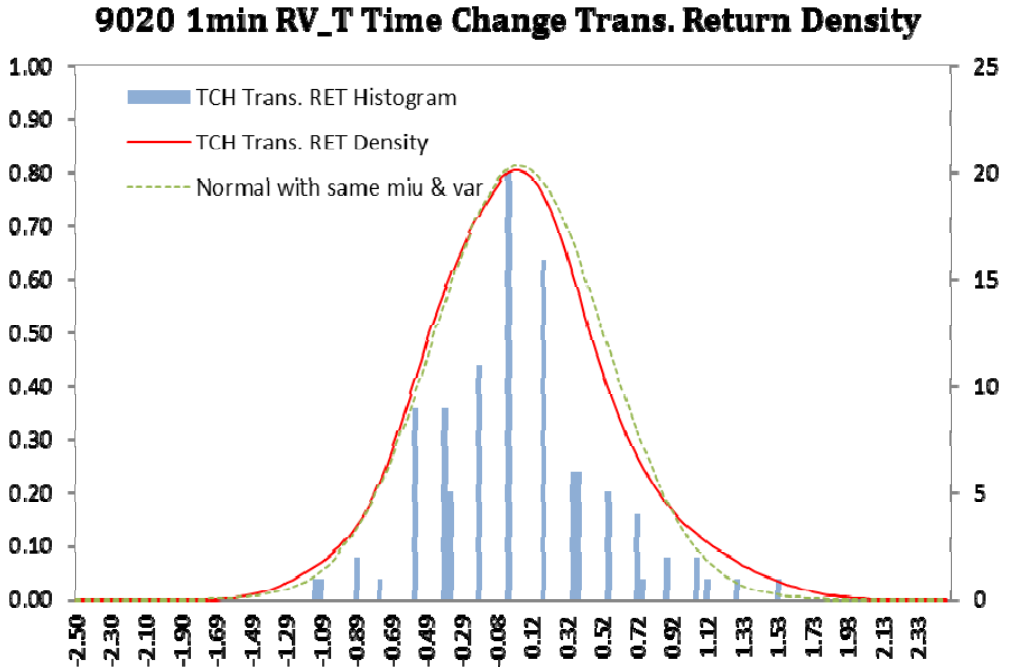




	H0 or H1	pval	JB stat	crit Val
Original Trans.RET	1	0.001	1,754.46	5.99
RV_T Cond.Trans.RET	0	0.427	1.39	5.52

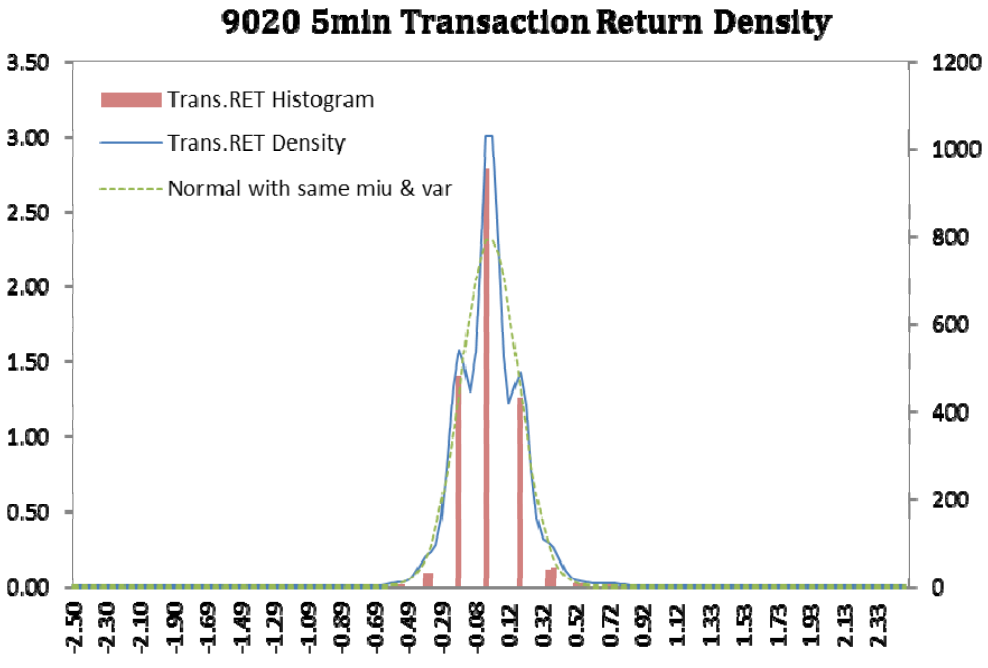
Return Density Comparison

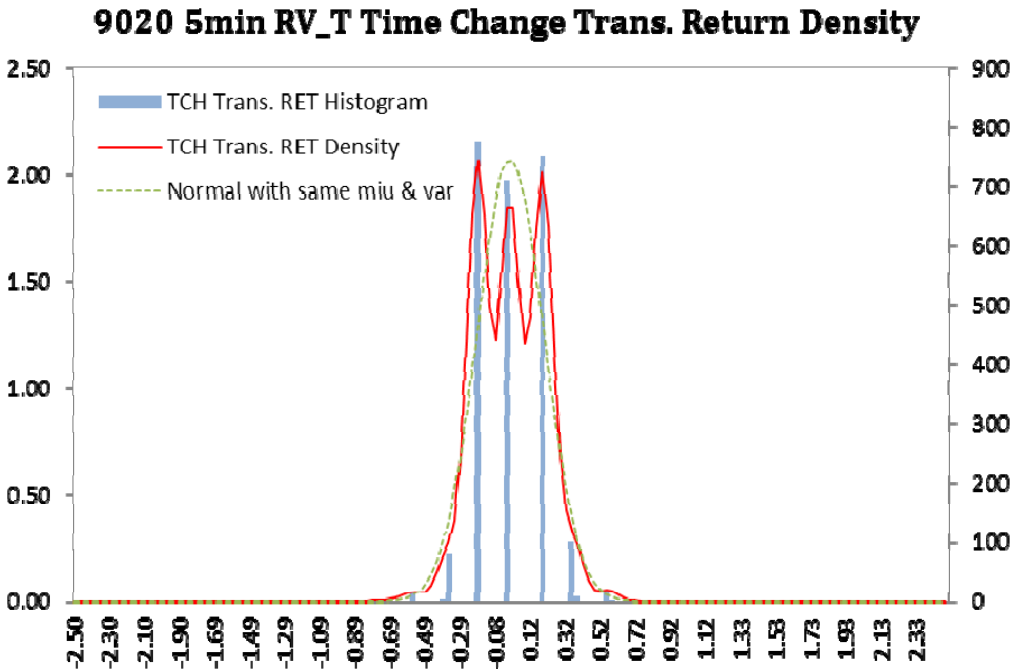




	H0 or H1	pval	JB stat	crit Val
Original Trans.RET	1	0.001	159.12	5.99
RV_T Cond.Trans.RET	0	0.168	2.77	5.45

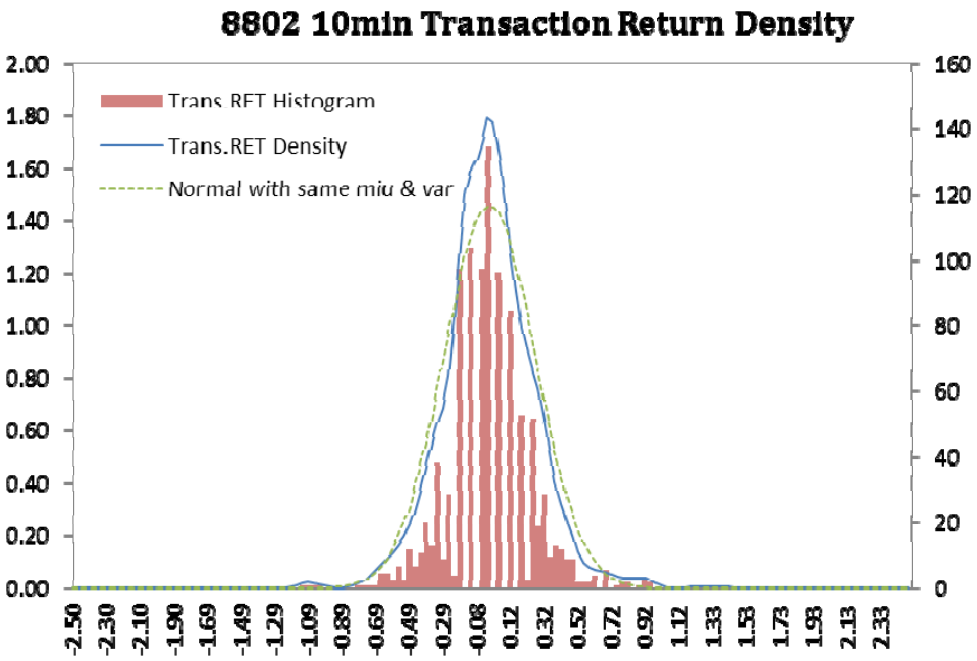
Return Density Comparison

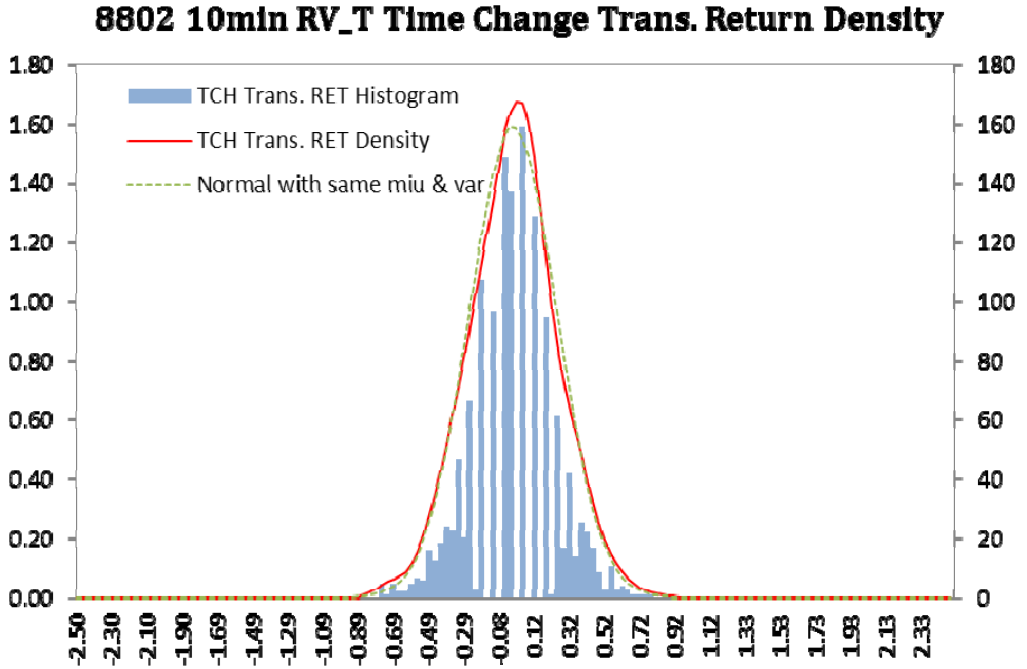




	H0 or H1	pval	JB stat	crit Val
Original Trans.RET	1	0.001	182.84	5.96
RV_T Cond.Trans.RET	0	0.129	4.03	5.97

Return Density Comparison





	H0 or H1	pval	JB stat	crit Val
Original Trans.RET	1	0.001	226.44	5.93
RV_T Cond.Trans.RET	0	0.294	2.39	5.95

Conclusion

- Differ from previous studies over 10 years ago, currently the automated trading system facilitates high-frequency trading.
  - Neither cumulative volumes nor cumulative number of trades are good proxies of stochastic time change variable.
  - Realized volatility estimated from transaction prices became the better proxy of time change variable.
  - Normality of the returns distribution conditioning on time change variable can be recovered, but an adverse effect can be observed when the original returns distribution exhibits normality.

Future Research

- Alternative methods to estimate time change variable regardless of physical time intervals.
- Alternative methods to impose condition in estimation of conditional return.