Sources of Great Recession: A Bayesian Approach of a Data Rich DSGE model with Time-Varying-Volatility Shocks

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Abstract

In order to investigate sources of Great Recession (Dec. 2007 to Jun. 2009) of the US economy in late 2000's, we modify the standard New Keynesian DSGE model by embedding financial frictions in both banking and corporate sectors. Further, the structural shocks in the model are assumed to possess stochastic volatility (SV) with leverage effect. Then, we estimate the model using the Data-Rich estimation method and utilize up to 40 macroeconomic time series in the estimation. In the light of a DSGE model, we suggest the following three empirical evidences in Great Recession; (1) negative bank net worth shock has gradually outspreaded before corporate net worth shock has burst down, (2) the Data-Rich approach and structural shocks with SV evaluate the contribution of corporate net worth shock to the substantial portion of macroeconomic fluctuations after Great Recession, in contrast to a standard DSGE model, and (3) Troubled Asset Relief Program (TARP) would work to bail out financial institutions, whereas balance sheets in corporate sector could not have stopped deteriorating yet. Incorporating time-varying-volatilities of shocks into the DSGE model make their credible bands narrower than half of constant volatilities, implying it is a realistic assumption of dynamics of structural shocks. It is plausible that the tiny volatilities (or the uncertainty) in ordinary times change to extraordinary magnitude at the turning points of business cycles. We also estimate that monetary policy shock has opposite leverage effect of SV which implies tightening policy makes interest rate more volatile.

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1 Introduction

It is reported that the Great Recession began in December 2007 and ended in June 2009, by the U.S. National Bureau of Economic Research (NBER). The emergence of sub-prime loan losses in 2007 began the recession and exposed other risky loans and over-inflated asset prices. With loan losses mounting and the collapse of Lehman Brothers on September 15, 2008, a major panic broke out on the interbank loan market. In the recession the financial crisis played a significant role in the failure of key businesses, declines in consumer wealth estimated in trillions of US dollars, and a downturn in economic activity leading to the 2008–2012 global recession and contributing to the European sovereign-debt crisis. Paul Krugman commented on this as seemingly the beginning of "a second Great Depression" in New York Times in January 2009. The central debate about the origin of the recession has been focused on the respective parts played by the public monetary policy and by the practices of private financial institutions. The U.S. Senate's Levin–Coburn Report asserted that the financial institution crisis, one of the causes, was the result of "high risk, complex financial products; undisclosed conflicts of interest; the failure of regulators, the credit rating agencies, and the market itself to rein in the excesses of Wall Street." In order to strengthen the financial sector, the Troubled Asset Relief Program (TARP), in which assets and equity are purchased from financial institutions by the U.S. government, was enforced and originally authorized expenditures of $700 billion in October 2008.

The purpose of this study is to argue macroeconomic fluctuations and mutual relationship among macroeconomic and financial endogenous variables and to identify what structural exogenous shocks contribute in the Great Recession in the light of a dynamic stochastic general equilibrium (DSGE) model. Because we obtain broad consensus that solvency and liquidity problems of the financial institutions are chief among the fundamental factors causing the recession itself as described above, it is plausible to embed financial frictions in both banking and corporate sectors of a New Keynesian DSGE model. In fact, according to Ireland (2011) who firstly attempted to analyze the impact of the recession using a New Keynesian DSGE model, there are three sets of considerations which are premature for existing DSGE models. First, banking failures and liquidity dry-ups should be endogenously explained with other fundamental macroeconomic variables for producing economic insights. Second, most recessions have always been accompanied by an increase in bankruptcies among financial and nonfinancial firms alike. And recessions featured systematic problems in banking and loan industry. And third, declines in housing prices and problems in credit markets might have played an independent and causal role in the Great Recession's severity. By identifying structural shocks generated from two financial frictions in both financial and nonfinancial sectors into a DSGE model, our study will cope with the former two exercises. In addition, we will focus on the extreme change of volatility in financial markets and across economy as a whole in the recession, by estimating time varying volatility of these structural shocks.

To this end, we will follow Nishiyama et al. (2011) who have already studied the US economy using a New Keynesian DSGE model with these two financial frictions in a Data Rich environment. In this model with asymmetric information between borrowers and lenders, banks have two roles generating two agency costs: one is the lenders of corporate sector and the other is the borrowers from depositors. To decompose the effects of the two kinds of agency costs on macroeconomic fluctuations might be important for sounding the origin of the recession as well as measuring how bad they damage the US economy. The data rich approach is a useful method separating coherence of endogenous variables with mutual relationship and measurement errors from many macroeconomic panel data, inducing to identify structural shocks more robustly. This study will extend estimated sample period to 2012Q2 and incorporate stochastic volatility (hereafter SV) model with leverage effect, which has been recently developed for measuring volatilities of stock returns in financial market, as explaining dynamics of structural shocks into the DSGE model above. Because we will more efficiently extract structural shocks as well as model variables by adopting the data rich approach, we will able to relax specifications of structural shocks and measure the impact of financial shocks on the real economy in the Great Recession and after it.

We will consider four alternative cases depending on the number of observation variables (11 vs. 40 observable variables) and specification of volatilities of structural shocks (constant volatility vs. time-varying volatility). It is expected that by adopting forty macroeconomic time series as observable variables, data rich information makes decomposition between measurement errors and model variables from data more robust, and that relaxation of specifying the volatilities makes rapid change of shocks more detailed. Comparing the four cases, we will suggest the following three empirical evidences in Great Recession; (1) negative bank net worth shock has gradually outspreaded before corporate net worth shock has burst down, (2) the data-rich approach and structural shocks with SV evaluate the contribution of corporate net worth shock to the substantial portion of macroeconomic fluctuations after the Great Recession, in contrast to a standard DSGE model, and (3) Troubled Asset Relief Program (TARP) would work to bail out financial institutions, whereas balance sheets in corporate sector could not have stopped deteriorating yet. Incorporating time-varying-volatilities of shocks into the DSGE model make their credible bands narrower than half of constant volatilities, implying it is a realistic assumption of dynamics of structural shocks. It is plausible that the tiny volatilities (or the uncertainty) in ordinary times change to extraordinary magnitude at the turning points of business cycles. We also estimate that monetary policy shock has opposite leverage effect of SV which implies tightening policy makes interest rate more volatile.

Finally, we compare the achievements of our study with the earlier studies from the three aspects: financal frictions, time varying volatilities of structural shocks and data rich approach. First, there is large literature estimating a large scaled DSGE model adopting a New Keynesian framework with nominal rigidities proposed by Chirstiano, Eichenbaum and Evans (CEE)(2005), e.g., Smets and Wouters (2007). And one financial friction between bank and corporate sectors is developed by Bernanke et al. (1999) and incorporated into a CEE model by Christensen and Dib (2008). Meanwhile, the other financial friction between banks and depositors are recently proposed by Gertler and Karadi (2011) and Gertler and Kiyotaki (2010). To the best of our knowledge, there is no literature combing these two frictions with a CEE model except Nishiyama et al. (2011) which is our based model.

Second, there are some researches allowing for time variation in the volatility of the structural disturbances. In this respect, Justiniano and Primiceri (2008) focused on the Great Moderation using model with structural shocks including SV models but neither financial frictions nor leverage effect. And Liu et al (2011) estimated a DSGE model without financial frictions by another approach regime-switching the volatilities for analyzing sources of the Great Moderation. And third, there is few literature dealing with the data rich approach proposed by Boivin and Giannoni (2006) except Kryshko (2011), Nishiyama et al. (2011) and Iiboshi et al. (2012). Our study is the first attempt of combination of data rich approach and time varying volatilities of structural disturbances.

The paper is organized as follows. Section 2 provides the framework of the DSGE model including the data rich approach and structural SV shock with leverage effect. Section 3 illustrate two financial fictions of the DSGE model. Section 4 presents the estimation technique. In Section 5, preliminary setting of structural parameters and data description are dealt with. Section 6 discusses the estimation results and interpretation of the Great Recession in terms of New Keynesian model. Section 7 concudes.

2 Data Rich Approach with Stochastic Volatility Shocks

2.1 Stochastic Volatility with Leverage in DSGE models

A DSGE model is the system of equations which is a collection of constraints and first-order conditions derived from micro-founded models in which economic agents such as households and firms are assumed to solve intertemporal optimization problem based on their rational expectation under economic frictional environment. A log-linearized model is derived in the neighborhood of stady state of the DSGE model by the first-order approximation. Using, for example, Sims' (2002) method, the law of motion around steady state of the model solved from log-linear approximation is represented as below.

$$\underbrace{\boldsymbol{S}_t}_{\mathrm{N\times 1}} = \underbrace{\boldsymbol{G}(\boldsymbol{\theta})}_{\mathrm{N\times N}} \boldsymbol{S}_{t-1} + \underbrace{\boldsymbol{E}(\boldsymbol{\theta})}_{\mathrm{N\times M}} \underbrace{\boldsymbol{\varepsilon}_t}_{\mathrm{M\times 1}}, \tag{2.1}$$

where $\boldsymbol{S}_t$ is a $N \times 1$ vector of endogenous variables referred to as model variables, whereas $\boldsymbol{\varepsilon}_t$ is a $M \times 1$vector of exogenous disturbances represented structural shocks. $\boldsymbol{\theta}$ is structural parameters derived DSGE models based on macroeconomic theory. In particular, its core parameters are referred to as *deep parameters* which govern the rational behaviors of economic agents. Matrices $\boldsymbol{G}(\boldsymbol{\theta})$, and $\boldsymbol{E}(\boldsymbol{\theta})$ are the function of $\boldsymbol{\theta}$. So far, disturbance terms ε_t are assumed to be i.i.d. (independent and identically distributed) normal distributions in most of DSGE models. This study extends this assumption and replaces them with time varying variances as below.

$$\boldsymbol{\varepsilon}_t = \underbrace{\boldsymbol{\Sigma}_t}_{\mathrm{M\times M}} \underbrace{\boldsymbol{z}_t}_{\mathrm{M\times 1}}, \tag{2.2}$$

$$\boldsymbol{z_t} \sim \text{i.i.d.}\, \boldsymbol{N}\,(\,0,\ \boldsymbol{I_M}\,),$$

$$\boldsymbol{\Sigma_t} = \text{diag}(\sigma_{1,t},\ \sigma_{2,t}\ \cdots\ \sigma_{M,t}),$$

where $\boldsymbol{z}_t$ is a $M \times 1$ vector with all elements following standard normal distribution. $\boldsymbol{I}_M$ is a $M \times M$ identity matrix. And, $\boldsymbol{\Sigma}_t$ is standard deviation; or volatility, of disturbance shocks $\boldsymbol{\varepsilon}_t$, and represented as a diagonal matrix with elements such as $\sigma_{1,t}. \cdots, \sigma_{M,t}$ which move following a stochastic volatility model.

$$\log \sigma_{i,t+1} = \mu_i + \phi_i(\log \sigma_{i,t} - \mu_i) + \eta_{i,t}, \quad i = 1, 2, \cdots, M, \tag{2.3}$$

$$\begin{pmatrix} z_{i,t} \\ \eta_{i,t} \end{pmatrix} \sim \text{i.i.d.}\ N(0,\ \Omega_i), \quad \Omega_i = \begin{bmatrix} 1 & \rho_i\omega_i \\ \rho_i\omega_i & \omega_i^2 \end{bmatrix} \tag{2.4}$$

Because volatility σ_{it} necessarily has positive value, it is converted into logarithmic value in order to throw off the limitation of sign. μ_i denotes the mean of volatility $\sigma_{i,t}$ of i-th shock. And ϕ_i is a coefficient of persistence of the i-th volatility. In this SV model, leverage effect of volatility $\sigma_{i,t}$ is introduced as equation (2.4) which measures the correlation between the sign of disturbance terms and the size of volatility. Typically, the correlation ρ is negative signifying that a negative stock return ($z_{i,t} < 0$) tends to increase the volatility of a stock price ($\eta_{i,t} > 0$). We try to verify whether volatility of net-worth shock in firms and bank sectors increases when they are negative. Although Justiniano and Primiceri (2008) inserted a SV model into a DSGE model, they did not consider leverage effect of volatility. This extension is one of advantages of our model.

2.2 Data Rich DSGE Models

2.2.1 Significance of Data Rich DSGE models

A data rich DSGE model is developed by Boivin and Giannoni (2006), and Kryshko (2011)[1] and composed from combination of a DSGE model and a dynamic factor model (DFM). This combination is basically possible since their frameworks are based on same state space representation. As result, it enjoys the advantages of two existing models, and compensates the drawbacks of these two models. We consider these advantages and properties of the combination before explaing the two models.

(1) Drawbacks of a standard DSGE model, and overcome by a DFM

In general speaking, it is difficult to identify model variables S_t and measurement error e_t from observable varibles X_t in a standard DSGE model. However, it is plausible that data X_t is composed from comovement (or systematic) components and idiosyncratic components indicating measurement errors or noise which are not correlated with systematic movements. That is,

$$\text{data} = \text{ common (or systematic) component} + \text{ idiosyncratic component},$$

[1]Recently, Schorfheide et al. (2010) was published as an empirical study applying a data-rich DSGE approach to forecasting economic fluctuations.

where two components are unobservable. Accordingly, it is assumed that there is no idiosyncratic component or measurement errors and "data = systematic components" in a standard DSGE model. A separation approach of these two factors is a DFM. In this model, comovement is likely to be a component explained from economic system affected by multi-variables with their mutual impacts. It indicates that a dynamic equation of comovement might be appropriate to be represented as a VAR model. On the other hand, idiosyncratic components should be expressed as univariable AR process, since they are thought to independently fluctuate.

(2) Drawbacks of a DFM, and overcome by a standard DSGE model

A DFM is nowadays focused on as a model decomposing comovement and idiosyncratic errors from data. However, is it possible to decompose them only from a statistical method? Conventionally, we just focused on generic correlation among macroeconomic variables but not on causal association among them from the viewpoint of an economic model. But, according to Woodford (2009), new neo-classical synthesis providing the theoretical foundation for much of contemporary mainstream macroeconomics asserts that models built out of theory should be focused on instead of looking at more generic correlations among data. In a DFM, comovement tends to be measured from a conventional VAR model in which it is difficult to interpret their coefficients from economic theory. Instead, a DSGE model expresses comovement of multi-variables from causal association and theoretical coherence, based on a micro-founded dynamic model, following the spirit of new neo-classical synthesis. That is, converting from a conventional VAR model to a DSGE model in a systematic component part of a DFM indicates that

$$\text{comovement}\,(\text{systematic variation}) = \text{genetic correlation}\ ,$$

$$\Rightarrow\ \text{comovement} = \text{causal association}$$

which induces resolution of drawback of a DFM.

(3) Synergetic effect of combination

According to Stock and Watson (2002a,b), consistency of DFM suggests that increasing the number of observation series X_t is expected to increase the certainty that idiosyncratic components not explained by an economic system, are removed from data using a DFM. It improves accuracy of measuring the comovement S_t and exogenous structural shocks ε_t. If the estimation of structural shocks ε_t successfully explain actual business cycles, this will indicate validity of the structural shocks and in addition, that of the DSGE model. And in data rich framework, same data set is applicable even for DSGE models with different model variables, so that the possibility of model selection among many alternative models emanates. It implies that data rich approach is expected to contribute the evaluation and selection among DSGE models from the point of view of validity of structural shocks and marginal likelihood (or Bayes factor).

2.2.2 Dynamic Factor Model (DFM)

Recently, the estimation method of DFMs are rapidly developed and applied for many fields of macroeconomics and finance. The DFM, which are a statistical model estimating common factors of business cycles, are proposed by Sargent and Sims (1977) and empirically applied by Stock and Watson (1989) who extract one unobserved common factor of business fluctuation from many macroeconomic time series using Kalman filter.[2]

The DFMs are represented by state space models composed from following three linear equations. Let $\boldsymbol{F}_t$ denote the $N \times 1$ vector of unobserved common factor, and $\boldsymbol{X}_t$ denote the $J \times 1$ vector of massive panel of macroeconomic and financial data. Note that $J \gg N$.

$$\underbrace{\boldsymbol{X}_t}_{J\times 1} = \underbrace{\Lambda}_{J\times N}\underbrace{\boldsymbol{F}_t}_{N\times 1} + \underbrace{\boldsymbol{e}_t}_{J\times 1}, \tag{2.5}$$

$$\underbrace{\boldsymbol{F}_t}_{N\times 1} = \underbrace{\mathrm{G}}_{N\times N}\underbrace{\boldsymbol{F}_{t-1}}_{N\times 1} + \underbrace{\boldsymbol{\varepsilon}_t}_{N\times 1}, \qquad \boldsymbol{\varepsilon}_t \sim \text{i.i.d.}\, N(\boldsymbol{0},\, \mathrm{Q}), \tag{2.6}$$

$$\underbrace{\boldsymbol{e}_t}_{J\times 1} = \underbrace{\boldsymbol{\Psi}}_{J\times J}\underbrace{\boldsymbol{e}_{t-1}}_{J\times 1} + \underbrace{\boldsymbol{\nu}_t}_{J\times 1} \qquad \boldsymbol{\nu}_t \sim \text{i.i.d.}\, N(\boldsymbol{0},\, \mathrm{R}), \tag{2.7}$$

where Λ is $J \times N$ matrix of factor loadings, $\boldsymbol{e}_t$ is the idiosyncratic components (or measrement errors) which is allowed to be serially correlated as equation (2.7). G is $N \times N$ matrix, and common factor $\boldsymbol{F}_t$ is following AR process (2.6). Matrices, $\boldsymbol{\Psi}$, Q and R are assumed to be diagonal in an *exact* DFM as Stock and Watson (2005). Equation (2.5) is a measurement equation, and equations (2.6) and (2.7) are transition equations. A state space model is composed from the two kinds of equations (2.5), (2.6) and (2.7).

The property of the model is to decompose common components, $\Lambda \boldsymbol{F}_t$, and idiosyncratic component $\boldsymbol{e}_t$ from massive panel of macroeconomic and financial data $\boldsymbol{X}_t$ in (2.5). Meanwhile, it is difficult to make an interpretation of factor $\boldsymbol{F}_t$ in terms of economic theory, since above equations are statistically estimated by a conventional VAR model (2.6) and the parameters are not derived from a structural model with micro-foundation.

2.2.3 Data-Rich DSGE Model

The idea of data-rich approach is to extract the common factor $\boldsymbol{F}_t$ from *massive panel* of macroeconomic and financial time series data $\boldsymbol{X}_t$ and to match the model variable $\boldsymbol{S}_t$ to the common factor $\boldsymbol{F}_t$. A virtue of this approach is that even if a model variable $\boldsymbol{S}_t$ and observed data $\boldsymbol{X}_t$ are slightly detached, one can estimate the DSGE model by matching model variables to the common factors extracted

[2]Stock and Watson (2002a,b) developed *approximate* DFMs using principal component analysis, extracting several common factors from more than one hundred macroeconomic time series and verifying that these factors include useful information on forecasting of macroeconomic time series. Nowadays, there are many studies in the literature concerning theoretical and empirical studies of DFMs. For example, Boivin and Ng (2005, 2006), Stock and Watson (2002a, b, 2005). The survey of DFMS covering the latest studies is Stock and Watson (2006, 2010). Kose et al. (2003) tried to extract common factors of world-wide and regional business cycles using a Kalman filter and DFM.

from large panel data and expect improved efficiency in estimating the parameters and structual shocks of the model.

The DSGE model is known to be state space models and estimated using Kalman filter as well as the DFM. So we can apply the framework of the DFM to a DSGE model. But the big difference between a DFM and a DSGE model is the meaning of their parameters. The those of later is derived from structural parameters θ. A data -rich DSGE model is given as

$$
\begin{aligned}
\underbrace{\boldsymbol{X}_t}_{J\times 1} &= \underbrace{\boldsymbol{\Lambda}(\boldsymbol{\theta})}_{J\times N}\underbrace{\boldsymbol{S}_t}_{N\times 1} + \underbrace{\boldsymbol{e}_t}_{J\times 1}, && (2.8)\\
\underbrace{\boldsymbol{S}_t}_{N\times 1} &= \underbrace{\mathbf{G}(\boldsymbol{\theta})}_{N\times N}\underbrace{\boldsymbol{S}_{t-1}}_{N\times 1} + \underbrace{\mathbf{E}(\boldsymbol{\theta})}_{N\times M}\underbrace{\boldsymbol{\varepsilon}_t}_{M\times 1}, \qquad \boldsymbol{\varepsilon}_t \sim \text{i.i.d.}\, N(\mathbf{0},\, \mathbf{Q}(\boldsymbol{\theta})), && (2.9)\\
\underbrace{\boldsymbol{e}_t}_{J\times 1} &= \underbrace{\boldsymbol{\Psi}_t}_{J\times J}\underbrace{\boldsymbol{e}_{t-1}}_{J\times 1} + \underbrace{\boldsymbol{\nu}_t}_{J\times 1} \qquad \boldsymbol{\nu}_t \sim \text{i.i.d.}\, N(\mathbf{0},\, \mathbf{R}), && (2.10)
\end{aligned}
$$

where observable variables $\boldsymbol{X}_t$ are a $J\times 1$ vector, state variables $\boldsymbol{S}_t$ are a $N\times 1$vector, and structural shocks $\boldsymbol{\varepsilon}_t$ are a $M\times 1$ vector. In a data-rich DSGE model, the number of observable variables is much larger than that of state variables ($J \gg N$) as well as a DFM. (On the other hand, in a regular DSGE model $J \leq N$.) And idiosyncratic components $\boldsymbol{e}_t$, which are a $J \times 1$ vector, means measurement errors following AR (1) process in (2.10) . (2.8) shows measurement equation which splits off components of common factors $\boldsymbol{S}_t$ and idiosyncratic components $\boldsymbol{e}_t$, from massive panel of macroeconomic indicators $\boldsymbol{X}_t$ and which consists common factors$\boldsymbol{S}_t$ with economic concepts.[3] A transition equation (2.9) indicates AR(1) process of common factors (or

[3]Alternatively, representing matrix-base equation (2.8) as element-base equation as below, the framework of a data-rich DSGE model might be more understandable. As can be seen from the second row of matrix $\boldsymbol{\Lambda}(\boldsymbol{\theta})$: $[\lambda_{y2}\ 0\ \cdots\ 0]$, observable variables are directly relation with only one specified model variable$\boldsymbol{S}_t$. And in order to identify the magnitude of each linkage model variable$\boldsymbol{S}_t$., the value of λ of just one variable of data indicators $\boldsymbol{X}_t$ is unity as the first $N^{\#1}$ rows of matrix $\boldsymbol{\Lambda}$: $N^{\#1} \times N^{\#1}$ identity matrix. Meanwhile, the parameters of information indicators (the remaining part of $\boldsymbol{X}_t$) can be represented as full elements in matrix $\boldsymbol{\Lambda}$ such as $[\lambda_{11}\ \lambda_{12}\ \cdots\ \lambda_{1n}]$ which connects information indicators through many-to-many relation with all model variable $\boldsymbol{S}_t$.

$$
\underbrace{\begin{bmatrix}
\text{Output Gap series \#1}\\
\text{inflation series \#1}\\
\vdots\\
\text{Output Gap series \#2}\\
\text{inflation series \#2}\\
\vdots\\
\text{Output Gap series \#}n\\
\text{inflation series \#}n\\
\vdots\\
-------\\
\text{information series \#1}\\
\vdots\\
\text{information series \#}n_i
\end{bmatrix}}_{\boldsymbol{X}_t\ (J\times 1)}
=
\underbrace{\begin{bmatrix}
1 & 0 & \cdots & 0\\
0 & 1 & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots\\
\lambda_{y2} & 0 & \cdots & 0\\
0 & \lambda_{\pi 2} & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots\\
\lambda_{yn} & 0 & \cdots & 0\\
0 & \lambda_{\pi n} & \cdots\vdots & 0\\
\vdots & \vdots & \cdots & \vdots\\
--- & -- & -- & ---\\
\lambda_{11} & \lambda_{12} & \cdots & \lambda_{1N}\\
\vdots & \vdots & \vdots & \vdots\\
\lambda_{ni1} & \lambda_{ni2} & \cdots & \lambda_{niN}
\end{bmatrix}}_{\boldsymbol{\Lambda}(\boldsymbol{\theta})\ (J\times N)}
\underbrace{\begin{bmatrix}
\hat{y}_t\\
\hat{\pi}_t\\
\vdots\\
\bar{S}^{non}_{N,t}
\end{bmatrix}}_{\bar{\boldsymbol{S}}_t\ (N\times 1)}
+
\underbrace{\begin{bmatrix}
e_{y1\ t}\\
e_{\pi 1\ t}\\
\vdots\\
e_{y2\ t}\\
e_{\pi 2\ t}\\
\vdots\\
e_{yn\ t}\\
e_{\pi n\ t}\\
\vdots\\
--\\
e_{i1\ t}\\
\vdots\\
e_{in\ t}
\end{bmatrix}}_{\boldsymbol{e}_t\ (J\times 1)},
$$

where $\hat{y}_t$ is the concept of output gap, $\hat{\pi}_t$ is the concept of inflation.

model concepts) $\boldsymbol{S}_t$ with structural shocks ε_t and also dynamics converging to a rational expectation equilibrium determined by a macroeconomic model. From (2.8) and (2.9), we can see that the model varaible $\boldsymbol{S}_t$ is realized from inter-correlation of data indicators $\boldsymbol{X}_t$ whose movement is coherent with each economic concept derived from economic theory. In contrast, measurement errors $\boldsymbol{e}_t$ fluctuate from only specific factors of each observable variables $\boldsymbol{X}_t$ but do not depend on economic theory and other data indicators. [4]

Structural shocks ε_t and disturbance terms $\boldsymbol{\nu}_t$ of measurement errors $\boldsymbol{e}_t$ follow normal distributions, i.e., $\varepsilon_t \sim$ i.i.d. $N(0,\, \mathrm{Q}(\boldsymbol{\theta}))$ and $\boldsymbol{\nu}_t \sim$ i.i.d. $N(\mathbf{0},\, \mathrm{R})$, respectively. And their variance covariance matrix $\mathrm{Q}(\boldsymbol{\theta})$, and R are positive definite and diagonal matrix. Coefficients Ψ of AR(1) process (2.10) is also diagonal matrix. These indicate that measurement errors e_t are independent with each other in terms of cross section but dependent with their lag variables in terms of time series restriction. Matrices $\mathrm{G}(\boldsymbol{\theta})$, $\mathrm{E}(\boldsymbol{\theta})$ and $\mathrm{Q}(\boldsymbol{\theta})$ are functions of structural parameters $\boldsymbol{\theta}$.

2.3 Data Rich DSGE models with Stochastic Volatility

2.3.1 Stochastic Volatility in a Data-Rich DSGE models

In the previous subsection, structural shocks ε_t of the DSGE model are assumed to follow i.i.d. normal distribution. Since adoption of data-rich approach is expected to improve the accuracy of structural shocks, the next task of the approach is thought to specify the structural shocks. In this paper, we consider relaxation of assumption of the shocks by inserting a SV model with leverage effects into a data-rich DSGE model. If the flexibility of the shocks changes to interpret sources of Great Recession described in Section 6, to insert this will be valuable for analyzing business cycles from the DSGE model's views. Combining a SV model described in Section 2.1 and a data-rich DSGE model, our model is represented as

$$\underbrace{\boldsymbol{X}_t}_{J\times 1} = \underbrace{\Lambda(\boldsymbol{\theta})}_{J\times N}\underbrace{\boldsymbol{S}_t}_{N\times 1} + \underbrace{\boldsymbol{e}_t}_{J\times 1}, \tag{2.11}$$

$$\underbrace{\boldsymbol{S}_t}_{N\times 1} = \underbrace{\mathrm{G}(\boldsymbol{\theta})}_{N\times N}\underbrace{\boldsymbol{S}_{t-1}}_{N\times 1} + \underbrace{\mathrm{E}(\boldsymbol{\theta})}_{N\times M}\underbrace{\varepsilon_t}_{M\times 1}, \tag{2.12}$$

$$\underbrace{\boldsymbol{e}_t}_{J\times 1} = \underbrace{\Psi_t}_{J\times J}\underbrace{\boldsymbol{e}_{t-1}}_{J\times 1} + \underbrace{\boldsymbol{\nu}_t}_{J\times 1} \qquad \boldsymbol{\nu}_t \sim \text{i.i.d. } N(\mathbf{0},\, \mathbf{R}), \tag{2.13}$$

$$\boldsymbol{\varepsilon_t} = \underbrace{\boldsymbol{\Sigma_t}}_{\mathrm{M}\times\mathrm{M}}\ \underbrace{\boldsymbol{z_t}}_{\mathrm{M}\times 1}, \tag{2.14}$$

$$\boldsymbol{z_t} \sim \text{i.i.d.}\, \boldsymbol{N}\,(\,0,\, \boldsymbol{I_M}\,),$$

$$\boldsymbol{\Sigma_t} = \mathrm{diag}(\sigma_{1,t},\, \sigma_{2,t}\, \cdots\, \sigma_{M,t}),$$

[4]Measurement errors play important role, since we could expect that they remove some degree of undesirable relation between observable variables and model concept variable influenced by model misspecification and mismatch of model concepts into observable variables. In addition, stochastic singularities can be avoided with measurement errors in a data-rich environment.

$$\log \sigma_{i,t+1} = \mu_i + \phi_i(\log \sigma_{i,t} - \mu_i) + \eta_{i,t}, \quad i = 1, 2, \cdots, M, \tag{2.15}$$

$$\begin{pmatrix} z_{i,t} \\ \eta_{i,t} \end{pmatrix} \sim \text{i.i.d. } N(0,\ \Omega_i), \quad \Omega_i = \begin{bmatrix} 1 & \rho_i\omega_i \\ \rho_i\omega_i & \omega_i^2 \end{bmatrix} \tag{2.16}$$

where the notations of system of equations is the same as the previous sections 2.1 and 2.2, so that we omit the explanation of them here. Compared to Justiniano and Primiceri (2008), this study extends their model to data rich approach and adds leverage effect to the SV model. Meanwhile, we combine the SV model with the data-rich model, in contrast to Boivin and Giannoni (2006)'s approach. In addition, these two models consider only nominal rigidities of goods and labor markets, whereas our model considers two financial frictions of bank sector as well as nominal rigidities of goods and labor markets. These extensions in terms of both economic and econometric approaches are thought to be appropriate for analyzing the sources of Great Recession.

2.3.2 Transformation into Estimated State Space Model

It is difficult to directly estimate state space representation (2.11), (2.12) and (2.13) as shown above for applying to large panel data set, since the size of matrix in transition equations (2.12) and (2.13) is equal to the total number of model variables $\boldsymbol{S}_t$ and measurement errors $\boldsymbol{e}_t$. This framework induces a dramatically increase of the matrix as the number of data $\boldsymbol{X}_t$ is increasing. To avoid this situation, we try to transform to small size for the transition equations as following. That is, we eliminate AR process of measurement errors of (2.10) and express from only $\boldsymbol{\nu}_t$ with i.i.d. process for measurement errors. Inserting (2.13) into (2.11), measurement equation is transformed as

$$(\mathbf{I} - \boldsymbol{\Psi} L)\boldsymbol{X}_t = (\mathbf{I} - \boldsymbol{\Psi} L)\boldsymbol{\Lambda}\boldsymbol{S}_t + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim \text{i.i.d. } N(\mathbf{0}, \mathbf{R}).$$

where L is lag operator. By using notations $\tilde{\boldsymbol{X}}_t = \boldsymbol{X}_t - \boldsymbol{\Psi}\boldsymbol{X}_{t-1}$,and $\tilde{\boldsymbol{S}}_t = [\boldsymbol{S}_t' \ \boldsymbol{S}_{t-1}']'$, this equation can be rewritten as

$$\tilde{\boldsymbol{X}}_t = \underbrace{[\boldsymbol{\Lambda} \quad -\boldsymbol{\Psi}\boldsymbol{\Lambda}]}_{\tilde{\boldsymbol{\Lambda}}} \underbrace{\begin{bmatrix} \boldsymbol{S}_t \\ \boldsymbol{S}_{t-1} \end{bmatrix}}_{\tilde{\boldsymbol{S}}_t} + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim \text{i.i.d. } N(\mathbf{0}, \mathbf{R}). \tag{2.17}$$

In the similar way, transition equation (2.12) is also rewritten as

$$\underbrace{\begin{bmatrix} \boldsymbol{S}_t \\ \boldsymbol{S}_{t-1} \end{bmatrix}}_{\tilde{\boldsymbol{S}}_t} = \underbrace{\begin{bmatrix} \mathbf{G}(\boldsymbol{\theta}) & \mathbf{O} \\ \mathbf{I} & \mathbf{O} \end{bmatrix}}_{\tilde{\mathbf{G}}} \underbrace{\begin{bmatrix} \boldsymbol{S}_{t-1} \\ \boldsymbol{S}_{t-2} \end{bmatrix}}_{\tilde{\boldsymbol{S}}_{t-1}} + \underbrace{\begin{bmatrix} \mathbf{E}(\boldsymbol{\theta}) \\ \mathbf{O} \end{bmatrix}}_{\tilde{\mathbf{E}}} \boldsymbol{\varepsilon}_t, \tag{2.18}$$

where $\mathbf{I}$ is a $N \times N$ identity matrix. Estimation method of data rich DSGE model is explained using state space model (2.17) and (2.18), and we estimate this model using Bayesian Method via MCMC in Section 4. For convenience, we set parameters of measurement equation (2.17) as $\boldsymbol{\Gamma} = \{\boldsymbol{\Lambda}, \boldsymbol{\Psi}, \mathbf{R}\}$. And Bayesian estimation of prameters $\boldsymbol{\Gamma}$ are following Chib and Greenberg (1994) which is described in Appendices A3.

3 The DSGE model with Two Financial Frictions in Corporate and Banking Sectors

In order to model the balance sheets of the corporate and banking sectors in a DSGE framework, this paper combines the essence of Bernanke, Gertler, and Gilchrist (hereafter, BGG) (1999), Gertler and Karadi (2011), and Gertler and Kiyotaki (2010). We adopt the stylized DSGE model based on CEE (2005) and Smets and Wouters (2003, 2007), which focused on nominal rigidities of price level and wage as well as quadratic adjustment cost of investment, and embed the financial frictions of corporate and banking sectors to it. This section is the high light modeling the frictions in our DSGE model. The rest of our model is described in Appendix A4.

3.1 Financial Friction in Corporate Sector

3.1.1 Enterance and Exit of Entrepreneurs

Following BGG (1999), there is a continuum of entrepreneurs indexed by $j \in [0, 1]$ where each entrepreneur is risk-neutral and has a finite expected horizon.[5] Each entrepreneur faces an exogenous time-varying stochastic survival rate of γ_{t+1}^E from period t to $t+1$ which is common across all entrepreneurs.[6]

Between period t and $t+1$, after $1-\gamma_{t+1}^E$ fraction of entrepreneurs have exited from the business, exactly the same amount of new entrepreneurs will enter the business so that the population of entrepreneurs in the economy remains the same (i.e., fraction f^E of the total members of the household) from period t to $t+1$. Each entering entrepreneur receives a 'start-up' transfer from the household and the total 'start-up' transfer from the household will be equal to the constant fraction ξ^E of aggregate net worth available in the corporate sector, n_t^E, i.e., $\xi^E n_t^E$. For $1-\gamma_{t+1}^E$ fraction of entrepreneurs who happened to exit the business, they will first sell off the capital they purchased last period and retire all of their debts before maturity. And then, they will transfer their remaining net worth back to the household. The total amount of transfers from exiting entrepreneurs to the household will be $(1-\gamma_{t+1}^E)n_t^E$. Accordingly, net transfer, Ξ_{t+1}^E, that the household receives from entrepreneurs at period $t+1$ is $(1-\gamma_{t+1}^E-\xi^E)n_t^E$.

3.1.2 Individual Entrepreneur's Problem

Each entrepreneur produces homogenous intermediate goods, $y_t(j)$, and they are perfectly competitive when selling their products to retailers. The production function for the intermediate goods is given by

$$y_t(j) = \omega_t(j) A_t k_t(j)^\alpha l_t(j)^{1-\alpha}, \tag{3.1}$$

where $k_t(j)$ is capital inputs and $l_t(j)$ is labor inputs. The total factor productivity shock (hereafter, TFP shock), A_t, is common across all entrepreneurs. However, following Carlstrom and Fuerst (1997) and BGG (1999), we assume each

[5] These assumptions ensure that each entrepreneur will not accumulate enough net worth to self-finance their new capital.

[6] We assume that the stochastic process of γ_t^E is uncorrelated with any other shocks in the economy and has its mean equal to γ^E, i.e., $E[\gamma_t^E] = \gamma^E$.

entrepreneur is subject to an idiosyncratic shock, $\omega_t(j)$, which is a private information to entrepreneur j and assumed to be i.i.d. shock with mean equal to one, i.e., $E[\omega_t(j)] = 1$.

The balance sheet statement of each entrepreneur at the end of period t can be expressed as

$$q_t k_{t+1}(j) = b_t^E(j) + n_t^E(j) \tag{3.2}$$

where q_t is the real price of capital, $k_{t+1}(j)$ is the capital which will be used for production in period $t+1$ but purchased at the end of period t, $b_t^E(j)$ is the real debt issued at period t and $n_t^E(j)$ is the net worth at period t. With the assumption of risk-neutrality and finite planning horizon, net worth itself is never enough in financing the cost of capital purchase and, therefore, each entrepreneur will rely on external financing in equilibrium.

The income statement for entrepreneur j is specified as follow

$$n_t^E(j) = p_t^{mc}(j)y_t(j) - w_t l_t(j) - \frac{R_{t-1}^E(j)}{\pi_t} b_{t-1}^E(j) + q_t(1-\delta)k_t(j) \tag{3.3}$$

where $p_t^{mc}(j)$ is the real price of intermediate goods j, $R_{t-1}^E(j)/\pi_t$ is the real rate of borrowing cost ($R_{t-1}^E(j)$ is nominal borrowing rate and π_t is inflation rate) and δ is capital depreciation rate.

Each entrepreneur entering period t maximizes her discounted cash flow by choosing capital inputs, labor inputs and debt issuance subject to (3.1), (3.2), and (3.3).[7] The FOCs for each entrepreneur j are given by

$$\begin{aligned} w_t &= (1-\alpha)\frac{p_t^{mc}(j)y_t(j)}{l_t(j)} && (3.4) \\ E_t\left[\gamma_{t+1}^E \frac{R_t^E(j)}{\pi_{t+1}}\right] &= E_t\left[\gamma_{t+1}^E \frac{\alpha p_{t+1}^{mc}(j)y_{t+1}(j)/k_{t+1}(j) + (1-\delta)q_{t+1}}{q_t}\right]. && (3.5) \end{aligned}$$

(3.4) equates marginal cost of labor to marginal product of labor and, thus, can be thought of as labor demand function by entrepreneur j. (3.5) equates the expected marginal cost of capital financed by debt to the expected marginal return of capital financed by debt and can be thought of as capital demand function by entrepreneur j. Since stochastic survival rate, γ_{t+1}^E, is uncorrelated to any other shocks in the economy, (3.5) can be further rearranged as

$$E_t\left[\frac{R_t^E(j)}{\pi_{t+1}}\right] = E_t\left[\frac{\alpha p_{t+1}^{mc}(j)y_{t+1}(j)/k_{t+1}(j) + (1-\delta)q_{t+1}}{q_t}\right] \tag{3.6}$$

Under the assumption of risk-neutrality, introduction of stochastic survival rate will not alter the capital demand equation for any entrepreneur j compared to the case with constant survival rate as in BGG (1999).

[7]Each entrepreneur is a price-taker in the labor market, financial market, and capital market. At the beginning of period t, each entrepreneur will utilize capital, $k_t(j)$, and labor input, $l_t(j)$, to produce the intermediate goods, $y_t(j)$. Then, they will sell off the intermediate goods to retailers in a perfectly competitive manner and earn the revenue, $p_t^{mc}(j)y_t(j)$. After earning the revenue, each entrepreneur will pay the labor cost and also repay the debt. Finally, each entrepreneur will sell off a depreciated capital at the capital market. The net income after these activities are captured by n_t^E and will be a net worth for the entrepreneur j at the end of period t. Given this net worth, each entrepreneur will plan for the next period and decide how much capital to purchase and how much debt to issue at the end of period t which appears in the balance sheet equation (3.2).

3.1.3 Debt Contract

Each period, entrepreneur j issues a debt and engages in a debt contract with an arbitrary chosen financial intermediary m where m is an indexed number uniformly distributed from 0 to 1. Debt contract is for one period only and if entrepreneur j needs to issue a debt again next period, another arbitrary financial intermediary m' will be chosen next period. Following BGG (1999), idiosyncratic TFP shock, $\omega_t(j)$, is private information of entrepreneur j that there exists asymmetric information between entrepreneur j and financial intermediary m. Due to costly state verification, financial intermediary m cannot observe entrepreneur j's output costlessly, but need to incur a monitoring cost to observe it. Entrepreneur j, after observing the project outcome, will decide whether to repay the debt or default at the beginning of period t. If the entrepreneur decides to repay, financial intermediary will receive repayment of $R^E_{t-1}(j)/\pi_t$ for each unit of credits outstanding, regardless of the realization of idiosyncratic shock. Otherwise, the financial intermediary will pay a monitoring cost to observe $y_t(j)$ and seize the project outcome from the entrepreneur.

Under the optimal debt contract, BGG (1999) shows that the external finance premium, $s_t(j)$, to be an increasing function of the leverage ratio. For estimation purpose, we follow Christensen and Dib's (2008) specification of the external finance premium as follow,

$$s_t(j) = \left(\frac{q_t k_{t+1}(j)}{n^E_t(j)} \right)^{\varphi} \tag{3.7}$$

where parameter $\varphi > 0$ can be interpreted as the elasticity of external finance premium with respect to the leverage ratio. In addition, discounting the external finance premium from the borrowing rate $R^E_t(j)$, the expected risk-adjusted nominal return for financial intermediary m from the debt contrat from period t to $t+1$ can be expressed as

$$E_t R^F_{t+1}(m) = \frac{R^E_t(j)}{s_t(j)}. \tag{3.8}$$

3.1.4 Aggregation

Since bankruptcy cost is constant-return-to-scale and leverage ratio are equal for all entrepreneur j, the external finance premium is equal across all solvent entrepreneurs in equilibrium, i.e., $s_t = s_t(j)$ for all j. Since (3.6) holds in aggregate level, the nominal borrowing rates across all solvent entrepreneurs become equal, i.e., $R^E_t = R^E_t(j)$ for all j. Consequently, because $R^E_t = R^E_t(j)$ and $s_t = s_t(j)$ for all j, the expected risk-adjusted nominal return for banker m becomes equal across all bankers, i.e.,

$$E_t \left[R^F_{t+1}(m) \right] = \frac{R^E_t}{s_t} \text{ for all } m. \tag{3.9}$$

Next, we derive the law of motion of the aggregate net worth of corporate sector. As for notation, aggregate variable is expressed by suppressing the argument j. Aggregating over income statement (3.3) and taking into account the entrance and exit of entrepreneurs from period t to $t+1$, we obtain the following aggregate net worth transition equation

$$n_{t+1}^{E} = \gamma_{t+1}^{E}\left[r_{t+1}^{k} q_t k_{t+1} - \frac{R_t^E}{\pi_{t+1}} b_t^E\right] + \xi^E n_t^E \tag{3.10}$$

where r_{t+1}^k is realized gross return from capital investment at period $t+1$ and is defined as

$$r_{t+1}^k \equiv \frac{\alpha p_{t+1}^{mc} \overline{y}_{t+1}/k_{t+1} + (1-\delta) q_{t+1}}{q_t}. \tag{3.11}$$

Here, $\overline{y}_{t+1}$ is the average of project outcomes, $y_{t+1}(j)$, across all entrepreneurs. Thus, idiosyncratic factor stemming from $\omega_t(j)$ is averaged away and r_{t+1}^k only reflects the aggregate factors in the economy. Using entrepreneur's balance sheet (3.2), the aggregate net worth transition (3.10) can be rearranged as

$$n_{t+1}^{E} = \gamma_{t+1}^{E}\left[\left(r_{t+1}^{k} - \frac{R_t^E}{\pi_{t+1}}\right) q_t k_{t+1} + \frac{R_t^E}{\pi_{t+1}} n_t^E\right] + \xi^E n_t^E. \tag{3.12}$$

Notice how the realization of r_{t+1}^k can affect the aggregate net worth next period. Ex-ante, by the rational expectation equilibrium condition (3.6), the expected return from capital investment and borrowing cost are equalized. Ex-post, however, realized return from capital investment can exceed or fall below the borrowing cost depending on the realizations of the aggregate shocks and it affects the evolution of the aggregate net worth. This is a case where forecast error has an actual effect on the economy. Another factor that affects the evolution of the aggregate net worth is the realization of stochastic survival rate γ_{t+1}^E. At the micro-level, γ_{t+1}^E has an interpretation of stochastic survival rate of entrepreneur j from period t to $t+1$. At the aggregate level, γ_{t+1}^E is interpreted as an exogenous shock to the aggregate net worth in corporate sector. In our paper, we interpret it as an aggregate corporate net worth shock.

3.2 Financial Friction in Banking Sector

3.2.1 Entrance and Exit of Bankers

Following Gertler and Karadi (2011) as well as Gertler and Kiyotaki (2010), there is a continuum of bankers indexed by $m \in [0,1]$ where each banker is risk-neutral and has a finite horizon. We assume that each banker faces exogenous time-varying stochastic survival rate of γ_{t+1}^F from period t to $t+1$ which is common to all bankers. By the same token as in corporate sector, the stochastic process of γ_t^F is uncorrelated with any other shocks in the economy and has it mean equal to γ^F, i.e., $E[\gamma_t^F] = \gamma^F$.

After $1-\gamma_{t+1}^F$ fraction of bankers have exit between period t and $t+1$, exactly the same number of new bankers will enter the banking business from the household. Each banker entering the baking business will receive a 'start-up' transfer from the household, while each banker exiting the business will transfer his net worth back to the household. In aggregate, 'start up' transfer is assumed to be the constant fraction ξ^F of aggregate net worth available in the banking sector, n_t^F, i.e., $\xi^F n_t^F$

and the aggregate transfer from the exiting bankers is equal to $\gamma^F_{t+1} n^F_t$. Thus, net transfer from the banking sector to the household, Ξ^F_t, is equal to $(1 - \gamma^F_{t+1} - \xi^F) n^F_t$.

3.2.2 Individual Banker's Problem

We now describe the individual banker's problem. The treatment here basically follows that of Gertler and Karadi (2011) and perfect inter-bank market version of Gertler and Kiyotaki (2010). The balance sheet equation of the individual banker m is given by

$$b^E_t(m) = n^F_t(m) + b^F_t(m) \tag{3.13}$$

where $b^E_t(m)$ is the asset of banker m which is lent out to an aribitrarily chosen entrepreneur j at period t, $n^F_t(m)$ is the net worth of banker m, and $b^F_t(m)$ is the liability of banker m which is also a deposit made by the household at period t.

By receiving deposits $b^F_t(m)$ from household at period t, banker m pledges to pay the deposit rate of R_t/π_{t+1} in real terms next period. As a result of the banking business, the net worth transition for banker m at period $t+1$ is given by $n^F_{t+1}(m) = r^F_{t+1}(m) b^E_t(m) - r_{t+1} b^F_t(m)$ where $r^F_{t+1}(m) \equiv R^F_{t+1}(m)/\pi_{t+1}$ and $r_{t+1} \equiv R_t/\pi_{t+1}$. Using the balance sheet equation (3.13), the net worth transition equation can be reformulated as follow

$$n^F_{t+1}(m) = \left(r^F_{t+1}(m) - r_{t+1}\right) b^E_t(m) + r_{t+1} n^F_t(m). \tag{3.14}$$

As shown by Gertler and Kiyotaki (2010), with the agency cost present between banker m and depositor, the expected spread between $r^F_{t+1}(m)$ and real deposit rate r_{t+1} becomes strictly positive, i.e., $E_t\left[r^F_{t+1}(m) - r_{t+1}\right] > 0$. However, of course, whether the net worth of banker m increases or decreases next period depends on the realization of $r^F_{t+1}(m)$.

Given the above net worth transition equation, risk-neutral banker m will maximize the net worth accumulation by maximizing the following objective function with respect to bank lending, $b^E_t(m)$,

$$V^F_t(m) = E_t \sum_{i=0}^{\infty} \beta^i (1 - \gamma^F_{t+1}) \gamma^F_{t+1,t+1+i} \left[\left(r^F_{t+1+i}(m) - r_{t+1+i}\right) b^E_{t+i}(m) + r_{t+1+i} n^F_{t+i}(m) \right] \tag{3.15}$$

where $\gamma^F_{t+1,t+1+i} \equiv \prod_{j=0}^{i} \gamma^F_{t+1+j}$. Now, since the expected spread between risk-adjusted bank lending rate and deposit rate is strictly positive, it is in the interest on banker m to lend out infinite amount to an entrepreneur by accepting infinite amount of deposits from the depositor.

In order to avoid the infinite risk-taking by the banker, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) impose a moral hazard/costly enforcement problem between the banker and depositor. Each period, the banker has a technology to divert fraction λ of his asset holding to the household and exit from the banking business. However, by doing so, the banker is forced to file bankruptcy and fraction $(1 - \lambda)$ of his asset will be seized by the depositors. Thus, in order for the banker to continue business and depositors to safely deposit their funds to the banker, the following incentive constraint must be met each period,

$$V_t^F(m) \geq \lambda b_t^E(m). \tag{3.16}$$

In other words, the net present value of the banking business needs to always exceed the reservation value retained by the banker.[8]

Now, assuming that the incentive constraint (3.16) to be binding each period and by maximizing the objective function (3.15) subject to the constraint (3.16), Gertler and Kiyotaki (2010) shows that the value function of the banker can be expressed as follow

$$V_t^F(m) = \nu_t b_t^E(m) + \eta_t n_t^F(m) \tag{3.17}$$

where

$$\nu_t \equiv E_t\left[\left(1-\gamma_{t+1}^F\right)\beta\left(r_{t+1}^F(m) - r_{t+1}\right) + \beta\gamma_{t+1}^F \frac{b_{t+1}^E(m)}{b_t^E(m)}\nu_{t+1}\right] \tag{3.18}$$

$$\eta_t \equiv E_t\left[\left(1-\gamma_{t+1}^F\right) + \beta\gamma_{t+1}^F \frac{n_{t+1}^F(m)}{n_t^F(m)}\eta_{t+1}\right]. \tag{3.19}$$

Now, from incentive constraint (3.16) and the value function (3.17), it follows that

$$\frac{b_t^E(m)}{n_t^F(m)} \leq \frac{\eta_t}{\lambda - \nu_t} \equiv \phi_t \tag{3.20}$$

which states that the leverage ratio of banker m cannot exceed the (time-varying) threshold ϕ_t. By the assumption that incentive constraint to bind every period, in equilibrium, the asset and the net worth by banker m have a following relationship

$$b_t^E(m) = \phi_t n_t^F(m). \tag{3.21}$$

3.2.3 Aggregation

Gertler and Karadi (2011) and Gertler and Kiyotaki (2010) show that time-varying threshold ϕ_t does not depend on banker-specific factors and is common across all bankers. Consequently, from eq. (3.21), aggregate asset and net worth in banking sector can be expressed as

$$b_t^E = \phi_t n_t^F \tag{3.22}$$

where $b_t^E \equiv \int_0^1 b_t^E(m)dm$ and $n_t^F = \int_0^1 n_t^F(m)dm$. Now, from individual banker's net worth transition (3.14) and taking into account entrance and exit of bankers, the aggregate net worth transition equation of banking sector is given by

$$n_{t+1}^F = \gamma_{t+1}^F\left[\left(\overline{r}_{t+1}^F - r_{t+1}\right)b_t^E + r_{t+1}n_t^F\right] + \xi^F n_t^F \tag{3.23}$$

[8] To see how this constraint binds, consider the case where the banker increases the asset enormously. Then, the reservation value by the banker (right-hand side of inequality (3.16)) will exceed the net present value of the banking business (left-hand side of inequality (3.16)) that the banker will decide to divert the assets to the household. As a steakholder, the depositors will not allow this reckless behavior by the banker and ask the banker to keep his asset, $b_t^E(m)$, low enough (or, equivalently, by not supplying the deposits beyond the incentive constraint) so that the incentive for the banker to remain in business is met.

where $\bar{r}^F_{t+1}$ stands for the average of realized risk-adjusted returns, $r^F_{t+1}(m)$, across all bankers. From the optimal debt contract specified in (3.8) and using the aggregate condition in (3.9), $\bar{r}^F_{t+1}$ is related to the borrowing rate, external finance premium, and inflation rate as follow

$$\bar{r}^F_{t+1} = \frac{R^E_t}{\pi_{t+1} s_t}. \tag{3.24}$$

As can be seen from the above equation, idiosyncratic factor pertaining to banker m is averaged away and, thus, realization of risk-adjusted return of banking sector (i.e., $\bar{r}^F_{t+1}$) only depends on aggregate factors in the economy. Now, by using (3.22), the aggregate net worth transition equation becomes

$$n^F_{t+1} = \gamma^F_{t+1} \left[\left(\bar{r}^F_{t+1} - r_{t+1} \right) \phi_t + r_{t+1} \right] n^F_t + \xi^F n^F_t. \tag{3.25}$$

3.3 Incorporation of the two Frictions within the DSGE model

To incorporate the two financial frictions into a stylized DSGE model, we use twelve constraint and FOC equations, which consist of five and seven equations derived in corporate and banking sectors, respectively.[9] The five equations representing the financial friction in the corporate sector are (i) the balance sheet statement of corporate sector (3.2), (ii) the capital demand function (3.6), (iii) the external financial premium (3.7), (iv) the realized gross return from capital investment (3.11), and (v) the aggregate net worth transition equation of corporate sector (3.12). On the other hand, the seven equations expressing the financial friction in the banking sector are (vi) the balance sheet statement of banking sector (3.13), (vii) the dynamics of the weight on the lending volume for the value of the banking business, ν_t, (3.18), (viii) the dynamics of the weight on the bank net worth for the value of the banking business, η_t, (3.19), (ix) the definition of the threshold, ϕ_t, (3.20), (x) the banker's leverage ratio constraint (3.22), (xi) the relationship between the corporate nominal borrowing rate and the risk adjusted nominal lending rate of the banking sector (3.24), and (xii) the aggregate net worth transition equation of the banking sector (3.25).

To complete our model, we employ the CEE (2005) type medium scale DSGE model described in Appendix A4 with the equations above, as well as structural shocks. We set the following eight structural shocks, each of them having a specific economic interpretation; i.e., (1) TFP shock, (2) preference shock, (3) labor supply shock, (4) investment specific technology shock, (5) government spending shock, (6) monetary policy shock, (7) corporate net worth shock and (8) bank net worth shock. Except for monetary policy shock, all of the structural shocks are assumed to follow AR(1) stochastic processes. We devote the following two shocks out of the eight shocks to identifying fundamental factors causing the financial crisis.

[9] We have twelve model (or endogenous) variables corresponding to the twelve estimated equations pertain to the financial frictions. These variables are (1) capital, k_t, (2) the real price of capital, q_t, (3) asset of the corporate sector, b^E_t, (4) asset of the banking sector b^F_t, (5) the corporate net worth n^E_t, (6) the bank net worth n^F_t, (7) external financial premium, s_t, (8) the gross return from capital investment, r^k_t, (9) time varying weight of lending for the value of banking business, ν_t, (10) time varying weight of bank net worth for the value of banking business, η_t, (11) the corporate nominal borrowing rate, R^E_t, and (12) the risk-adjusted lending rate of banking sector, R^F_t.

Corporate net worth shock ε_t^E is inserted into AR(1) process of the survival rate of the corporate sector γ_t^E which is a component of equation (3.12), while bank net worth shock ε_t^F is done into AR(1) process of the survival rate of the banking sector γ_t^F which is that of equation (3.25). The two shocks are given as

$$\begin{aligned} \text{Corporate net worth shock} \quad &: \quad \hat{\gamma}_t^E = \rho^E \hat{\gamma}_{t-1}^E + \varepsilon_t^E, \\ \text{Bank net worth shock} \quad &: \quad \hat{\gamma}_t^F = \rho^F \hat{\gamma}_{t-1}^F + \varepsilon_t^F, \end{aligned}$$

where ρ is for the AR(1) coefficients for respective structural shocks. Both shocks indicating stochastic survival rate for entrepreneurs and bankers at micro-level can be interpreted as net worth shocks for corporate and banking sectors at aggregate level, respectively. Notice that each stochastic disturbance ε_t is assumed to follow time varying volatility using SV model as mentioned in Section 2.

4 Method of Estimation

In this study, a hybrid MCMC (it is also referred to as Metropolis-within-Gibbs) is employed as a estimation method of data-rich DSGE model following Boivin and Giannoni (2006), and Kryshko (2011). [10] The contribution of our study is to extend the data-rich DSGE to including SV shocks from i.i.d. shocks.

The benefit of employing a hybrid MCMC is not only to implement sampling of posterior of model variables $\boldsymbol{S}_t$ but also to implement sampling of posterior of structural shocks ε_t. Using the sampling of structural shocks, we compose historical decompositions. On the other hand, the MH algorithm—which is a general estimation method used for the regular DSGE model—has disadvantages for policy analysis because it cannot generate posterior structural shocks and induces the impossibility of generating a credible interval of estimation in policy simulation.

The objects of estimation in the state space model (2.17), and (2.18) are structural parameters $\boldsymbol{\theta}$, parameters in measurement equations $\boldsymbol{\Gamma}(=\{\boldsymbol{\Lambda},\ \boldsymbol{\Psi},\ \mathbf{R}\})$, model variables $\boldsymbol{S^T}(=\boldsymbol{S_1}, \boldsymbol{S_2}, \cdots,\ \boldsymbol{S_T})$, and stochastic volatilities $\boldsymbol{H}^T(=h_1,\ h_2,\ \cdots h_T)$. For convenience, let $\log \sigma_t$ denote h_t, hereafter. Notice that to estimate $\boldsymbol{\theta},\ \boldsymbol{\Gamma},\ \boldsymbol{S}^T\ \boldsymbol{H}^T$ is necessary and sufficient for estimating our model, because matrices $\mathbf{G}(\boldsymbol{\theta}),\ \mathbf{E}(\boldsymbol{\theta}),\ \mathbf{Q}(\boldsymbol{\theta})$ in the transition equation (2.18) is a function of structural parameters $\boldsymbol{\theta}$.

General speaking, Bayesian estimation of parameters $\boldsymbol{\theta},\ \boldsymbol{\Gamma}$,$\mathbf{H^T}$ is implemented as the following steps.

Step I. We set priors of paramters $\boldsymbol{\theta}, \boldsymbol{\Gamma}, \boldsymbol{H}^T$, i.e. $p(\boldsymbol{\theta}, \boldsymbol{\Gamma}, \boldsymbol{H}^T)$ where $p(\boldsymbol{\theta}, \boldsymbol{\Gamma}, \boldsymbol{H}^T) = p(\boldsymbol{\theta}|\boldsymbol{\Gamma}\boldsymbol{H}^T)p(\boldsymbol{\Gamma}|\boldsymbol{H}^T)p(\boldsymbol{H}^T)$, since $\boldsymbol{\theta}, \boldsymbol{\Gamma}, \boldsymbol{H}^T$ are assumed to be independent.

[10] The algorithm of MCMC for estimating DSGE models was developed by Schorfheide (2000). Our method extends Schorfheide (2000) to the hybrid MCMC. The algorithm of hybrid MCMC is described in chapter six of Gamerman and Lopes (2006) etc. We also adopt speed-up algorithm of sampling state variables by Jungbacker and Koopman (2008), but find that it drops the accuracy of estimates of state variables, so we omit the algorithm from MCMC roution.

Step II. Using Bayes theorem, posterior $p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T|\boldsymbol{X}^T)$ is derived from prior $p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T)$ and likelihood function $p(\boldsymbol{X}^T|\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T)$.

$$p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T|\boldsymbol{X}^T) = \frac{p(\boldsymbol{X}^T|\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T\ p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T)}{\int p(\boldsymbol{X}^T|\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T)p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T)d\boldsymbol{\theta}d\Gamma d\boldsymbol{H}^T}.$$

Step III. We obtain representative values (mean, median, credible band etc.) of parameters $\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T$ from posterir $p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T|\boldsymbol{X}^T)$ using numerical technique.

However, it is troublesome to sample directly joint posterior distribution $p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H^T}|\boldsymbol{X}^T)$of a state space model (2.17), (2.18) in step II. Instead, using Gibbs sampling, we obtain the joint posterior $p(\boldsymbol{\theta}, \Gamma, \boldsymbol{H}^T|\boldsymbol{X}^T)$ from conditional posterior $\boldsymbol{\theta}$, Γ and $\boldsymbol{H}^T$ as below,

$$p(\boldsymbol{\theta}|\Gamma, \boldsymbol{H}^T, \boldsymbol{X}^T), \qquad p(\Gamma|\boldsymbol{\theta}, \boldsymbol{H}^T, \boldsymbol{X}^T), \qquad p(\boldsymbol{H}^T|\boldsymbol{\theta}, \Gamma, \boldsymbol{X}^T)$$

In addition, since parameter Γ is dependent on model variable $\boldsymbol{S}_t$, we have to separate two conditional posterior $p(\boldsymbol{S}^T|\Gamma,\ \boldsymbol{\theta},\ \boldsymbol{H^T},\ \boldsymbol{X}^T)$ and $p(\Gamma|\boldsymbol{S}^T, \boldsymbol{\theta},\ \boldsymbol{H^T},\ \boldsymbol{X}^T)$ from above conditional posterior $p(\Gamma|\boldsymbol{\theta},\ \boldsymbol{H^T},\ \boldsymbol{X}^T)$ and insert $\boldsymbol{S}_t$ into posterior. We also adopt a forward-backward recursion for sampling from $p(\boldsymbol{S}^T|\Gamma, \boldsymbol{\theta},\ \boldsymbol{H^T},\ \boldsymbol{X}^T)$ and $p(\boldsymbol{H}^T|\Gamma, \boldsymbol{\theta},\ \boldsymbol{S^T},\ \boldsymbol{X}^T)$ as a data augmentation method, Gibbs sampling for sampling from $p(\Gamma|\boldsymbol{S}^T, \boldsymbol{\theta},\ \boldsymbol{H^T},\ \boldsymbol{X}^T)$, and MH algorithm for sampling from $p(\boldsymbol{\theta}|\Gamma,\ \boldsymbol{H^T},\ \boldsymbol{X}^T)$, respectively. In this way, different algorithms are employed for different parameters in a hybrid MCMC. In sum, we show six steps of hybrid MCMC for estimating a data rich DSGE model as follow.[11]

Step 1. Specify initial values of parameters $\boldsymbol{\theta}^{(0)}$, $\Gamma^{(0)}$, and $\boldsymbol{H}^{T(0)}$. And set iteration index $g = 1$.

Step 2. Solve the DSGE model numerically at $\boldsymbol{\theta}^{(g-1)}$ based on Sims' (2002) method and obtain matrices $\mathrm{G}(\boldsymbol{\theta}^{(g-1)})$, $\mathrm{E}(\boldsymbol{\theta}^{(g-1)})$, and $\mathrm{Q}(\boldsymbol{\theta}^{(g-1)})$ in equation (2.18).

Step 3. Draw $\Gamma^{(g)}$ from $p(\Gamma \mid \theta^{(g-1)}, \boldsymbol{H}^{T(g-1)}, \boldsymbol{X}^T)$.

(3.1) Generate model variables $\boldsymbol{S}_t^{(g)}$ and structural shocks $\boldsymbol{\varepsilon}_t^{(g)}$ from $p(\boldsymbol{S}^T, \boldsymbol{\varepsilon}^T \mid \Gamma^{(g-1)}, \boldsymbol{\theta}^{(g-1)}, \boldsymbol{H}^{T(g-1)}, \boldsymbol{X}^T)$ using simulation smoother by de Jong and Shephard (1995).

(3.2) Generate parameters $\Gamma^{(g)}$ from $p(\Gamma \mid \boldsymbol{S}^{T(g)}, \theta^{(g-1)}, \boldsymbol{H}^{T(g-1)}, \boldsymbol{X}^T)$ based on the sampled draw $\bar{\boldsymbol{S}}^{T(g)}$ using Gibbs sampling by Chib and Greenberg(1994).

Step 4. Draw $\boldsymbol{H}^{T(g-1)}$ from $p(\boldsymbol{H}^T \mid \theta^{(g-1)}, \Gamma^{(g)}, \boldsymbol{\varepsilon}^{T,(g)}, \boldsymbol{X}^T)$.

(4.1) Generate stochastic volatility $\boldsymbol{H}^{T(g)}$ from $p(\boldsymbol{H}^T \mid \Gamma^{(g)}, \boldsymbol{\theta}^{(g-1)}, \boldsymbol{\varepsilon}^{T,(g)}, u^{T(g-1)}, \Phi^{(g-1)}, \boldsymbol{X}^T)$, using a draw of $\boldsymbol{\varepsilon}^{T,(g)}$ at Step 3.1, and the forward-backward recursion by Cater and Kohn (1994).

[11]Bayesian estimations using MCMC for state space models are described in detail in textbooks such as Kim and Nelson (1999) and Bauwns et al. (1999).

(4.2) Generate the indicators of the mixture approximation $u^{T(g)}$ using descrete density proposed by Omori et al. (2007).

(4.3) Generate the coefficients $\Phi^{(g)}$ of stochastic volatility process using Metropolis step.

Step 5. Draw deep parameters $\boldsymbol{\theta}^{(g)}$ from $p(\boldsymbol{\theta} \mid \boldsymbol{\Gamma}^{(g)}, \boldsymbol{H}^{t(g)}, \boldsymbol{X}^T)$ using Metropolis step:

(5.1) Sample from proposal density $p(\boldsymbol{\theta}|\boldsymbol{\theta}^{(g-1)})$ and, using the sampled draw $\boldsymbol{\theta}^{(\text{proposal})}$, calculate the acceptance probability q as follows.

$$q = \min\left[\frac{p(\boldsymbol{\theta}^{(\text{proposal})} \mid \boldsymbol{\Gamma}^{(g)}, \boldsymbol{H}^{t(g)}, \boldsymbol{X}^T)\ p(\boldsymbol{\theta}^{(g-1)}|\boldsymbol{\theta}^{(\text{proposal})})}{p(\boldsymbol{\theta}^{(g-1)} \mid \boldsymbol{\Gamma}^{(g)}, \boldsymbol{H}^{t(g)}, \boldsymbol{X}^T)\ p(\boldsymbol{\theta}^{(\text{proposal})}|\boldsymbol{\theta}^{(g-1)})}, 1\right].$$

(5.2) Accept $\boldsymbol{\theta}^{(\text{proposal})}$ with probability q and reject it with probability $1-q$. Set $\boldsymbol{\theta}^{(g)} = \boldsymbol{\theta}^{(\text{proposal})}$ when accepted and $\boldsymbol{\theta}^{(g)} = \boldsymbol{\theta}^{(g-1)}$ when rejected.

Step 6. Set iteration index $g = g+1$ and return to Step 2 up to $g = G$.

The algorithm of sampling stochastic volatilities $\boldsymbol{H}^T$ in Step 4 is explained in the Appendix A1. And a simulation smoother in Step 3.1 and of sampling parameters $\boldsymbol{\Gamma}$ in Step 3.2 are in the appendix A2 and A3, respectively. [12]

[12]Here, we supplement Steps 1 and 5. On setting of initial values of parameters $\boldsymbol{\theta}^{(0)}$ and $\boldsymbol{\Gamma}^{(\mathbf{0})}$in Step 1, it is known that arbitrary initial values are eventually converged in MCMC. However, we require a huge number of iterations in MCMC simulation to converge to target posterior distributions in the case of a considerable number of parameters. Accordingly, following Boivin and Giannoni (2006) we set initial values as below for converging effectively. First, the posterior mode of structural parameters $\boldsymbol{\theta}$ in a regular DSGE model without measurement errors is derived from numerical calculation and set as initial values $\boldsymbol{\theta}^{(0)}$. Second, a implementing simulation smoother of state variables $\boldsymbol{S}_t$ using $\boldsymbol{\theta}^{(0)}$, we get initial value $\boldsymbol{S}_t^{(0)}$. Finally, initial values $\boldsymbol{\Gamma}^{(0)}$of measurement equations are obtained by OLS using $\boldsymbol{S}_t^{(0)}$and$\boldsymbol{X}^T$.

Next, on generating structural parameters$\boldsymbol{\theta}$ from proposal density in Step 5.1, we adopt a random walk MH algorithm following previous works. Proposal density $\boldsymbol{\theta}^{(\text{proposal})}$ is represented as

$$\boldsymbol{\theta}^{(\text{proposal})} = \boldsymbol{\theta}^{(g-1)} + \boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim N(\mathbf{0}, c\boldsymbol{\Sigma}),$$

where,$\boldsymbol{\Sigma}$ is variance covariance matrix of random walk process, and c is the adjustment coefficient. The matrix $\boldsymbol{\Sigma}$ is the Hessian $(-l''^{-1}(\hat{\boldsymbol{\theta}})$) of log posterior distribution ($l(\boldsymbol{\theta}) = \ln p(\boldsymbol{\theta}|\boldsymbol{\Gamma}, \boldsymbol{X}^T)$) when obtaining initial value $\boldsymbol{\theta}^{(0)}$. And the case of sampling in MH algorithm,

$$p(\boldsymbol{\theta}^{(g-1)}|\boldsymbol{\theta}^{(\text{proposal})}) = p(\boldsymbol{\theta}^{(\text{proposal})}|\boldsymbol{\theta}^{(g-1)})$$

is held in a stationary state of the target distribution, so that acceptance rate q is reduced to the following equation.

$$q = \min\left[\frac{f(\boldsymbol{\theta}^{(\text{proposal})})}{f(\boldsymbol{\theta}^{(g-1)})}, 1\right],$$

In this equation, acceptance rate q is not dependent on proposal density $p(\boldsymbol{\theta}|\boldsymbol{\theta}^{(g-1)})$. As a result, it is the advantage of a random walk MH algorithm that we do not need to adopt a proposal density close to posterior density. However, when proposal value $\boldsymbol{\theta}^{(\text{proposal})}$departs from the previous sample $\boldsymbol{\theta}^{(g-1)}$, acceptance rate q becomes small and efficiency of MCMC worsens. To avoid this,

5 Preliminary Settings and Data Description

5.1 Specifications of Four Alternative Cases

This study considers four alternative cases depending on the number of observation variables (11 vs. 40 observable variables) and specification of volatilities of structural shocks (constant vs. time-varying volatility) as summarized in Table 1. This is because we would like to verify whether data rich information makes decomposition between measurement errors and model variables from data more robust, and whether relaxation of specifying the volatilities depicts rapid change of shocks more detailed thanks to the data rich approach. The first case (referred to as Case A) deals with one of standard DSGE models which uses 11 observable variables in the measurement equation (2.11) and structural shocks with i.i.d. Normal distribution in the transition equation (2.12). In Case A, each observable variable connected with its specified model variable by one-to-one matching. The second case (Case B) is extended to data-rich approach with i.i.d shocks, including 40 observable variables which indicate more or less four observable variables corresponding to one specified model variable. The third case (Case C) extends to SV shocks from Case A. And the forth case (Case D) extends to data-rich approach with SV shocks from Case B. Nishiyama et al. (2011) have already studied the comparison of a standard DSGE approach (Case A) and a data rich approach (Case B). This study focus on the remaining two cases (C and D) with SV shock, using Case A as the reference model.

5.2 Calibrations and Priors of Parameters

We calibrate the subset of the structural parameters in the model that are not identifiable (i.e., the parameters that are only used to pin down the steady states) or are difficult to identify from the observed data. Calibrated parameters with their descriptions are reported in Table 2. We assume discount factor $\beta = 0.995$ so as to make the steady state real interest rate to be 2% (annual rate). We assume the profit margin of the retailers to be 10% in steady state and, thus, set elasticity of substitution of intermediate goods as $\epsilon = 11$. We have no reliable information regarding the new entry rate of entrepreneurs (i.e., ξ^E) and will simply set it equal to the calibration for new banker's entry rate by Gertler and Kiyotaki (2011). The rest of the calibrated parameter values are borrowed from Smets and Wouters (2003), Christensen and Dib (2008), and Gertler and Kiyotaki (2011).

Regarding the steady states, most of them are pinned down by equilibrium conditions of the model, but some others need to be calibrated. For the steady state value of external finance premium, we follow the calibration of Christensen and Dib (2008). For the steady state corporate borrowing rate (real, quarterly rate), we calculate the historical average of the yields of Moody's Baa-rated corporate bonds and set it as the steady state rate. In the same way, we calculate the historical aver-

the adjustment coefficient c should be small, but doing so narrows the range of sampling space of $\theta^{(\text{proposal})}$. Roberts et al. (1997) and Neal and Roberts (2008) reported that the optimal acceptance rate q of a random walk MH algorithm is around 25%. Accordingly, adjustment coefficient c of this study is set so that the acceptance rate is close to around 25 %.

age of the non-farm, non-financial business leverage ratio based on Flow of Funds and set it as the steady state of corporate leverage ratio. Finally, the government expenditure to output ratio in steady state is set to be 0.2 because of borrowed from Gertler and Kiyotaki's (2011) calibration.

Next, we turn to describe priors distribution of interest as a preamble for Bayesian estimation. The settings of priors are reported in Table 3. We set $\varphi = 0.05$ for the prior mean of this parameter, which controls the sensitivity of external finance premium with respect to corporate leverage ratio, following the calibration of BGG. For AR(1) persistence parameters for structural shocks, we set prior mean equal to 0.5 for all of them. For standard errors of structural shocks, we set prior mean equal to 1% for each standard error, except for monetary policy shock (where a change of policy rate for more than 25 basis point is rare). By the same token, we set prior mean equal to 1% for most of the measurement errors, except for the data related with interest rates.

5.3 Data Description

For adopting data rich approach, relatively large and quarterly panel data set as many as 40 observable variables is used as described in Data Appendix in detail. The sample period of estimation is between 1985Q2 and 2012Q2, because we avoid the effect on estimation result from the instability of monetary policy regime changes especially around the end of the 1970's and early 1980's; i.e., pre and post regimes by Volcker and Greenspan, (See Clarida et al. 2000, Lubik and Schorfheide 2004, and Boivin 2005) and from structural change of the Great moderation which began in mid-1980's (See Bernanke 2004, Stock and Watson 2002, Kim and Nelson 1999, and McConnell and Perez-Quiros 2000). And another reason why the sample period is determined is the availability of financial data, in which charge-off rates for banks are available only from 1985Q1.

In Cases A and C, we regard the following eleven series: (1) output, y_t, (2) consumption, c_t, (3) investment, i_t^k, (4) inflation, π_t, (5) real wage, w_t, (6) labor input, l_t, (7) nominal interest rate, R_t, (8) nominal corporate borrowing rate, R_t^E, (9) external finance premium, s_t, (10) corporate leverage ratio, $q_t k_t / n_t^E$, and (11) bank leverage ratio, b_t^E / n_t^F, as observable variables in the measurement equation (2.11). The first seven series are generally used in a large literature estimating DSGE models (see, for instance, Smets and Wouters, 2003 and 2007).[13] Using the four remain-

[13](1) Output is real GDP less net export. (2) Consumption and (3) investment are normalized respectively to personal consumption expenditures and fixed private domestic investment. Following Altig et al. (2003), Smets and Wouters (2003), and Boivin and Giannoni (2006), the nominal series for consumption and investment are deflated with the GDP deflator. (6) The labor input corresponds to hours worked per person. Average hours of nonfarm business sector are multiplied with civilian employment to represent the limited coverage of the nonfarm business sector, compared to GDP, as in Smets and Wouters (2003), and Boivin and Giannoni (2006). (5) The real wage is normalized with the hourly compensation for the nonfarm business sector, divided by the GDP deflator. We express these six series as percent deviations from steady states consistently with model concepts, taking the natural logarithm, extracting the linear trend by an OLS regression, and multiplying the resulting de-trended series by 100. (4) Inflation measures are obtained by taking the first difference of the natural logarithm of the GDP deflator, and multiplied by 400 for expressing the annualized percentages. (7) The nominal interest rate is the effective Federal funds rate. Both inflation and the interest rate are de-trended via Hodrick-Prescott filter (penalty parameter is 1600), indicating

ing financial time-series as observable variables is the feature of our DSGE model compared with existing models. These four actual series are selected for matching the model variables corresponding to the two financial frictions. (8) Entrepreneur's nominal borrowing rate, R_t^E, is the yield on Moody's Baa-rated corporate bonds, which is de-trended via Hodrick-Prescott filter for the same reason of inflation and the interest rate. To measure (9) the external financial premium, s_t, we employ the charge-off rates for all banks credit and issuer loans, measured as an annualized percentage of uncollectible loans. The charge-off rate is demeaned to be consistent with our model variable. (10 and 11) The two leverage ratios, $q_t k_t / n_t^E$ and b_t^E / n_t^F are calculated as their total asset divided by their net worth, respectively. We take natural logarithm for both leverage ratios, and then either demean for entrepreneur's leverage ratio, or detrend banking sector leverage ratio by Hodrick-Prescott filter, due to taking account of Basel Capital Accord Revision.

In Cases B and D which adopt the data rich approach indicating one model variable corresponding to four actual series, we employ additional 29 series, which consist of 18 series of key macroeconomics and 11 series of banking sector, with existing 11 series in Cases A and C. We select 18 data indicators for six key model variables used in a standard DSGE model such as (1) output, (2) consumption, (3) investment, (4) inflation, (5) real wage and (6) labor input except nominal interest rate as mentioned in Data Appendix, along the same line of Boivin and Giannoni (2006). On the other hand, 11 series of the banking sector are selected along the line concentrated on banking which is depart from previous work dealing with the data rich framework. Three additional banking indicators: (i) the core capital leverage ratio, (ii) the domestically chartered commercial banks' leverage ratio and (iii) the leverage ratio of brokers and dealers, are selected for data indicators corresponding to the model variable such as banking sector leverage ratio. Notice that as an observable variable of leverage ratio we use the inverse of the commonly-used ratio, i.e., bank asset over bank equity. [14] As data indicators for external financial premium, we select three kinds of charge-off rates of loans depending on different institutions, which are transformed as percentage deviations from trends using the same detrending methods as described above.

6 Empirical Results

In this section, we report results of our estimation and especially focus on the estimates of several key structural parameters, those of SV shocks, and historical decompositions of four principal model variables: (1) output, (2) investment, (3) bank leverage and (4) borrowing rate playing a significant role in the recession and the financial crisis of 2007-2008. Then, we discuss and remark the sources

time-varying targeting inflation rate.

[14] The core capital leverage ratio represents tier 1 (core) capital as a percent of average total assets. Tier 1 capital consists largely of equity. We use the inverse of the core capital leverage ratio, because of corresponding to the ratio of banks' asset to banks' net worth, before taking natural logarithm and detrended by Hodrick Prescott filter. Following Adrian and Shin (2010), we add the leverage ratio of brokers and dealers since investment banks are categorized as brokers and dealers in Flow of Funds (FOF) and the financial shock is caused mainly by the deterioration of investment banker's balance sheet condition.

of the recession in the light of the data rich approach. [15] Our estimation results are constructed from 300,000 draws of the hybrid MCMC algorithm as posterior distributions of interest above for every case. [16]

6.1 Key Structural Parameters

The estimates of structural parameters of Cases A and B are summarized in Table 9, and those of Cases C and D are done in Table 10. Table 11 notifies the estimates of parameters of the SV models concerning the eight structural shocks. Because the New-Keynesian DSGE model with the two financial frictions is estimated, we focus on interpretation of the seven key structural parameters, i.e., two financial friction, two nominal rigidities and three monetary policy parameters. Table 4 collects these parameters of the four cases to be easily compared with one another. The parenthesis in the table indicates the 90% credible interval of the posterior distribution of the structural parameters.

First of all, we consider two estimated parameters involved in the financial friction of the corporate sector; κ and φ. κ denotes the elasticity of the quadratic adjustment cost of investment in eq.(A20) described in Appendix A4, while φ is the elasticity of the external financial premium in eq.(3.7). According to Table 4, the posterior mean of κ in Case B (the data-rich approach with constant-volatility-shocks) is around 0.88, whereas those in the rest cases are between 0.56 and 0.63. The large elasticity, κ, in Case B implies that the corporate net worth shock more strongly amplifies fluctuation of business cycles via the channel of adjustment of the capital asset price (Tobin's marginal q). On the other hand, the posterior means of φ are less than 0.03 in Cases A and B (models with constant-volatility-shocks), whereas those on the counterpart cases with SV shocks are nearly 0.04. Because the elasticity of the external financial premium, φ, which reflects the size of agency cost of corporate sector, is the coefficient of the logarithm of corporate leverage ratio for explaining the aggregated level of external financial premium s_t as shown in eq.(3.7), the relatively large size of φ in Cases C and D incorporating the effect of SV shocks into the DSGE models suggests that variation of leverage ratio in the corporate sector is likely to be more influential in enlarging the external financial premium and to lead a more severe decline of investment, compared with results of Case A and B with structural shocks following i.i.d. normal. Notice that there are no parameters concerning the financial friction in banking sector of our model following Gertler and Kiyotaki (2010) and Gertler and Kradi (2011), so that we cannot show the comparison among the cases in banking sector. Instead, structural shock pertaining to banking net-worth will be compared among the cases in the following subsection.

Next, nominal rigidities of price level and wage are considered. In our study, these rigidities are governed by a Calvo type model explained in Appendix A4.

[15] We adopt Sims (2002) method in solving for our DSGE model and all estimation procedures are implemented using GAUSS.

[16] 300,000 iterations are implemented using the MH within Gibbs. We sample one draw out of every 10 replicates, for reducing the impact of auto-correlations between draws of interest on their target posterior distributions and store up total 30,000 samples. Then, we discard first 10,000 samples, and the remaining 20,000 samples are used for calculating moments of the posterior distributions.

Calvo price θ_P implies the ratio of intermediate goods firms facing monopolistic competition and reconciling to remain their price without optimally setting new price. The value of θ_P is theoretically real number between 0 and 1. As can be seen from Table4, the posterior mean of θ_P in Case B is nearly half size (0.37) against the rest three cases where those are relatively high normal rigidity (around 0.8) indicating that new price are set at one-time per every 5 quarter period. On the other hand, Calvo wage θ_W (the ratio of workers facing monopolistic competition and reconciling to remain their wage without optimally setting new wage) is nearly 0.43 in Case B and those in other cases are within 0.5 through 0.6. These values imply that wage are reset with frequency around one-time per half year.

Finally, Table 4 also reports the three parameters of Taylor type monetary policy rule, eq.(A24) described in Appendix A4. ρ^R denotes size of inertia of policy interest rate, i.e., federal fund rate. And, μ^π and μ^Y are Taylor coefficients in response to inflation gap and output gap, respectively. There are no big differences of the three parameters among the four cases. That is, ρ^R is between 0.61 and 0.67, μ^π is around 2.8 to 3.0, and μ^Y is tiny such as 0.006 through 0.010. These results imply that reactions for inflation gap are aggressively implemented, while those for output gap are not so by the central banker. However, the volatilities of monetary policy shocks are largely different among the four cases. We will see that time-varying volatilities of monetary policy shocks rapidly increase in the period of the recession in Section 6.2.

6.2 Structural Shocks and their Volatilities

Figure 1 shows the posterior mean and 90% credible interval of the eight structural shocks in Cases A and B which deal with models with constant-volatility-shocks, whereas Figure 2 dose those in Cases C and D estimating models with time-varying-volatility-shocks. In the panel (a) of each figure, shocks of model with 11 observable variables are drawn with deep blue solid line for posterior means and light blue shade for the 90% interval, while panel (b) shows estimates of shocks of the data-rich approach with deep red solid line and light red shade in the similar way. In each panel, the line and shade of its counterpart are underlain because of compared each other. And also Figure 3 depicts posterior means and 90% intervals of time varying volatility in Cases C and D. From Figures 1 and 2, two points are impressively observed. First, the fluctuation of each structural shock is different over the four cases depending on the number of observable series and specification of shocks, even though using the same DSGE model. This induces different interpretation of economic analysis for business cycle regardless of adopting the same models. Second, the structural shocks (red shade) estimated from the data-rich approach seem to fluctuate with bigger swing than those (blue shade) of the standard approach. In particular, it is distinguished that red shade in data-rich approach covers almost area of blue shade in Case C in Figure 2 dealing with models with SV shocks.

Next, we focus on the two structural shocks pertaining to the financial frictions in banking and corporate sectors. In Table 5, the timings of the peaks of the two shocks are described for the four cases. At first, the banking net-worth shocks have the exactly same peak at 2008Q3 for all cases. In this period, i.e., September and

October 2008, several major financial institutions were either failed, acquired under duress, or subject to government take over. These financial institutions include Lehman Brothers, Merrill Lynch, Fannie Mae, Freddie Mac, Washington Mutual, Wachovia, Citi group, and AIG. On the other hand, the timings of the peak of corporate net-worth shock are not consistent and divided into two periods, i.e., 2009Q1 in Cases A and B, and 2009Q2 in Cases C and D. Remark that corporate net-worth shocks have peak after banking sector shocks hit peak, whatever the case.

And we consider the accuracy of estimation for the eight shocks using average range of 90% credible interval over the all sample period as seen from Table 6. If we observe the 90% interval ranges are smaller, then it is thought that the shocks are likely to be identified more precisely. Compared among the four cases, five average intervals of shocks out of eights are smaller in Cases C and D than in Cases A and B. These five shocks are (1) preference, (2) banking net-worth, (3) labor supply, (4) government spending, (5) monetary policy. The intervals in the former three shocks are around half in the two cases with time-varying-volatility-shocks against the other two cases with constant ones. And those of government spending shock shrink to one eighth to one tenth by adopting SV shocks. These suggest that constant volatilities of shocks might be misspecified and that the shocks follow time-varying volatilities. In particular, the volatilities are expected to change to large values at the turning points of business cycles as seen later.

Figure 3 draws estimates of time-varying volatilities of shocks between Cases C and D. Surprisingly, the seven shocks except government spending shocks are very similar in both cases. In Figure 3, deep blue and deep red solid lines denote posterior means of Cases C and D, respectively. As seen from this figure, the six shocks except preference and labor supply shocks are very stable and level off between 1990Q1 and 2007Q3, while preference and labor supply shocks might play an important role of the boom around 2003 to 2005. After August 2007 when financial crisis of 2007 to 2009 began with the seizure in the banking system precipitated by BNP Paribas announcing that it was ceasing activity in three hedge funds that specialized in US mortgage debt, the volatilities of both banking and corporate net-worth, investment, and TFP rapidly increased. Although our DSGE model is not thought perfectly to capture macroeconomic fluctuations in the period of the Great Recession, the estimates show that the sizes of volatilities in this period look like extraordinary.

And Table 7 indicates average 90 % intervals of SV over the entire sample period in the two cases because of verifying whether the data rich approach contributes improvement of estimates of SV of shocks. As seen from Table 7 as well as Figure 3, there are no differences of means of interval ranges between Cases C and D except government spending shocks. However, it might be too hasty to conclude that the data rich method does not improve the accuracy of SV estimates. We would need to validate the further evidences in this question.

Next, we turn to discuss the leverage effects of SV shocks. Table 11 summarizes the estimation results of the parameters in the SV model defined in eq.(2.2) to eq.(2.4) and used in Cases C and D. The leverage effect is represented by the sign of the correlation coefficient ρ_σ of each shocks. If ρ_σ is negative, the shock has leverage effect which implies that the negative shock at the present period amplifies its volatility at the next period, and vice versa. Table 8 sums up the sign of the correlation coefficient ρ_σ of each shocks in terms of 90% credible interval. The mark "-" indicates negative of ρ_σ (leverage effect) at 90% credible degree of posterior probability, while the mark "+" does positive of ρ_σ (opposite leverage effect) in similar way. The mark "0" implies that we do not judge the sign of ρ_σ and leverage effect of each shock because zero is within 90% interval of ρ_σ. According to many financial empirical studies, leverage effect are often observed in financial time series such like stock price. Our question is whether banking and corporate net-worth shock have the leverage effect which implies that decline of net-worth shock lead to extend its volatility or its uncertainty at next period. However, we cannot observe leverage effects for these two shocks as Table 8. This result might be derived from either the number of observation, specification of our DSGE model, or something else. To answer this question we need to continue development of econometric method further as well as selecting and accumulating data.

Finally, we remark the monetary policy in the period of the Great Recession, although we adopt liner-type Taylor rule and estimate it for the sample period including QE1 (round 1 of quantitative easing by FRB, between 2008Q4 and 2010Q2) and QE2 (2010Q4 to 2011Q2). Monetary policy shocks in Figures 1 and 2 seem to have two big negative spikes after 2007. The first negative spike is observed at 2007Q4 when BNP Paribas announcement impacts on global financial market. And the second one is observed at 2008Q3 immediately before an unconventional monetary policy (QE1) was conducted by the FRB. In particular, the magnitudes of these two negative shocks are distinguished in the cases of time-varying-volatility as Figure 2. Figure 3 also captures rapidly appreciation of these volatilities of policy shocks in the period between 2007Q4 and 2008Q3. Figure 8 shows monetary policy has opposite leverage effect over the entire sample periods even though FRB took tremendous monetary easing policies in the recession. That is, tightening policy is likely to be conducted more boldly without hesitation, while easing policy might be done more carefully, according to the results with 90 % credible degree of posterior probability.

6.3 Historical Decompositions

To investigate the sources of the Great Recession, we focus on the historical decompositions of four observable variables; (1) real GDP as output gap, (2) gross private domestic investment (fixed investment) as investment, (3) Moody's bond index (corporate Baa) as corporate borrowing rate, (4) commercial banks leverage ratio as bank leverage ratio which are described in detail in Data Appendix. Each of Figures 4 to 7 draws four decompositions of each observable variable based on the four cases for the period between 2000Q1 and 2012Q2, and light blue shade denotes the period of Great Recession (2007Q3 to 2009Q2). To facilitate visualization and focus on contributions of two financial frictions, technology and monetary policy shocks for the recession, we collect the remaining four miscellaneous shocks as one bundle in these figures.

At first we consider real activities. Figures 4 and 5 show historical decompositions of real GDP and gross private domestic investment, respectively. Because the decompositions of these real activities' variables have similar properties, we discuss them as a whole. Although the signs of contribution of each shock are the same in every case of the two variables at each period, we can see that the sizes of the contribution of shocks are different depending on the cases. In Case A (standard DSGE model), TFP shock accounts for large portion of sources of the Great Recession (light blue shade), while decline of bank net-worth impact on small part of drops. And positive corporate net-worth increases and contributes upward these variables at significant portion during the recession in this case. On the other hand, the remaining three cases shows that positive effect of corporate net-worth shock are small, and that bank net-worth shock accounts for bigger place of downturn of these real activities in the period. Even in the period of recovery of the US economy, Case A shows the different picture with the other cases. The main source of blocking recovery is derived from negative TFP shocks in Case A, whereas TARP works and prominently improves bank's balance sheet so that positive bank net-worth shock contributes upward real activities in the three cases. In addition, decompositions of these cases suggest deterioration of corporate's balance sheet is likely to be main source blocking recovery after the recession.

In Figure 6, Moody's bond index (corporate Baa) is decomposed as corporate borrowing rate. According to the figure, a sharp rise of the rate might be derived from mainly negative bank net-worth shock as well as a fall of TFP shock, whereas positive firm net-worth shock contributes the downward rate in the recession. And then, firm net-worth shock turns to be remarkably negative, seriously deteriorates its balance sheet and accounts for large portion of rise of the rate after the recession. On the other hand, TARP might work well and make bank net-worth shock change to positive, and this contributes the downward borrowing rate after 2010Q1. In particular, we can see these findings in Cases B, C and D.

Figure 7 depicts decomposition of commercial banks leverage ratio defined as the inverse of the commonly-used ratio, i.e., bank asset over bank net-worth. As can be seen from this figure, countercyclical of this inverse ratio is observed and the contributions of shocks to the fluctuations are explained. Both financial shocks in banking and corporate sectors with conflicting direction, i.e., negative banking balance sheet shock and positive corporate balance sheet shock, contribute an increase in the ratio at the almost same proportion in the recession. Soon after that, an increase in bank equity by conducting TARP makes its balance sheet improve, while negative firm net-worth shock makes firm balance sheet much worse, leading to a sharp reduction of loan by bank. These both things bring the ratio downward, since the numerator of the ratio consists of both loan and equity in banks and the denominator is only its equity. These findings are observed from every case. However, the dynamics of countercyclical of this inverse ratio is not generated from Gertler and Kiyotaki (2011). Recently, Adrian et al. (2012) try to explained countercyclical of this inverse ratio following two conflicting movements of banking loan and bond financing of firms., i.e., loan declines and bond increases in the recession. Our findings about both conflicting financial shocks in the recession are consist with Adrian et al. (2012).

6.4 Observations and Interpretation

Overall, we can make three important observations based on our empirical results.

First, as for the timing of the financial shocks during the period of Great Recession shown in Figure 1 and 2, we observed that the bank net worth shock occured earlier than the corporate net worth shock. Putting it differently, two financial shocks did not occur concurrently, but the corporate net worth shock occured just shortly after the bank net worth shock. This timing pattern (not concurrent, but proximate timing) may points to the possibility of endogenous relationshhip between the balance sheet conditions of the banking sector and the corporate sector. For instance, in reality, it is possible for the corporate sector to hold financial sector's equity as an asset and the devaluation of the financial sector's asset may affect the balance sheet condition of the corporate sector. Unfortunately, however, the model in this paper does not allow the corporate sector to hold banking sector's equity as an asset and further assumes the two financial shocks to be independent with each other.[17] Thus, it is inappropriate to interpret the endogenous relationship between two financial shocks in the context of the model assumed in this paper. Yet, the timing of the two financial shocks during Great Recession is worth noting.

Second, through the historical decomposition results shown in Figure 4 to Figure 7, we observed that the corporate net worth shock during Great Recession to be relatively weak in Case A, compared to the those in Case B, C, and D. This results may point to the possibility of underestimation of the importance of corporate net worth shock when the model is estimated by a plain-vanilla Bayesian estimation method – i.e., without data-rich estimation or stochastic volatility. Moreover, an accurate estimation of corporate net worth shock during Great Recession is crucially important in accounting for the economic recovery of the U.S. economy in recent years. For instance, in Case A, a slow recovery of output is mainly accounted by negative productivity shock, while in Case B, C, and D, it is mainly accounted by a prolonged negative corporate net worth shock. A slow recovery of the U.S. economy after Great Recession remain as an important puzzle and persuasive explanation of this puzzle calls for an accurate estimation of the structural shocks. For accurate estimation of the structural shocks (especially for corporate net worth shock), data-rich estimation with stochastic volatility may be more reliable than a plain-vanilla Bayesian estimation method.

Third, another important observation from the historical decomposition results is the behavior of bank net worth shock. Bank net worth shock declines sharply during Great Recession and is the main source of the sharp decline in output and investment as shown in Figure 4 and 5. But then, right after Great Recession period, bank net worth shock quickly reverses its direction and contributes positively to output and investment. Considering the timing of this reversal, it is quite possible that the implementation of TARP is behind this reversal. In other words, implementation of TARP may have successfully countered the negative bank net worth shock. Interpreting further, considering the positive contribution of bank net worth shock to ouput and investment right after Great Recession period, implementation of TARP may be one of the major reasons in stopping the spell of Great Recession and contributing to the recovery (albeit weak) of the U.S. economy in recent years.

[17] In this paper, the corporate sector is assumed to hold the asset fully in the form of physical capital.

7 Conclusion

According to the NBER, the Great Recession, in which the financial crisis played a significant role in the failure of key businesses, declines in consumer wealth estimated in trillions of US dollars, and a downturn in economic activity leading to the 2008–2012 global recession and contributing to the European sovereign-debt crisis, is reported to begin in December 2007 and end in June 2009. The purpose of this study is to argue macroeconomic fluctuations and mutual relationship among macroeconomic and financial endogenous variables and to identify what structural exogenous shocks contribute in the Great Recession in the light of a DSGE model. Because we obtain broad consensus that solvency and liquidity problems of the financial institutions are chief among the fundamental factors causing the recession itself, it is plausible to embed financial frictions in both banking and corporate sectors of a New Keynesian DSGE model. To this end, we followed Nishiyama et al. (2011) who have already studied the US economy using a New Keynesian DSGE model with these two financial frictions in a Data Rich environment. In this model with asymmetric information between borrowers and lenders, banks have two roles generating two agency costs: one is the lenders of corporate sector and the other is the borrowers from depositors. Further, the structural shocks in the model are assumed to possess SV with leverage effect. Then, we estimated the model using the Data-Rich estimation method and utilize up to 40 macroeconomic time series in the estimation. Our study is the first attempt of combination of data rich approach and time varying volatilities of structural disturbances.

We considered four alternative cases depending on the number of observation variables (11 vs. 40 variables) and specification of volatilities of structural shocks (constant volatility vs. time-varying-volatility). Compared with four cases, we suggested the following three empirical evidences in the Great Recession; (1) negative bank net worth shock has gradually outspreaded before corporate net worth shock has burst down, (2) the Data-Rich approach and structural shocks with SV evaluated the contribution of corporate net worth shock to the substantial portion of macroeconomic fluctuations after the Great Recession, in contrast to a standard DSGE model, and (3) Troubled Asset Relief Program (TARP) would work to bail out financial institutions, whereas balance sheets in corporate sector could not have stopped deteriorating yet.

Incorporating time-varying-volatilities of shocks into the DSGE model make their credible bands narrower than half of constant volatilities, implying it is a realistic assumption of dynamics of structural shocks. It is plausible that the tiny volatilities (or the uncertainty) in ordinary times change to extraordinary magnitude at the turning points of business cycles. We also estimated that monetary policy shock has opposite leverage effect of SV which implies tightening policy makes interest rate more volatile.

A Appendix

A.1 Sampling Stochastic Volatility with Leverage

Step 4 of MCMC procedure described in Section 4 employs the algorithm of Omori et al. (2007) which is the extenstion of Kim et al. (1998) toward a SV model with leverage effect. This subsection is based on Justiniano and Primiceri (2008) who employed Kim et al. (1998) for drawing the stochastic volatilites.

Accoding to Omori et al. (2007), the key idea of MCMC algorithm of a SV model with leverage effect is to obtain a draw from an approximate linear and Gaussian state space form such as

$$\begin{pmatrix} \sigma_t^* \\ h_{t+1} \end{pmatrix} = \begin{pmatrix} h_{t+1} \\ \mu + \phi(h_t - \mu) \end{pmatrix} + \begin{pmatrix} z_t^* \\ \nu_t \end{pmatrix}, \tag{A.1}$$

$$\left\{ \begin{pmatrix} z_t^* \\ \nu_t \end{pmatrix} \mid d_{i,t},\, u_{it} = k,\, \rho_i,\, \omega_i \right\} = \begin{pmatrix} m_k + v_k \zeta_t \\ d_t\, \rho\, \omega\, (a_k + b_k v_k \zeta_t)\, \mathbf{exp}(m_k/2) + \omega\sqrt{1-\rho^2}\zeta_t^* \end{pmatrix}, \tag{A.2}$$

$$\begin{pmatrix} \zeta_t \\ \zeta_t^* \end{pmatrix} \sim i.i.d.\, N(0, I_2),$$

where $\sigma_{i,t}^* = \log \sigma_{i,t} = h_{i,t} + z_{i,t}^*$, $h_{it} = \log \sigma_{i,t}$, and $z_{it}^* = \log (z_{it}{}^2)$. And d_{it}, and $\eta_{i,t}$ are denoted as

$$d_{i,t} = \mathbf{I}(z_{i,t} \geq 0) - \mathbf{I}(z_{i,t} < 0),$$
$$\eta_{i,t} = (h_{i,t} - \mu) - \phi(h_{i,t-1} - \mu),$$

where, $\mathrm{I}(\cdot)$ is an indicator function which indicates $d_{i,t} = 1$ when $z_{i,t} > 0$, or otherwise:$d_{i,t} = -1$.

Suppose that the MCMC algorithm has implemented iteration g, generating samples $\Phi_i^{(g)}(= (\phi_i,\, \rho_i\,,\, \omega_i)\,)$ and $\boldsymbol{H}^{T,(g)}$. In iteration $g+1$, the following four steps are used to a set of new draws.

Step 1: Draw the structural shocks $\varepsilon_t^{(g+1)}$.

In order to generate a new sample of stochastic volatilities, we need to obtain a new sample of structural shocks. This can be done using simulation smoother debeloped by de Jong and Shephard (1995) whose algorithm is described in Appendix A2. We obtain a new draw of structural shocks from eq.(A.12) of Appendix A2.

Step 2: Draw the stochastic volatilites $\boldsymbol{H}^{T\,(g+1)}$with leverage effect

With a draw of Shocks in hand, nonlinear measurent equations (2.2) in Section 2.1, which is represented as eq.(A.3) for each structural shock, can be easily converted in linear one such as eq.(A.4) by squaring and taking logarithms of every elements. This induces the following approximating state space repsentation (A.4) and (A.5).

$$\varepsilon_{i,t} = \sigma_{i,t}\, z_{i,t}, \;\; \text{i=1,2,}\cdots\text{, M}, \tag{A.3}$$

$$\tilde{\varepsilon}_{i,t} = 2h_{i,t} + z_{i,t}^*, \tag{A.4}$$

$$h_{i,t} = \mu + \phi(h_{i,t-1} - \mu) + \nu_{i,t},\ \nu_{i,t} \sim \text{i.i.d. N}(0, \omega_i^2) \tag{A.5}$$

where $\tilde{\varepsilon}_{i,t} = \log[\,(\,\varepsilon_{i,t}\,)^2 + \bar{c}]$; $\bar{c}$ is the offset constant (set to 0.001); $h_{it} = \log \sigma_{i,t}$ and $z^*_{it} = \log(z_{it}{}^2)$. M is the number of structural shocks. Since the squared shocks $\varepsilon^2_{i,t}$ is very small, an offset constant is used to make the estimation procedure more robust. Eqs.(A.4) and (A.5) are linear, but non-Gaussian state space form, because z^*_{it} are distributed as a log $\chi^2(1)$. In order to transform the system in a Gaussian state space form, a mixture of normals approximation of the log $\chi^2(1)$ distribution is used, as decribed in Kim et al. (1998) and Omori et al. (2007). A draw of $z^*_{i,t}$ is implemented from the mixture normal distribution given as

$$f(z^*_{i,t}) = \sum_{k=1}^{K} q_k f_N(z^*_{i,t} \mid u_{i,t} = k),\ i = 1, \cdots\cdots, M, \tag{A.6}$$

where $u_{i,t}$ is the indicator variable selecting which member of the mixture of normals has to be used at period t for shock i. And q_k is the probability of $u_{i,k} = k$; $q_k = \Pr(u_{i,t} = k)$, and $f_N(\cdot)$ denotes the probability density function of normal distribution. Omori et al (2007) select a mixture of ten normal densities ($K = 10$) with component probabilities q_k, means m_k, and variances v_j^2 , for $k = 1, 2, \cdots 10$, chosen to match a number of moment of the log $\chi^2(1)$ distribution. The constant $\{q_k, m_k, v_k^2\}$ are reported as Table blow.

Table of Selection Probability Function (q_k, m_k, v_k^2, a_k, b_k)
K=10

k	q_k	m_k	v_k^2	a_k	b_k
1	0.00609	1.92677	0.11265	1.01418	0.50710
2	0.04775	1.34744	0.17788	1.02248	0.51124
3	0.13057	0.73504	0.26768	1.03403	0.51701
4	0.20674	0.02266	0.40611	1.05207	0.52604
5	0.22715	-0.85173	0.62699	1.08153	0.54076
6	0.18842	-1.97278	0.98583	1.13114	0.56557
7	0.12047	-3.46788	1.57469	1.21754	0.60877
8	0.05591	-5.55246	2.54498	1.37454	0.68728
9	0.01575	-8.68384	4.16591	1.68327	0.84163
10	0.00115	-14.65000	7.33342	2.50097	1.25049

Using generator of the mixture normal distribution above, the system has an approximate linear and Gaussian state space form. Therefore, a new draw of the stochastic volatilites $\boldsymbol{H}^{T\,(g+1)}$ can be obtained recursively with standard Gibbs sampler for state space form using the algorithm of Carter and Kohn (1994).

Step 3: Draw the indicators of the mixture approximation $u^{T\,(g+1)}$

In the case of SV with leverage effect, we need to modify the indicator $u_{i,t}$ for the mixture normal described in Step 2, compared with Justiniano and Primiceri (2008). We follow the algorithm proposed by Omori et al. (2007), and obtain a new draw of indicators $u_{i,t}$ which is generated condtional on $\varepsilon^{(g+1)}_{i,t}$, $\boldsymbol{H}^{T,(g+1)}$ by independently sampling each from the discrete density defined by

$$\pi(u_{i,t} = k \mid \varepsilon_{i,t},\, h_{it},\, \Phi) \;\propto\; \pi(u_{i,t} = k \mid \sigma^*_{i,t},\, d_{it},\, h_{it},\, \Phi) \;\propto\; \pi(u_{it} = k \mid z^*_{i,t},\, \eta_{i,t},\, d_{i,t},\, \Phi)$$

$$\propto q_k\, v_k^{-1} \exp\left\{ -\frac{(z_{i,t}^{*}-m_k)^2}{2v_k^2} - \frac{[\eta_{i,t} - d_{i,t}\,\rho_i\,\omega_i \exp{(m_k/2)}\{a_k + b_k\,(z_{i,t}^{*}-m_k)\}]^2}{\omega_i^2(1-\rho_i^2)} \right\} \quad \text{(A.7)}$$

Step 4: Draw the coefficients $\Phi_i^{(g+1)}(=(\phi_i,\,\rho_i\,,\,\omega_i))$ of stochastic volatility processess.

Having generated a sample $H^{T,(g+1)}$, we sample the elements of vector $\Phi_i^{(g+1)}$ from the density

$$p(\,\Phi_i\,|\,\sigma_{i,t}^{*},\,d_{i,t}\,u_{i,t}\,) \propto p(\sigma_{i,t}^{*}\,|\,d_{it},\,u_{i,t}\,\Phi_i\,)p(\Phi_i).$$

The density $p(\sigma_{i,t}^{*}\,|\,d_{it},\,u_{i,t}\,\Phi_i\,)$ is found from the output of Kalman filter recursion applied to the state space model (A.1) and (A.2). For the sampling we rely on the Metropolis-Hasting algorithm with a proposal density based on random walk such as

$$\boldsymbol{\theta}^{(\text{proposal})} = \boldsymbol{\theta}^{(g-1)} + \boldsymbol{u}_t, \quad \boldsymbol{u}_t \sim N(\mathbf{0}, c\boldsymbol{\Sigma}),$$

where c is an adjustment constant.

A.2 Simulation Smoother of Model Variable

Step 3.1 of algorithm of data-rich DSGE described in Section 4 employs simulation smoother (de Jong and Shephard, 1995) which generate sampling of model variables $\boldsymbol{S}_t$ from conditional posterior distribution, $p(\boldsymbol{S}^T|\boldsymbol{\Gamma}^{(g-1)}, \boldsymbol{\theta}, \boldsymbol{X}^T)$. [18]On the other hand, Boivin and Giannoni (2006), and Kryshko (2011) employ smoothing method proposed by Carter and Kohn (1994). But their method does not apply only to sample positive definite matrix as variance covariance matrix of state variables so that their method discontinue on the way of sampling in MCMC pointed out by Chib (2001, p.3614). As a result, Kryshko (2011) transforms to ad hoc variance covariance matrix of state variables. To avoid this problem, our algorithm employs simulation smoother instead of Carter and Kohn's (1994) algorithm. Accordingly, our algorithm accomplishes generalization of estimating data-rich DSGE model.

To simplify representation of algorithm of simulation smoother, we rewrite state space model of (2.17) and (2.18) described in Section 2.1 into ($A.8$), and ($A.9$) as below.

$$\tilde{\boldsymbol{X}}_t = \tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{S}}_t + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim N(\mathbf{0},\, \mathbf{R}), \tag{A.8}$$

$$\tilde{\boldsymbol{S}}_t = \tilde{\mathbf{G}}\tilde{\boldsymbol{S}}_{t-1} + \tilde{\mathbf{E}}\boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim N(\mathbf{0},\, \mathbf{Q}(\boldsymbol{\theta})), \tag{A.9}$$

[18]Another simulation smoother has been invented by Durbin and Koopman (2002). The advantage of their method is to make code easily because of using existing Kalman smoother and not coding new algorithm, while simulation smoother of Carter and Kohn (1994) and de Jong and Shephard (1995) need to made new code of their algorithm. However, since our model is medium-size DSGE model and it requests long computing time for MCMC processing, we adopt more speeding algorithm of de Jong and Shephard (1995), instead of Durbin and Koopman (2002).

The following four steps are conducted to generate a new draw of model variables.

Step 1: Kalman filter for state space model is implemented.

Kalman filter is represented as

$$\boldsymbol{\eta}_t = \tilde{\boldsymbol{X}}_t - \tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{S}}_{t|t}, \qquad \mathbf{F}_t = \tilde{\boldsymbol{\Lambda}}\tilde{\mathbf{P}}_{t|t}\tilde{\boldsymbol{\Lambda}}' + \mathbf{R}, \qquad \mathbf{K}_t = \tilde{\mathbf{G}}\tilde{\mathbf{P}}_{t|t}\tilde{\boldsymbol{\Lambda}}'\mathbf{F}_t^{-1},$$

$$\mathbf{L}_t = \tilde{\mathbf{G}} - \mathbf{K}_t\tilde{\boldsymbol{\Lambda}}, \qquad \tilde{\boldsymbol{S}}_{t+1|t+1} = \tilde{\mathbf{G}}\tilde{\boldsymbol{S}}_{t|t} + \mathbf{K}_t\boldsymbol{\eta}_t, \qquad \tilde{\mathbf{P}}_{t+1|t+1} = \tilde{\mathbf{G}}\tilde{\mathbf{P}}_{t|t}\mathbf{L}_t' + \tilde{\mathbf{E}}\mathbf{Q}(\boldsymbol{\theta})\tilde{\mathbf{E}}',$$

where $\boldsymbol{\eta}_t$ is forecasting errors, $\mathbf{K}_t$ is Kalman gain, $\tilde{\mathbf{P}}_t$ is variance covariance matrix of state variables $\tilde{\boldsymbol{S}}_t$. Filtering of $\tilde{\boldsymbol{S}}_{t|t}$, $\tilde{\mathbf{P}}_{t|t}$ iterates forward for period $t = 1, 2, \cdots, T$. And for initial value $\tilde{\boldsymbol{S}}_{1|1}, \tilde{\mathbf{P}}_{1|1}$, we set $\tilde{\boldsymbol{X}}_1 = \tilde{\boldsymbol{\Lambda}}\tilde{\boldsymbol{S}}_1$, and $\tilde{\mathbf{P}}_{1|1} = \tilde{\mathbf{G}}\tilde{\mathbf{P}}_{1|1}\tilde{\mathbf{G}}' + \tilde{\mathbf{E}}\mathbf{Q}(\boldsymbol{\theta})\tilde{\mathbf{E}}'$, where subscript $t|t$ of $\tilde{\boldsymbol{S}}_{t|t}$ denotes conditional expected value of $\tilde{\boldsymbol{S}}_t$ up to information on $\boldsymbol{X}_1, \cdots, \boldsymbol{X}_t$, thus, $E(\tilde{\boldsymbol{S}}_t|\boldsymbol{X}_1, \boldsymbol{X}_2, \cdots, \boldsymbol{X}_t)$.

Step 2: Generate values of $\boldsymbol{r}_{t-1}$, $\mathbf{N}_{t-1}$by implementing simulation smoother.

This algorithm is iterated backward from period: $t = T, \cdots, 2, 1$ using values obtained from Kalman filter, as following equations $(A.10)$, $(A.11)$.

$$\boldsymbol{r}_{t-1} = \tilde{\boldsymbol{\Lambda}}'\mathbf{F}_t^{-1}\boldsymbol{\eta}_t - \mathbf{W}_t'\mathbf{C}_t^{-1}\boldsymbol{d}_t + \mathbf{L}_t'\boldsymbol{r}_t, \tag{A.10}$$

$$\mathbf{N}_{t-1} = \tilde{\boldsymbol{\Lambda}}'\mathbf{F}_t^{-1}\tilde{\boldsymbol{\Lambda}} + \mathbf{W}_t'\mathbf{C}_t^{-1}\mathbf{W}_t + \mathbf{L}_t'\mathbf{N}_t\mathbf{L}_t, \tag{A.11}$$

where $\mathbf{W}_t$ and $\mathbf{C}_t$ are obtained from the equations such as

$$\mathbf{W}_t = \mathbf{Q}(\boldsymbol{\theta})\tilde{\mathbf{E}}'\mathbf{N}_t\mathbf{L}_t,$$

$$\mathbf{C}_t = \mathbf{Q}(\boldsymbol{\theta}) - \mathbf{Q}(\boldsymbol{\theta})\tilde{\mathbf{E}}'\mathbf{N}_t\tilde{\mathbf{E}}\mathbf{Q}(\boldsymbol{\theta}),$$

and random variable $\boldsymbol{d}_t$ is generated from $N(\mathbf{0}, \mathbf{C}_t)$. Initial value $\boldsymbol{r}_T$ and $\mathbf{N}_T$ are set at $\boldsymbol{r}_T = \mathbf{0}$, and $\mathbf{N}_T = \mathbf{O}$.

Step 3: Smoothing of structural shocks $\hat{\varepsilon}_{t|T}$, are implemented backward iteration using the equation (A.12).

Subscript $t|T$ of $\hat{\varepsilon}_{t|T}$ denotes expected value conditional on total sample period such as $E(\varepsilon_t|\boldsymbol{X}_1, \boldsymbol{X}_2, \cdots, \boldsymbol{X}_T)$.

$$\hat{\varepsilon}_{t|T} = \mathbf{Q}(\boldsymbol{\theta})\tilde{\mathbf{E}}'\boldsymbol{r}_t + \boldsymbol{d}_t \qquad \boldsymbol{d}_t \sim N(\mathbf{0}, \mathbf{C}_t), \qquad t = T, \cdots, 2, 1 \tag{A.12}$$

Step 4: Generate model variables $\tilde{S}_t$ by forward iteration of the equation (A.13).

$$\tilde{\boldsymbol{S}}_{t+1|T} = \tilde{\mathbf{G}}\tilde{\boldsymbol{S}}_t + \mathbf{E}\hat{\boldsymbol{\varepsilon}}_{t|T}, \qquad t = 1, 2, \cdots, T, \tag{A.13}$$

where initial value $\tilde{\boldsymbol{S}}_{1|T}$ is obtained from $\tilde{\boldsymbol{S}}_{1|T} = \tilde{\boldsymbol{S}}_{1|1} + \tilde{\mathbf{P}}_{1|1}\boldsymbol{r}_0$.

The algorithm described above is procedure generating model variables$\tilde{\boldsymbol{S}}_t(t = 1, 2, \cdots, T)$ from conditional posterior distribution $p(\boldsymbol{S}^T|\boldsymbol{\Gamma}^{(g-1)}, \boldsymbol{\theta}, \boldsymbol{X}^T)$ which is implemented in Step 3.1 of Section 4.

A.3 Sampling of Parameters Set Γ of Measurement Equation (2.11)

In Step 3.2 of MCMC algorithm in Section 4, we sample parameters $\mathbf{\Gamma} = \{\mathbf{\Lambda}, \mathbf{R}, \mathbf{\Psi}\}$ of measurement equation obtained from (2.11) and (2.13) . To do so, (2.11) is transformed by substituing (2.13) into it as

$$(\mathbf{I} - \mathbf{\Psi}L)\boldsymbol{X}_t = (\mathbf{I} - \mathbf{\Psi}L)\mathbf{\Lambda}\boldsymbol{S}_t + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim i.i.d.\, N(\mathbf{0}, \mathbf{R}),$$

where $\mathbf{I}$ denotes identity matrix. The sampling of parameters $\mathbf{\Gamma} = \{\mathbf{\Lambda}, \mathbf{R}, \mathbf{\Psi}\}$ from conditional posterior distribution $p(\mathbf{\Gamma}|\boldsymbol{S}^{T(g)}, \boldsymbol{\theta}^{(g-1)}, \boldsymbol{X}^T)$ given the unobserved model variables $\boldsymbol{S}^T$and deep parameters $\boldsymbol{\theta}$, is conducted following the approach by Chib and Greenberg (1994) who proposed Bayesian estimation method of linear regression model with AR (1) errors such like (2.11) and (2.13).

For estimating above model, Chib and Greenberg (1994) divided it into two linear regression models. First, by using notations, $X^*_{k,t} = X_{k,t} - \Psi_{kk}X_{k,t-1}$, and $\boldsymbol{S}^*_{k,t} = \boldsymbol{S}_{k,t} - \Psi_{kk}\boldsymbol{S}_{k,t-1}$ where subscript k is k-th indicator of data set $\boldsymbol{X}_t$, above equation is represented as

$$\boldsymbol{X}^*_t = \mathbf{\Lambda}\boldsymbol{S}^*_t + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim i.i.d.\, N(\mathbf{0}, \mathbf{R}),$$

Second, by using notation $e_{k,t} = X_{k,t} - \mathbf{\Lambda}_k\boldsymbol{S}_t$ which means measurement errors, the equation is also rewritten as

$$\boldsymbol{e}_k = \Psi_{kk}\boldsymbol{e}_{k,-1} + \boldsymbol{\nu}_k,$$

where $\boldsymbol{e}_k = [e_{k,2}, \ldots, e_{k,T}]'$, $\boldsymbol{e}_{k,-1} = [e_{k,1}, \ldots, e_{k,T-1}]'$. We sample parameter $(\mathbf{\Lambda}, \mathbf{R})$ given parameter $\mathbf{\Psi}$ from the first equation, and parameter $\mathbf{\Psi}$ given $(\mathbf{\Lambda}, \mathbf{R})$ from the second equation sequencially based on the following two-step algorithm.

Step 1. **Sampling** (Λ_k, R_{kk}) **from conditional posterior distribution** $p(\Lambda_{kk}, R_{kk} \mid \Psi_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T)$ **for estimating equation**

$$\boldsymbol{X}^*_t = \mathbf{\Lambda}\boldsymbol{S}^*_t + \boldsymbol{\nu}_t, \qquad \boldsymbol{\nu}_t \sim i.i.d.\, N(\mathbf{0}, \mathbf{R}).$$

The posterior density of (Λ, R_{kk}) given the unobserved state variables $\boldsymbol{S}^T$and deep parameters $\boldsymbol{\theta}$ is represented as

$$p(\mathbf{\Lambda}_k, R_{kk}|\Psi_{kk}, \boldsymbol{S}^T, \boldsymbol{X}^T) \;\; \propto \;\; p(\boldsymbol{X}^T|\boldsymbol{S}^T, \mathbf{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta})\; p(\Lambda_{kk}, R_{kk}),$$

where $p(\boldsymbol{X}^T|\boldsymbol{S}^T, \mathbf{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta})$ is likelifood function and $p(\Lambda_{kk}, R_{kk})$ is prior density.

As shown by Chib and Greenberg (1994), the above likelifood function is proportional to a Normal -Inverse-Gamma density as

$$p(\boldsymbol{X}^T|\boldsymbol{S}^T, \mathbf{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta}) \;\; \propto \;\; p_{NIG}(\mathbf{\Lambda}_k, R_{kk} \mid \hat{\mathbf{\Lambda}}_k, (\boldsymbol{S}^{*\prime}\boldsymbol{S}^*),\; s,\; T - N - 2)$$

where

$$\begin{aligned}
\hat{\mathbf{\Lambda}}_k &= (\boldsymbol{S}^{*\prime}\boldsymbol{S}^*)^{-1}\,\boldsymbol{S}^{*\prime}\boldsymbol{X}_k^* \\
s &= \boldsymbol{X}_k^{*\prime}\Big(\mathbf{I}_T - \boldsymbol{S}^*(\boldsymbol{S}^{*\prime}\boldsymbol{S}^*)^{-1}\boldsymbol{S}^{*\prime}\Big)\boldsymbol{X}_k^* \;=\; \boldsymbol{X}_k^{*\prime}\Big(\boldsymbol{X}_k^* - \boldsymbol{S}^*\hat{\mathbf{\Lambda}}_k\Big).
\end{aligned}$$

Since above prior $p(\Lambda_{kk},\ R_{kk})$ is assumed to be Normal-Inverse-Gamma $p_{NIG}(\mathbf{\Lambda}_k, R_{kk}| \mathbf{\Lambda}_{k,0}, \mathrm{M}_{k,0}, s_0, \nu_0)$, the resulting conditional posterior density is also Normal -Inverse-Gamma as following.

$$\begin{aligned}
p(\mathbf{\Lambda}_k, R_{kk}|\Psi_{kk}, \boldsymbol{S}^T, \boldsymbol{X}^T) &\propto p_{NIG}(\mathbf{\Lambda}_k, R_{kk} \mid \hat{\mathbf{\Lambda}}_k, (\boldsymbol{S}^{*\prime}\boldsymbol{S}^*), s, T-N-2) \\
&\times\ p_{NIG}(\mathbf{\Lambda}_k, R_{kk} \mid \mathbf{\Lambda}_{k,0}, \mathrm{M}_{k,0}, s_0, \nu_0) \\
&\propto p_{NIG}(\mathbf{\Lambda}_k, R_{kk} \mid \bar{\mathbf{\Lambda}}_k, \bar{\mathrm{M}}_k, \bar{s}, \bar{\nu})
\end{aligned}$$

where

$$\begin{aligned}
\bar{\mathrm{M}}_k &= \mathrm{M}_{k,0} + \Big(\boldsymbol{S}^{*\prime}\boldsymbol{S}^*\Big) \\
\bar{\mathbf{\Lambda}}_k &= \bar{\mathrm{M}}_k^{-1}\Big(\mathrm{M}_{k,0}\ \mathbf{\Lambda}_{k,0} + (\boldsymbol{S}^{*\prime}\boldsymbol{S}')\hat{\mathbf{\Lambda}}_k\Big) \\
\bar{s} &= s_0 + s + (\boldsymbol{\lambda}_{k,0} - \hat{\mathbf{\Lambda}}_k)'\Big[\mathrm{M}_{k,0}^{-1} + (\boldsymbol{S}^{*\prime}\boldsymbol{S}^*)^{-1}\Big]^{-1}(\boldsymbol{\lambda}_{k,0} - \hat{\mathbf{\Lambda}}_k) \\
\bar{\nu} &= \nu_0 + T
\end{aligned}$$

and $\mathbf{\Lambda}_{k,0}$, $\mathrm{M}_{k,0}$, s_0, and ν_0 are parameters of the prior density.

We sample factor loading $\mathbf{\Lambda}_k$ and the variance of measuremenent error R_{kk} sequencially from

$$\begin{aligned}
R_{kk}|\Psi_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T &\sim IG(\bar{s}, \bar{\nu}) \\
\mathbf{\Lambda}_k|R_{kk}, \Psi_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T &\sim N(\bar{\mathbf{\Lambda}}_k, R_{kk}, \bar{\mathrm{M}}_k^{-1})
\end{aligned}$$

Step 2. **Sampling Ψ_{kk} from conditional posterior distribution $p(\Psi_{kk}|\mathbf{\Lambda},\ R_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T)$ for estimating equation of measurement errors**

$$\boldsymbol{e}_k = \Psi_{kk}\boldsymbol{e}_{k,-1} + \boldsymbol{\nu}_k.$$

The conditional posterior density :$p(\Psi_{kk}|\mathbf{\Lambda}, R_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T)$ is given as

$$p(\Psi_{kk}|\mathbf{\Lambda}_k, R_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T) \;\propto\; p(\boldsymbol{X}_k^T|\boldsymbol{S}^T, \mathbf{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta})\ p(\Psi_{kk}),$$

where $p(\boldsymbol{X}_k^T|\boldsymbol{S}^T, \mathbf{\Lambda}_k, R_{kk}, \Psi_{kk}, \boldsymbol{\theta})$ is likelihood function and $p(\Psi_{kk})$ is prior density.

Then, above likelifood function is proportional to the normal density such as

$$p(\boldsymbol{X}_k^T|\boldsymbol{S}^T, R_{kk}, \Psi_{kk}, \boldsymbol{\theta}) \propto \exp\left[-\frac{1}{2R_{kk}}(\Psi_{kk} - \hat{\Psi}_{kk})'\boldsymbol{e}'_{k,-1}\boldsymbol{e}_{k,-1}(\Psi_{kk} - \hat{\Psi}_{kk})\right].$$

And above prior density of coefficient of AR (1) errors Ψ_{kk} is also normal density but truncated at less than unity because dynamic of errors keep to be stationary. So, prior densty is assumed to be such as

$$p(\Psi_{kk}) \quad \propto \quad \exp\left[-\frac{1}{2\sigma^2_{\Psi,0}}(\Psi_{kk} - \Psi_0)^2\right] \times \mathbf{1}_{\{|\Psi_{kk}|<1\}},$$

where $\mathbf{1}_{\{|\Psi_{kk}|<1\}}$ denotes indicator function which is unity if $|\Psi_{kk}| < 1$, otherwise zero.

The conditional posterior density is proportional to a product of above two normal densities, and represented as

$$\begin{aligned} p(\Psi_{kk}|R_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T) \quad &\propto \quad \exp\left[-\frac{1}{2R_{kk}}(\Psi_{kk} - \hat{\Psi}_{kk})'\boldsymbol{e}'_{k,-1}\boldsymbol{e}_{k,-1}(\Psi_{kk} - \hat{\Psi}_{kk})\right] \\ &\times \quad \exp\left[-\frac{1}{2\sigma^2_{\Psi,0}}(\Psi_{kk} - \Psi_0)^2\right] \times \mathbf{1}_{\{|\Psi_{kk}|<1\}}. \end{aligned}$$

Hence, we sample coefficient of AR (1) errors Ψ_{kk} from trancasted normal such as

$$\begin{aligned} &\Psi_{kk}|R_{kk}, \boldsymbol{S}^T, \boldsymbol{\theta}, \boldsymbol{X}^T \sim N(\bar{\Psi}_{kk}, \bar{V}_{\Psi_{kk}}) \times \mathbf{1}_{\{|\Psi_{kk}|<1\}}, \\ \text{where} \quad &\bar{V}_{\Psi_{kk}} = \left[(R_{kk}(\boldsymbol{e}'_{k,-1}\boldsymbol{e}_{k,-1})^{-1})^{-1} + (\sigma^2_{\Psi,0})^{-1}\right]^{-1}, \\ &\bar{\Psi}_{kk} = \bar{V}_{\Psi_{kk}}\left[(R_{kk}(\boldsymbol{e}'_{k,-1}\boldsymbol{e}_{k,-1})^{-1})^{-1}\hat{\Psi}_{kk} + (\sigma^2_{\Psi,0})^{-1}\Psi_0\right]. \end{aligned}$$

A.4 The Remaining Framework of the DSGE model

In this section, the remaining structure of our DSGE model described in Section 3 is dealt with.

A.4.1 Household Sector

There is a continuum of members in the household where the total population measures to one. Within the household, there are fractions of f^E entrepreneurs, f^F financial intermediaries (or "bankers"), and $1 - f^E - f^F$ workers. Entrepreneurs engage in a business where they produce intermediate goods and transfer the net worth back to the household when they exit from the business. Now, each financial intermediary manages a bank where it accepts the deposits from the household sector and lend to entrepreneurs. When financial intermediaries exit from their business, they also transfer their net worth back to the household sector. Finally, remaining fraction of the members of the household become workers. Workers supply labor input to earn wage and they transfer their wage earnings to the household each period. Within the household, each member shares the risk perfectly.

The representative household maximizes her expected discounted sum of utility over time and their objective function is specified as follow;

$$E_t \sum_{i=0}^{\infty} \beta^i \chi_{t+i}^c \left[\frac{(c_{t+i} - hC_{t+i-1})^{1-\sigma^c}}{1-\sigma^c} - \chi_{t+i}^L \frac{(l_{t+i})^{1+\sigma^L}}{1+\sigma^L} \right] \tag{A.14}$$

where β is the discount rate, h is the habit persistence,σ^c is the inverse of intertemporal elasticity of substitution, c_t is final goods consumption, C_{t-1} represents the external habit formation, σ^L is the inverse of Frisch labor supply elasticity and l_t is the supply of aggregate labor by workers. Now, there are two structural shocks embedded in the function. χ_t^c represents an intertemporal preference shock, while χ_t^L represents labor disutility shock relative to consumption.

Next, turning to the budget constraint of the representative household, they make a deposit, b_t, at period t and earn real interest rate, R_t/π_{t+1}, next period where R_t is risk-free gross nominal interest rate at period t and π_{t+1} is gross inflation rate at period $t+1$. In addition, the household pays lump sum tax of τ_t to the government. Now, they receive a lump-sum transfer of wage incomes from workers which is expressed as $\int_0^1 w_t(x) l_t(x) dx$, where $w_t(x)$ and $l_t(x)$ are real wage and labor supply by individual worker x, respectively.[19] Finally, the household earns the combined dividend of Ξ_t^{div} from retailers, earns the net transfer of Ξ_t^E from entrepreneurs, and the net transfer of Ξ_t^F from bankers each period. Thus, the representative household's budget constraint at period t can be expressed as, in real terms, as follow, ,

$$c_t + b_t = \frac{R_{t-1}}{\pi_t} b_{t-1} - \tau_t + \Xi_t^{div} + \Xi_t^E + \Xi_t^F. \tag{A.15}$$

Consumption and Deposit Decision The first-order conditions (hereafter, FOCs) of the household with respect to c_t and b_t as follows;

$$\zeta_t^H = \chi_t^c (c_t - hc_{t-1})^{-\sigma^c} \tag{A.16}$$

$$\zeta_t^H = \beta E_t \zeta_{t+1}^H \frac{R_t}{\pi_{t+1}}. \tag{A.17}$$

where ζ_t^H is Lagrangian multiplier associated with the budget constraint. (A.16) is the FOC of consumption which equates the marginal utility of consumption to the shadow price of the final goods. (A.17) is the FOC of deposit decision.

Wage Setting Decision by Workers Following Erceg, Henderson, and Levin (2000) (hereafter, EHL), each worker indexed by $x \in [0,1]$ supplies differentiated labor input, $l_t(x)$, monopolistically and sells this service to the labor union who

[19] Here, the real wage set by worker x is defined as $w_t(x) \equiv W_t(x)/P_t$, where $W_t(x)$ stands for the nominal wage set by worker x and P_t stands for the price index of final goods. The formulation of $W_t(x)$ and P_t will be described later in this section.

is perfectly competitive.[20] Each worker sets his nominal wage according to Calvo style sticky price setting where fraction θ^w of the entire workers cannot freely adjust the wages at their discretion. For fraction θ^w of workers, the partial indexation of the nominal wage is assumed.[21] Due to the perfect risk-sharing assumed in the model, each worker maximizes the objective function (A.14) by choosing the amount of individual labor supply, $l_t(x)$, while taking the amount of consumption, c_t, as given. Under this setting, $(1-\theta^w)$ fraction of workers maximize their objective function by setting the nominal wage, $\widetilde{W}_t$, such that

$$E_t \sum_{i=0}^{\infty} \beta^i (\theta^w)^i \left[\frac{\widetilde{W}_t}{P_{t+i}} \left(\frac{P_{t-1+i}}{P_{t-1}} \right)^{\iota^w} \chi^c_{t+i} \left(c_{t+i} - hc_{t+i-1} \right)^{-\sigma^c} - (1+\psi^w) \chi^c_{t+i} \chi^L_{t+i} \left(l_{t+i}(x) \right)^{\sigma^L} \right] l_{t+i}(x) = 0. \tag{A.18}$$

The law of motion of the aggregate wage index can be shown to be as follow,

$$W_t^{-1/\psi^w} = \theta^w \left[W_{t-1} \left(\frac{W_{t-1}}{W_{t-2}} \right)^{\iota^w} \right]^{-1/\psi^w} + (1-\theta^w) \widetilde{W}_t^{-1/\psi^w}. \tag{A.19}$$

Finally, the real wage index in the economy is defined as $w_t \equiv W_t / P_t$.

A.4.2 Capital Production Sector

Capital producers are identical, perfectly competitive, and risk neutral. They purchase i^k_t units of final goods from the retailer, convert them to i^k_t units of capital goods, and combine them with existing capital stock, $(1-\delta)k_t$, to produce new capital stock, k_{t+1}. Capital producers will, then, sell off new capital stock to entrepreneurs in a perfectly competitive manner. Capital producers have linear production technology in converting final goods to capital goods. In addition, they will incur quadratic investment adjustment cost when they change the production capacity of capital goods from previous period. Each capital producer maximizes the expected discounted cash flow with respect to i^k_t.[22] The FOC is given by

$$q_t = \frac{1}{A^k_t} \left[1 + \kappa \left(\frac{i^k_t}{i^k_{t-1}} - 1 \right) \frac{i^k_t}{i^k_{t-1}} + \frac{\kappa}{2} \left(\frac{i^k_t}{i^k_{t-1}} - 1 \right)^2 \right] - \beta \frac{\kappa}{A^k_{t+1}} \left(\frac{i^k_{t+1}}{i^k_t} - 1 \right) \left(\frac{i^k_{t+1}}{i^k_t} \right)^2. \tag{A.20}$$

[20]The labor union transforms labor services to an aggregate labor input, l_t using the Dixit and Stiglitz type aggregator function. The factor demand function for $l_t(x)$ is given by $l_t(x) = (W_t(x)/W_t)^{-(1+\psi^w)/\psi_w} l_t$ where ψ^w is the wage markup, $W_t(x)$ is the nominal wage set by worker x and W_t is the aggregate nominal wage index which is given as $W_t = \left[\int_0^1 W_t(x)^{-1/\psi_w} dx \right]^{-\psi_w}$.

[21]The lagged inflation indexation is specified as $W_t(x) = (P_{t-1}/P_{t-2})^{\iota^w} W_{t-1}(x)$ where ι^w controls the degree of nominal wage indexation to past inflation rate.

[22]The profit function for each capital producer at period t can be expressed as follows,

$$E_t \sum_{i=0}^{\infty} \beta^i \left\{ q_{t+i} i^k_{t+i} - \frac{1}{A^k_{t+i}} \left[i^k_{t+i} + \frac{\kappa}{2} \left(\frac{i^k_{t+i}}{i^k_{t+i-1}} - 1 \right)^2 i^k_{t+i} \right] \right\}$$

where A_t^k is the investment-specific technology shock common across all capital producers and κ is the investment adjustment cost parameter. Finally, aggregate capital accumulation equation is given by

$$k_{t+1} = i_t^k + (1-\delta)k_t. \tag{A.21}$$

A.4.3 Retailing Sector

Retailers $z \in [0,1]$ purchase intermediate goods from the entrepreneur at perfectly competitive price and resale them monopolistically in the retail market.[23] We assume Calvo type sticky price setting for the retailer where, for any given period t, fraction θ^p of the entire retailers cannot freely revise their prices. Further, θ^p fraction of the retailers who did not receive a 'signal of price change' will partially index their nominal prices to lagged inflation rate of price index.[24] Under this setting, for $(1-\theta^p)$ fraction of the retailers who received a 'price changing signal' at period t, they maximize their expected discounted sum of profits by setting the nominal price, $\tilde{p}_t$, such that

$$E_t \sum_{i=0}^{\infty} \beta^i (\theta^p)^i \left[\frac{\widetilde{p}_t}{P_{t+i}} \left(\frac{P_{t-1+i}}{P_{t-1}} \right)^{\iota^p} - \left(\frac{\epsilon}{\epsilon - 1} \right) p_{t+i}^{mc} \right] y_{t+i}(z) = 0. \tag{A.22}$$

From the definition of aggregate price index, the law of motion of P_t can be shown to be as follow,

$$(P_t)^{1-\epsilon} = \theta^p \left[P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\iota^p} \right]^{1-\epsilon} + (1-\theta^p)\tilde{p}_t^{1-\epsilon}. \tag{A.23}$$

A.4.4 The Rest of the Economy

In closing the model, we describe the rest of the model structure here. The central bank is assumed to follow a standard Taylor-type monetary policy rule,

$$\hat{R}_t = \rho^R \hat{R}_{t-1} + (1-\rho^R) \left[\mu^\pi \hat{\pi}_t + \mu^y \hat{Y}_t \right] + \varepsilon_t^R \tag{A.24}$$

where ρ^R controls the magnitude of interest smoothing, μ^π is Taylor coefficient in response to inflation gap, μ^y is Taylor coefficient in response to output gap, and ε_t^R is i.i.d. monetary policy shock.

The government budget constraint is simply specified as

$$g_t = \tau_t. \tag{A.25}$$

[23] The demand function for retail goods sold by retailer z is given by $y_t(z) = (P_t(z)/P_t)^{-\epsilon} Y_t$, where Y_t is aggregated final goods, $p_t(z)$ is nominal price of retail goods $y_t(z)$, P_t is aggregate price index of final goods, and ϵ is the price elasticity of retail goods. Specifically, aggregated final goods, Y_t, and the aggregate price index, P_t, are given as follows; $Y_t \equiv \left[\int_0^1 y_t(z)^{(\epsilon-1)/\epsilon} dz \right]^{\epsilon/(\epsilon-1)}$ and $P_t \equiv \left[\int_0^1 p_t(z)^{(\epsilon-1)/\epsilon} dz \right]^{\epsilon/(\epsilon-1)}$.

[24] The lagged inflation indexation is specified as $p_t(z) = (P_{t-1}/P_{t-2})^{\iota^p} p_{t-1}(z)$ where ι^p controls for the magnitude of price indexation to past inflation rate.

The government expenditure, g_t, is financed solely by lump-sum tax, τ_t. In our model, we simply assume that the government expenditure to follow stochastic AR(1) process.

Finally, the market clearing condition for final goods is given as follow,

$$Y_t = c_t + i_t^k + g_t. \tag{A.26}$$

A.4.5 Structural Shocks in the Model

There are eight structural shocks in the model, each of them having a specific economic interpretation as below. Except for monetary policy shock, all of the structural shocks are assumed to follow AR(1) stochastic processes where ρ is for the AR(1) coefficients for respective structural shocks.

$$\begin{aligned}
\text{TFP shock} &: \hat{A}_t = \rho^A \hat{A}_{t-1} + \varepsilon_t^A \\
\text{Preference shock} &: \hat{\chi}_t^c = \rho^c \hat{\chi}_{t-1}^c + \varepsilon_t^c \\
\text{Labor supply shock} &: \hat{\chi}_t^L = \rho^L \hat{\chi}_{t-1}^L + \varepsilon_t^L \\
\text{Investment specific technology shock} &: \hat{A}_t^K = \rho^K \hat{A}_{t-1}^K + \varepsilon_t^K \\
\text{Government spending shock} &: \hat{g}_t = \rho^G \hat{g}_{t-1} + \varepsilon_t^G \\
\text{Monetary policy shock} &: \varepsilon_t^R \\
\text{Corporate net worth shock} &: \hat{\gamma}_t^E = \rho^E \hat{\gamma}_{t-1}^E + \varepsilon_t^E \\
\text{Bank net worth shock} &: \hat{\gamma}_t^F = \rho^F \hat{\gamma}_{t-1}^F + \varepsilon_t^F
\end{aligned}$$

Notice that each stochastic disturbance ε_t including monetary policy shock is assumed to follow time varying volatility using SV model as mentioned in Section 2.

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Data Appendix

No.	Variables	Code	Series description	Unit of data	Source
Case A and Case D: The standard one-to-one matchning estimation method					
1	R	6	Interest rate: Federal Funds Effective Rate	% per annum	FRB
2	Y_1	5	Real gross domestic product (excluding net export)	Billion of chained 2000	BEA
3	C_1	5^*	Gross personal consumption expenditures	Billion dollars	BEA
4	I_1	5^*	Gross private domestic investment - Fixed investment	Billion dollars	BEA
5	π_1	8	Price deflator: Gross domestic product	2005Q1 = 100	BEA
6	w_1	2	Real Wage (Smets and Wouters)	1992Q3 = 0	SW (2007)
7	L_1	1	Hours Worked (Smets and Wouters)	1992Q3 = 0	SW (2007)
8	RE_1	6	Moody's bond indices - corporate Baa	% per annum	Bloomberg
9	Lev_1^F	7	Commercial banks leverage ratio	Total asset/net worth ratio	FRB
10	Lev_1^E	3	Nonfarm nonfin corp business leverage ratio	Total asset/net worth ratio	FRB
11	s_1	1	Charge-off rates for all banks credit and issuer loans	% per annum	FRB
Case B and Case D: The data-rich estimation method					
12	Y_2	4	Industrial production index: final products	Index 2007 = 100	FRB
13	Y_3	4	Industrial production index: total index	Index 2007 = 100	FRB
14	Y_4	4	Industrial production index: products	Index 2007 = 100	FRB
15	C_2	5^*	PCE excluding food and energy	Billions of dollars	BEA
16	C_3	5	Real PCE, quality indexes; nonduable goods	Index 2005 = 100	BEA
17	C_4	5	Real PCE, quality indexes; services	Index 2005 = 100	BEA
18	I_2	5	Real gross private domestic investment	Billions of Chained 2005	BEA
19	I_3	5^*	Gross private domestic investment: fixed nonresidential	Billions of dollars	BEA
20	I_4	5	Manufactures' new orders: nondefence capital goods	Millions of dollars	DOC
21	π_2	8	Core CPI excluding food and energy	Index 2005 = 100	BEA
22	π_3	8	Price index - PCE excluding food and energy	Index 2005 = 100	BEA
23	π_4	8	Price index - PCE - Service	Index 2005 = 100	BEA
24	w_2	4^*	Average hourly earnings: manufacturing	Dollars	BLS
25	w_3	4^*	Average hourly earnings: construction	Dollars	BLS
26	w_4	4^*	Average hourly earnings: service	Dollars	BLS
27	L_2	4	Civillian Labor Force: Employed Total	Thous.	BLS
28	L_3	4	Employees, nonfarm: total private	Thous.	BLS
29	L_4	4	Employees, nonfarm: goods-producing	Thous.	BLS
30	RE_2	6	Bond yield: Moody's Baa industrial	% per annum	Bloomberg
31	RE_3	6	Bond yield: Moody's A corporate	% per annum	Bloomberg
32	RE_4	6	Bond yield: Moody's A industrial	% per annum	Bloomberg
33	Lev_2^F	9	Core capital leverage ratio PCA all insured institutions	Core capital/total asset	FDIC
34	Lev_3^F	7	Domestically chartered commercial banks leverage ratio	Total asset/net worth	FRB
35	Lev_4^F	7	Brokers and dealers leverage ratio	Total asset/net worth	FOF
36	Lev_2^E	3	Nonfarm nonfinancial non-corporate leverage ratio	Total asset/net worth	FOF
37	Lev_3^E	3	Nonfarm corporate leverage ratio	Total asset/net worth	FRB
38	s_2	1	Charge-off rate on all loans and leases all commercial banks	% per annum	FRB
39	s_3	1	Charge-off rate on all loans all commercial banks	% per annum	FRB
40	s_4	1	Charge-off rate on all loans banks 1st to 100th largest by assets	% per annum	FRB

Note: The format is: series number; transformation code; series description; unit of data and data source. The transformation codes are: 1 - demeaned; 2 - linear detrended; 3 - logarithm and demeaned; 4 - logarithm, linear detrend, and multiplied by 100; 5 - log per capita, linear detrended and multiplied by 100; 6 - detrended via HP filter; 7 - logarithm, detrended via HP filter, and multiplied by 100; 8 - first difference logarithm, detrended via HP filter, and multiplied by 400; 9- the reciprocal number, logarithm, detrended via HP filter, and multiplied 100. A $*$ indicate a series that is deflated with the GDP deflator. "PCE" and "SW (2007)" in this table denote personal consumption expenditure and Smets and Wouters (2007), respectively.

Table 1: Specifications of Four Alternative Cases

	Case A	Case B	Case C	Case D
Number of Observation	11	40	11	40
Model Variable to Obs.	1 to 1	1 to 4	1 to 1	1 to 4
Structural Shock	i.i.d. Normal	i.i.d. Normal	SV with Leverage	SV with Leverage

Note: Item *Number of Observatio*n in the first column denotes the number of data indicators used for estimating the model of each case. Item *Model Variable to Ob*s denote the ratio what number of observations per one model variable are adopted. In the case of a standard DSGE model, we adopt one to one matching between model variables and obsevations. In data rich approach, one to many matching are adopted between model variables and obsevations. Item *Structural Shock* denotes specification of stochastic process of shocks. SV is abbreviation of stochastic volatility.

Table 2: Calibrated Parameters and Key Steady States

Calibrated Param.	Description	Value	Source
β	Discount factor	0.995	Our setting
δ	Depreciation rate	0.025	Christensen and Dib (2008)
α	Capital share	0.33	Gertler and Kiyotaki (2010)
γ_{ss}^E	Survival rate of entrepreneur in steady state	0.972	Christensen and Dib (2008)
γ_{ss}^F	Survival rate of banker in steady state	0.972	Gertler and Kiyotaki (2010)
λ	Bank's participation constraint parameter	0.383	Gertler and Kiyotaki (2010)
ψ^w	Wage markup	0.05	Smets and Wouters (2003)
ϵ	Elasticity Substitution of intermediate goods	11	Our setting
ξ^E	New entrepreneur entry rate	0.003	Our setting
ξ^F	New banker entry rate	0.003	Gertler and Kiyotaki (2010)
Key Steady State	Description	Value	
mc_{ss}	Steady state marginal cost	$\frac{\epsilon-1}{\epsilon}$	-
S_{ss}	Steady state external financial premium	1.0075	Christensen and Dib (2008)
rr_{ss}^E	Steady state corp. borrowing rate (real, QPR)	1.0152	From data (1980Q1-2010Q2)
rr_{ss}^F	Steady state bank lending rate (real, QPR, ex-premium)	rr_{ss}^E/S_{ss}	-
rr_{ss}	Steady state real interest	$1/\beta$	-
ν_{ss}	Steady state Nu	$\frac{(1-\gamma_{ss}^F)\beta(rr_{ss}^F-rr_{ss})}{(1/\beta-\gamma_{ss}^F)}$	-
η_{ss}	Steady state Eta	$\frac{1-\gamma_{ss}^F}{1-\beta\gamma_{ss}^F}$	-
Lev_{ss}^F	Steady state leverage ratio of banker	$\frac{\eta_{ss}}{\lambda-\nu_{ss}}$	-
K_{ss}/N_{ss}^E	Steady state leverage ratio of entrepreneur	1.919	From data (1980Q1-2010Q2)
K_{ss}/Y_{ss}	Steady state capital/output ratio	$\frac{\alpha mc_{ss}}{rr_{ss}^E-(1-\delta)}$	-
I_{ss}/Y_{ss}	Steady state investment/output ratio	$\delta K_{ss}/Y_{ss}$	-
G_{ss}/Y_{ss}	Steady state government expenditure/output ratio	0.2	Gertler and Kiyotaki (2010)
C_{ss}/Y_{ss}	Steady state consumption/output ratio	$1-I_{ss}/Y_{ss}-G_{ss}/Y_{ss}$	-

Table 3: Prior Settings of Structural Parameters

Parameter	Description	Density	Prior Mean	Prior SE
Structural Parameters				
Parameter	Description	Density	Prior Mean	Prior SE
κ	Investment adjustment cost	Gamma	1.000	0.500
h	Habit formation	Beta	0.500	0.250
σ^C	IES of consumption	Gamma	1.500	0.500
σ^L	Inverse Frisch elasticity of labor supply	Gamma	1.500	0.500
φ	Elasticity of premium to leverage ratio	Inv. Gamma	0.050	4.000
ι_P	Price indexation	Beta	0.500	0.100
ι_W	Wage indexation	Beta	0.500	0.250
θ_P	Calvo parameter for goods pricing	Beta	0.500	0.250
θ_W	Calvo parameter for wage setting	Beta	0.500	0.250
ρ_R	Moneatary policy persist. param.	Beta	0.500	0.250
μ_π	Taylor coefficient for inflation	Gamma	1.500	0.500
μ_Y	Taylor coefficient for output gap	Gamma	0.500	0.250
Persistence Parameters for Structural Shocks				
Parameter	Description	Density	Prior Mean	Prior SE
ρ_A	Persistent parameter for TFP shock	Beta	0.500	0.250
ρ_C	Persistent parameter for preference shock	Beta	0.500	0.250
ρ_K	Persistent parameter for investment tech. shock	Beta	0.500	0.250
ρ_E	Persistent parameter for entrepreneur net worth shock	Beta	0.500	0.250
ρ_F	Persistent parameter for banking sector net worth shock	Beta	0.500	0.250
ρ_G	Persistent parameter for government expenditure shock	Beta	0.500	0.250
ρ_L	Persistent parameter for labor supply shock	Beta	0.500	0.250
Standard Errors for Structural Shocks				
Parameter	Description	Density	Prior Mean	Prior SE
e_A	SE of TFP shock	Inv. Gamma	0.707	4.000
e_C	SE of preference shock	Inv. Gamma	0.707	4.000
e_E	SE of entrepreneur net worth shock	Inv. Gamma	0.707	4.000
e_F	SE of banking sector net worth shock	Inv. Gamma	0.707	4.000
e_G	SE of government expenditure shock	Inv. Gamma	0.707	4.000
e_K	SE of investment specific technology shock	Inv. Gamma	1.000	4.000
e_L	SE of labor supply shock	Inv. Gamma	0.707	4.000
e_R	SE or monetary policy shock	Inv. Gamma	0.224	4.000

Table 4: Estimates of Key Structural Parameters

Parameters	Case A	Case B	Case C	Case D
	Parameters for Financial Friction in Corporate Section			
κ	0.614 [0.547, 0.689]	0.877 [0.818, 0.938]	0.627 [0.526, 0.762]	0.562 [0.470, 0.661]
φ	0.027 [0.024, 0.030]	0.025 [0.023, 0.026]	0.042 [0.036, 0.048]	0.041 [0.036, 0.046]
	Parameters for Nominal Rigidities			
θ_P	0.854 [0.811, 0.895]	0.374 [0.305, 0.440]	0.778 [0.701, 0.859]	0.760 [0.697, 0.822]
θ_W	0.589 [0.531, 0.649]	0.428 [0.351, 0.500]	0.525 [0.450, 0.598]	0.516 [0.452, 0.580]
	Parameters for Monetary Policy Rule			
ρ_R	0.670 [0.581, 0.758]	0.643 [0.582, 0.707]	0.613 [0.528, 0.690]	0.632 [0.590, 0.675]
μ_π	2.805 [2.767, 2.842]	2.820 [2.790, 2.848]	2.984 [2.974, 2.995]	2.986 [2.977, 2.995]
μ_Y	0.006 [0.000, 0.014]	0.010 [0.000, 0.020]	0.009 [0.000, 0.020]	0.008 [0.000, 0.018]

Note: The parenthesis in the table indicates 90% credible interval of structural parameters. 300,000 iterations are implemented using algorithm of MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions.

Table 5: Timings of Peaks of the Financial Shocks

	Structural Shock			
	Case A	Case B	Case C	Case D
Corp. Net Worth	2009Q1	2009Q1	2009Q2	2009Q2
Bank Net Worth	2008Q3	2008Q3	2008Q3	2008Q3
	Stochastic Volatilities			
	Case A	Case B	Case C	Case D
Corp. Net Worth	-	-	2009Q2	2009Q2
Bank Net Worth	-	-	2009Q3	2009Q3

Table 6: Average Ranges of 90% Credible Interval of Structural Shocks over the entire sample peiods

Structural Shocks	Case A	Case B	Case C	Case D
TFP	0.635	0.353	0.462	0.539
Preference	1.593	1.633	0.897	0.824
Corp. Net Worth	0.141	0.148	0.226	0.216
Bank Net Worth	1.902	1.433	0.805	0.907
Government Expenditure	2.207	2.018	0.201	0.322
Investment	0.983	0.236	1.133	1.107
Labor Supply	2.516	3.133	1.686	1.430
Monetary Policy	0.121	0.178	0.125	0.126

Note: This table reports the average of the difference between the upper and the lower bounds of 90% credible interval of the structural shock over the entire sample periods (1985Q2-2012Q2), depicted in Figures 1 and 2.

Table 7: Average Ranges of 90% Credible Interval of Stochastic Volatilities in the entire sample peiods

Structural Shocks	Case C	Case D
TFP	0.385	0.384
Preference	0.994	0.857
Corp. Net Worth	0.222	0.219
Bank Net Worth	0.837	0.908
Government Expenditure	0.202	0.769
Investment	0.621	0.592
Labor Supply	1.403	1.378
Monetary Policy	0.086	0.095

Note: This table reports the average value in the entire sample periods (1985Q2-2012Q2) of the difference between the upper bound and the lower bound of 90% credible interval on the stochastic volatiliy for the structural shock depicted in Figure 3.

Table 8: Leverage Effect of Structural Shocks: Correlation between the Sign of Shock and its Volatility

Structural Shocks	Case C	Case D
TFP	0	0
Preference	+	+
Corp. Net Worth	0	0
Bank Net Worth	0	0
Government Expenditure	0	0
Investment	0	0
Labor Supply	0	0
Monetary Policy	+	+

Note: The mark "-" indicates negative of ρ_σ (leverage effect) at 90% credible degree of posterior probability, while the mark "+" does positive of ρ_σ (opposite leverage effect) in similar way. The mark "0" implies that we do not judge the sign of ρ_σ and leverage effect of each shock because zero is within 90% credible interval of ρ_σ.

Table 9: Posterior Mean: Case A and Case B

	Case A				Case B			
	Key Strucutral Parameters							
Parameter	Mean	SD	90% CI		Mean	SD	90% CI	
κ	0.614	0.043	[0.547	0.689]	0.877	0.038	[0.818	0.938]
h	0.464	0.045	[0.396	0.537]	0.597	0.040	[0.535	0.661]
σ_C	1.628	0.036	[1.578	1.688]	1.404	0.032	[1.356	1.451]
σ_L	0.939	0.071	[0.819	1.052]	0.417	0.072	[0.323	0.524]
φ	0.027	0.002	[0.024	0.030]	0.025	0.001	[0.023	0.026]
ι_P	0.521	0.027	[0.478	0.566]	0.358	0.017	[0.330	0.386]
ι_W	0.422	0.009	[0.408	0.437]	0.450	0.007	[0.440	0.459]
θ_P	0.854	0.026	[0.811	0.895]	0.374	0.041	[0.305	0.440]
θ_W	0.589	0.037	[0.531	0.649]	0.428	0.048	[0.351	0.500]
ρ_R	0.670	0.055	[0.581	0.758]	0.643	0.038	[0.582	0.707]
μ_π	2.805	0.025	[2.767	2.842]	2.820	0.018	[2.790	2.848]
μ_Y	0.006	0.005	[0.000	0.014]	0.010	0.007	[0.000	0.020]
	Persisitence Parameters for Strucutral Shocks							
Parameter	Mean	SD	90% CI		Mean	SD	90% CI	
ρ_A	0.975	0.007	[0.964	0.986]	0.975	0.005	[0.966	0.983]
ρ_C	0.636	0.093	[0.504	0.788]	0.088	0.053	[0.004	0.166]
ρ_K	0.391	0.044	[0.323	0.462]	0.998	0.001	[0.996	0.999]
ρ_E	0.907	0.022	[0.873	0.944]	0.976	0.012	[0.959	0.996]
ρ_F	0.031	0.024	[0.000	0.064]	0.016	0.011	[0.000	0.031]
ρ_G	0.798	0.047	[0.733	0.864]	0.671	0.012	[0.652	0.686]
ρ_L	0.933	0.041	[0.876	0.995]	0.967	0.009	[0.953	0.982]
	Standard Errors for Structural Shocks							
Parameter	Mean	SD	90% CI		Mean	SD	90% CI	
e_A	0.564	0.043	[0.492	0.629]	0.398	0.030	[0.347	0.447]
e_C	1.475	0.161	[1.242	1.716]	1.729	0.189	[1.441	1.986]
e_K	0.238	0.016	[0.212	0.265]	0.286	0.020	[0.254	0.318]
e_E	0.787	0.072	[0.689	0.918]	1.423	0.042	[1.358	1.491]
e_F	0.757	0.057	[0.690	0.843]	0.890	0.058	[0.811	0.979]
e_G	0.520	0.050	[0.439	0.603]	0.895	0.119	[0.751	1.102]
e_L	0.881	0.110	[0.722	1.060]	1.383	0.040	[1.325	1.448]
e_R	0.228	0.016	[0.201	0.255]	0.245	0.019	[0.215	0.274]

Note: 300,000 iterations are implemented using MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions. Items *SD* and *90% CI* denote the standard deviations and 90% credible intervals of the posterior distributions of the structural parameters, respectively.

Table 10: Posterior Mean: Case C and Case D

	Case C				Case D			
	Key Strucutral Parameters							
Parameter	Mean	SD	90% CI		Mean	SD	90% CI	
κ	0.627	0.072	[0.526	0.762]	0.562	0.058	[0.470	0.661]
h	0.210	0.037	[0.149	0.271]	0.221	0.038	[0.161	0.282]
σ_C	1.588	0.012	[1.568	1.608]	1.605	0.017	[1.574	1.627]
σ_L	0.577	0.019	[0.548	0.610]	0.597	0.017	[0.569	0.626]
φ	0.042	0.003	[0.036	0.048]	0.041	0.003	[0.036	0.046]
ι_P	0.513	0.010	[0.496	0.529]	0.503	0.009	[0.490	0.520]
ι_W	0.494	0.001	[0.492	0.496]	0.489	0.001	[0.487	0.491]
θ_P	0.778	0.050	[0.701	0.859]	0.760	0.038	[0.697	0.822]
θ_W	0.525	0.045	[0.450	0.598]	0.516	0.039	[0.452	0.580]
ρ_R	0.613	0.049	[0.528	0.690]	0.632	0.026	[0.590	0.675]
μ_π	2.984	0.007	[2.974	2.995]	2.986	0.006	[2.977	2.995]
μ_Y	0.009	0.008	[0.000	0.020]	0.008	0.007	[0.000	0.018]
	Persisitence Parameters for Strucutral Shocks							
Parameter	Mean	SD	90% CI		Mean	SD	90% CI	
ρ_A	0.956	0.017	[0.930	0.982]	0.956	0.014	[0.933	0.979]
ρ_C	0.908	0.028	[0.862	0.953]	0.909	0.025	[0.868	0.952]
ρ_K	0.823	0.061	[0.726	0.922]	0.776	0.056	[0.682	0.864]
ρ_E	0.854	0.136	[0.606	0.983]	0.918	0.036	[0.867	0.971]
ρ_F	0.156	0.025	[0.126	0.211]	0.167	0.012	[0.151	0.191]
ρ_G	0.661	0.016	[0.630	0.683]	0.619	0.005	[0.612	0.627]
ρ_L	0.974	0.014	[0.953	0.997]	0.982	0.012	[0.965	0.998]

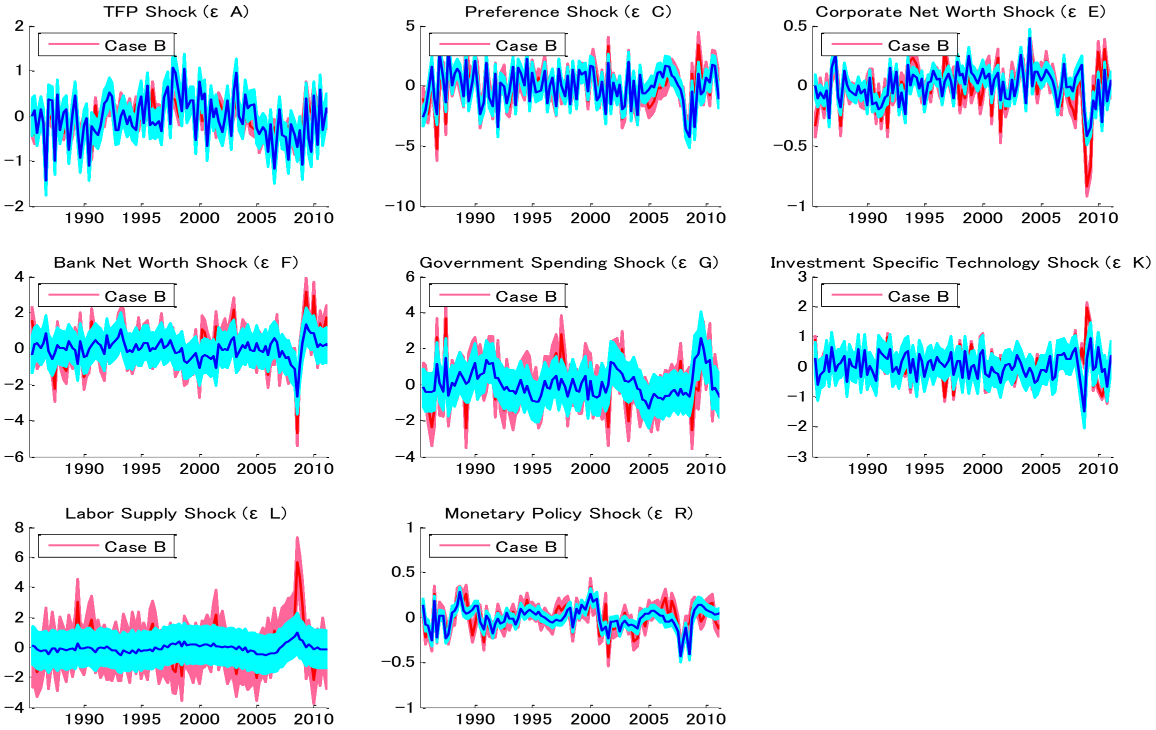
Note: 300,000 iterations are implemented using MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions. Items *SD* and *90% CI* denote the standard deviations and 90% credible intervals of the posterior distributions of the structural parameters, respectively.

Table 11: Posterior Mean of Parameters of SVs: Case C and Case D

	Case C				Case D			
Parameter	Mean	SD	90% CI		Mean	SD	90% CI	
	Parameters of Stochasitc Volatilities for TFP Shock							
σ_A	0.345	0.108	[0.186	0.500]	0.338	0.120	[0.158	0.500]
$\rho_{\sigma A}$	0.227	0.490	[-0.507	0.990]	0.347	0.390	[-0.186	0.989]
ϕ_A	0.540	0.192	[0.240	0.865]	0.509	0.184	[0.213	0.810]
μ_A	0.518	0.031	[0.465	0.567]	0.429	0.049	[0.349	0.501]
	Parameters of Stochasitc Volatilities for Preference Shock							
σ_C	0.488	0.012	[0.472	0.500]	0.476	0.023	[0.447	0.500]
$\rho_{\sigma C}$	0.426	0.168	[0.160	0.714]	0.481	0.141	[0.226	0.696]
ϕ_C	0.927	0.077	[0.842	0.990]	0.958	0.037	[0.919	0.990]
μ_C	0.623	0.038	[0.573	0.692]	0.933	0.055	[0.844	1.026]
	Parameters of Stochasitc Volatilities for Corporate Net Worh Shock							
σ_E	0.477	0.026	[0.452	0.500]	0.412	0.073	[0.303	0.500]
$\rho_{\sigma E}$	0.349	0.220	[-0.019	0.724]	0.280	0.329	[-0.217	0.869]
ϕ_E	0.741	0.147	[0.538	0.990]	0.758	0.186	[0.493	0.990]
μ_E	0.166	0.011	[0.151	0.185]	0.194	0.013	[0.173	0.212]
	Parameters of Stochasitc Volatilities for Bank Net Worth Shock							
σ_F	0.230	0.052	[0.147	0.315]	0.445	0.041	[0.395	0.500]
$\rho_{\sigma F}$	0.130	0.231	[-0.264	0.483]	0.218	0.199	[-0.132	0.498]
ϕ_F	0.933	0.059	[0.872	0.990]	0.894	0.066	[0.804	0.990]
μ_F	0.862	0.049	[0.789	0.942]	0.893	0.050	[0.783	0.959]
	Parameters of Stochasitc Volatilities for Government Expenditure Shock							
σ_G	0.200	0.055	[0.112	0.286]	0.440	0.048	[0.373	0.500]
$\rho_{\sigma G}$	0.358	0.388	[-0.294	0.896]	0.044	0.367	[-0.536	0.670]
ϕ_G	0.512	0.250	[0.110	0.904]	0.517	0.246	[0.071	0.891]
μ_G	0.497	0.049	[0.401	0.557]	0.570	0.031	[0.519	0.627]
	Parameters of Stochasitc Volatilities for Investment Specific Technology Shock							
σ_K	0.441	0.049	[0.371	0.500]	0.452	0.063	[0.335	0.500]
$\rho_{\sigma K}$	0.020	0.261	[-0.361	0.493]	0.128	0.246	[-0.215	0.540]
ϕ_K	0.181	0.199	[0.000	0.453]	0.219	0.214	[0.000	0.548]
μ_K	0.343	0.032	[0.298	0.393]	0.406	0.049	[0.324	0.476]
	Parameters of Stochasitc Volatilities for Labor Supply Shock							
σ_L	0.462	0.032	[0.412	0.500]	0.482	0.016	[0.458	0.500]
$\rho_{\sigma L}$	0.238	0.190	[-0.066	0.554]	0.232	0.178	[-0.071	0.517]
ϕ_L	0.685	0.265	[0.242	0.990]	0.903	0.084	[0.813	0.990]
μ_L	0.962	0.079	[0.821	1.085]	1.461	0.078	[1.351	1.580]
	Parameters of Stochasitc Volatilities for Monetary Policy Shock							
σ_R	0.456	0.038	[0.408	0.500]	0.464	0.034	[0.407	0.500]
$\rho_{\sigma R}$	0.476	0.214	[0.114	0.825]	0.479	0.211	[0.122	0.797]
ϕ_R	0.703	0.128	[0.496	0.929]	0.727	0.122	[0.540	0.941]
μ_R	0.101	0.006	[0.092	0.112]	0.112	0.013	[0.092	0.131]

Note: 300,000 iterations are implemented using MH within Gibbs described in Section 4. We sample one draw out of every 10 replicates and discard first 10,000 samples. The remaining 20,000 samples are used for calculating moments of the posterior distributions. Items *SD* and *90% CI* denote the standard deviations and 90% credible intervals of the posterior distributions of the structural parameters, respectively.

(a) Case A



(b) Case B

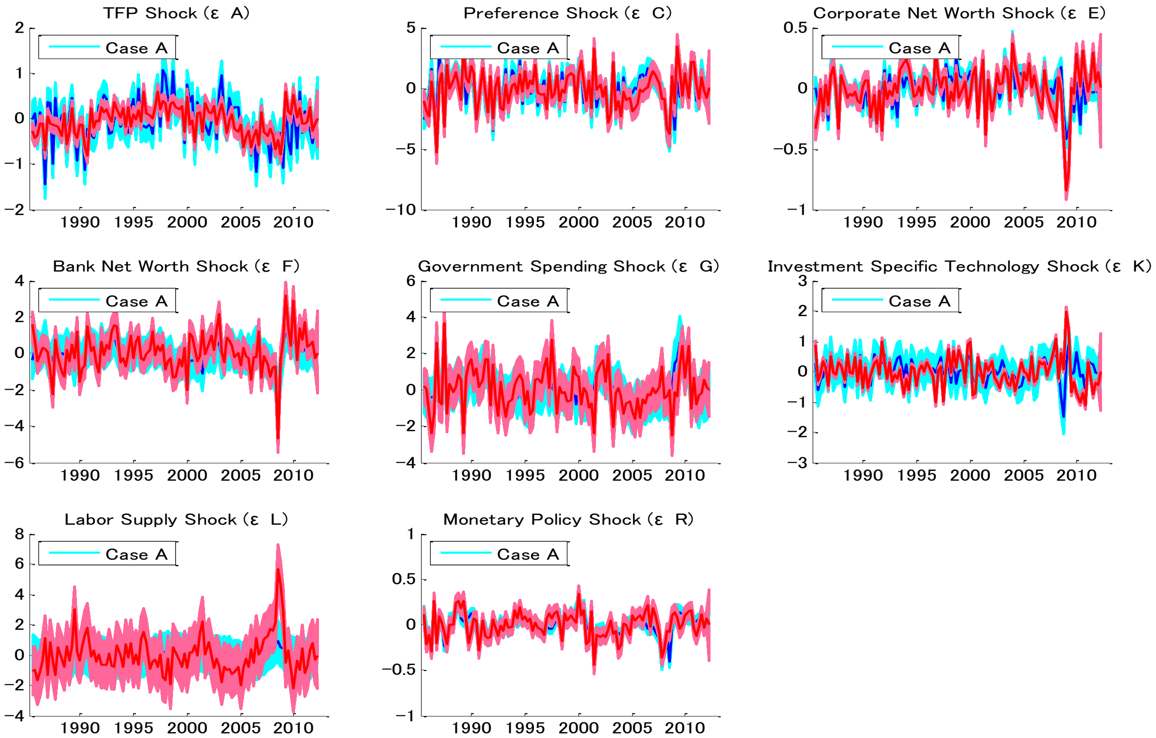
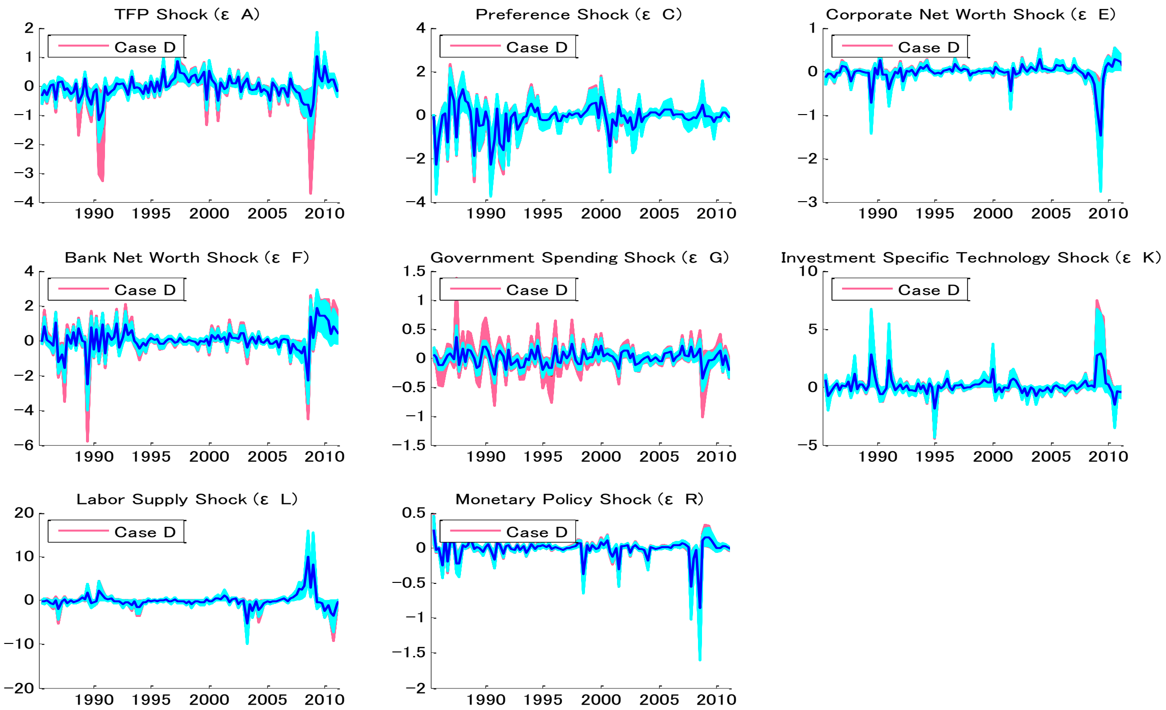


Figure 1: Structural Shocks with i.i.d. Normal in Cases A and B

Note: Deep blue line and blue shade are posterior mean and 90% credible interval of structural shocks in Case A, respectively. And deep red line and red shade are posterior mean and 90% credible interval of structural shocks in Case B.

(a) Case C



(b) Case D

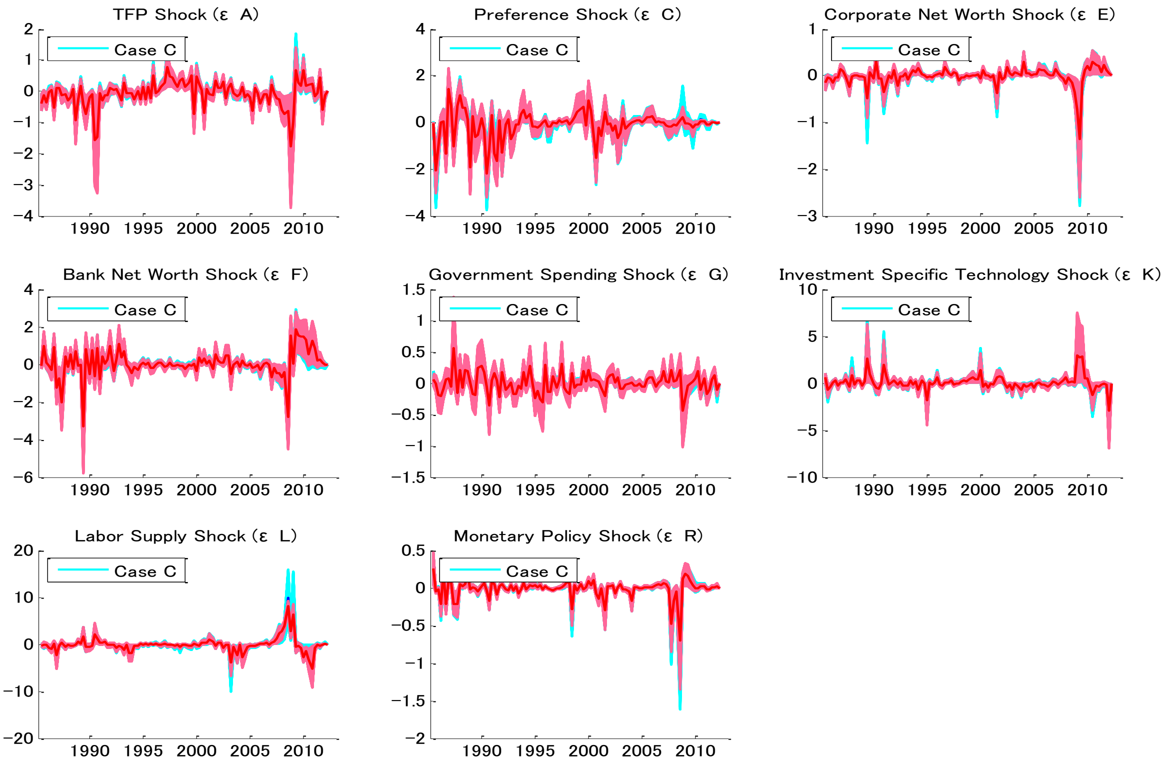
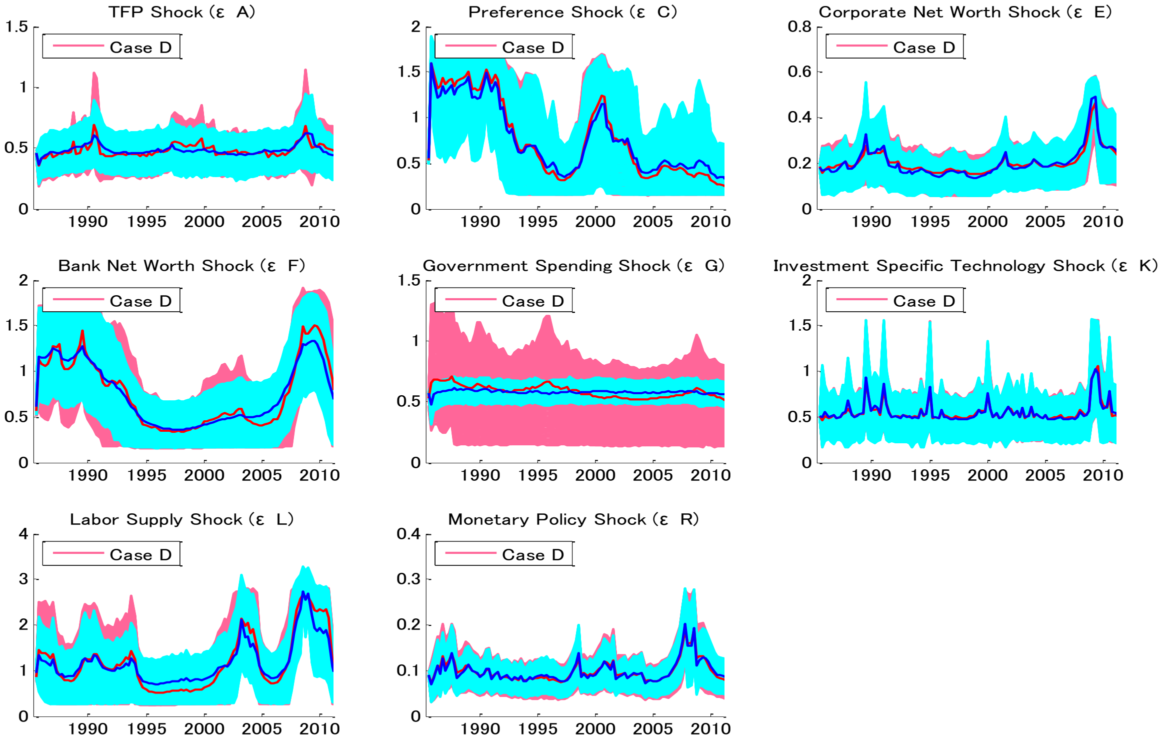


Figure 2: Structural Shocks with SV in Cases C and D

Note: Deep blue line and blue shade are posterior mean and 90% credible interval of structural shocks in Case C, respectively. And deep red line and red shade are posterior mean and 90% credible interval of structural shocks in Case D.

(a) Case C



(b) Case D

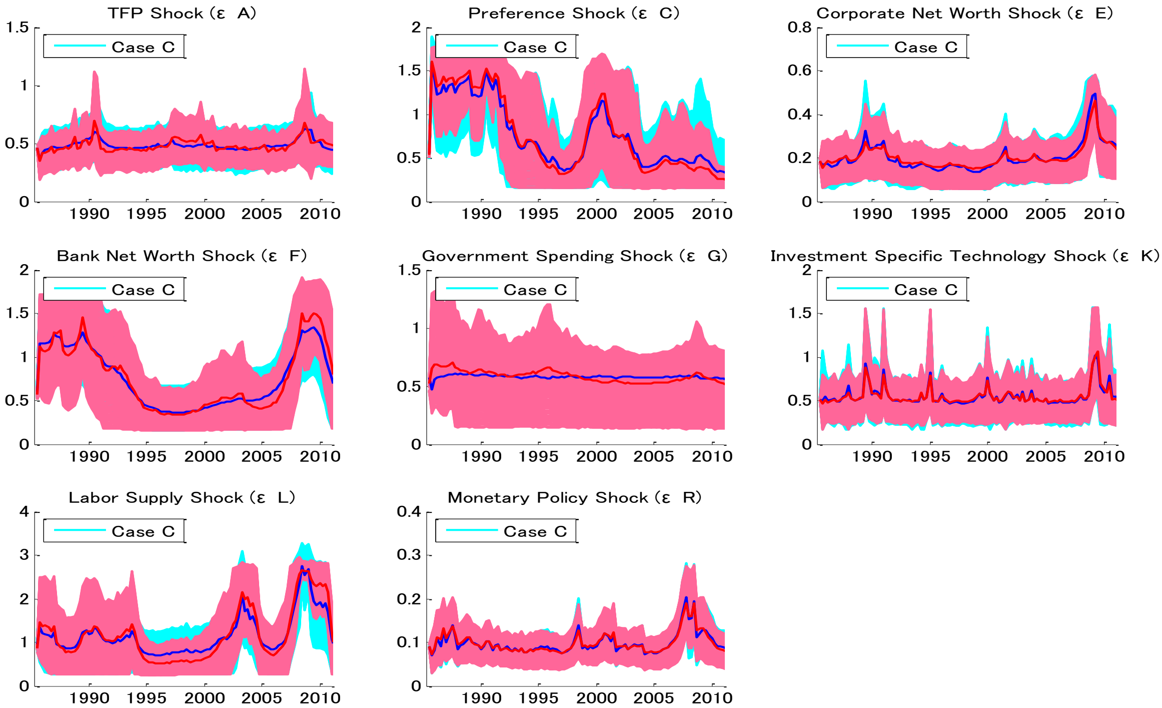


Figure 3: Stochastic Volatilities of Structural Shocks in Cases C and D

Note: Deep blue line and blue shade are posterior mean and 90% credible interval of structural shocks in Case C, respectively. And deep red line and red shade are posterior mean and 90% credible interval of structural shocks in Case D.

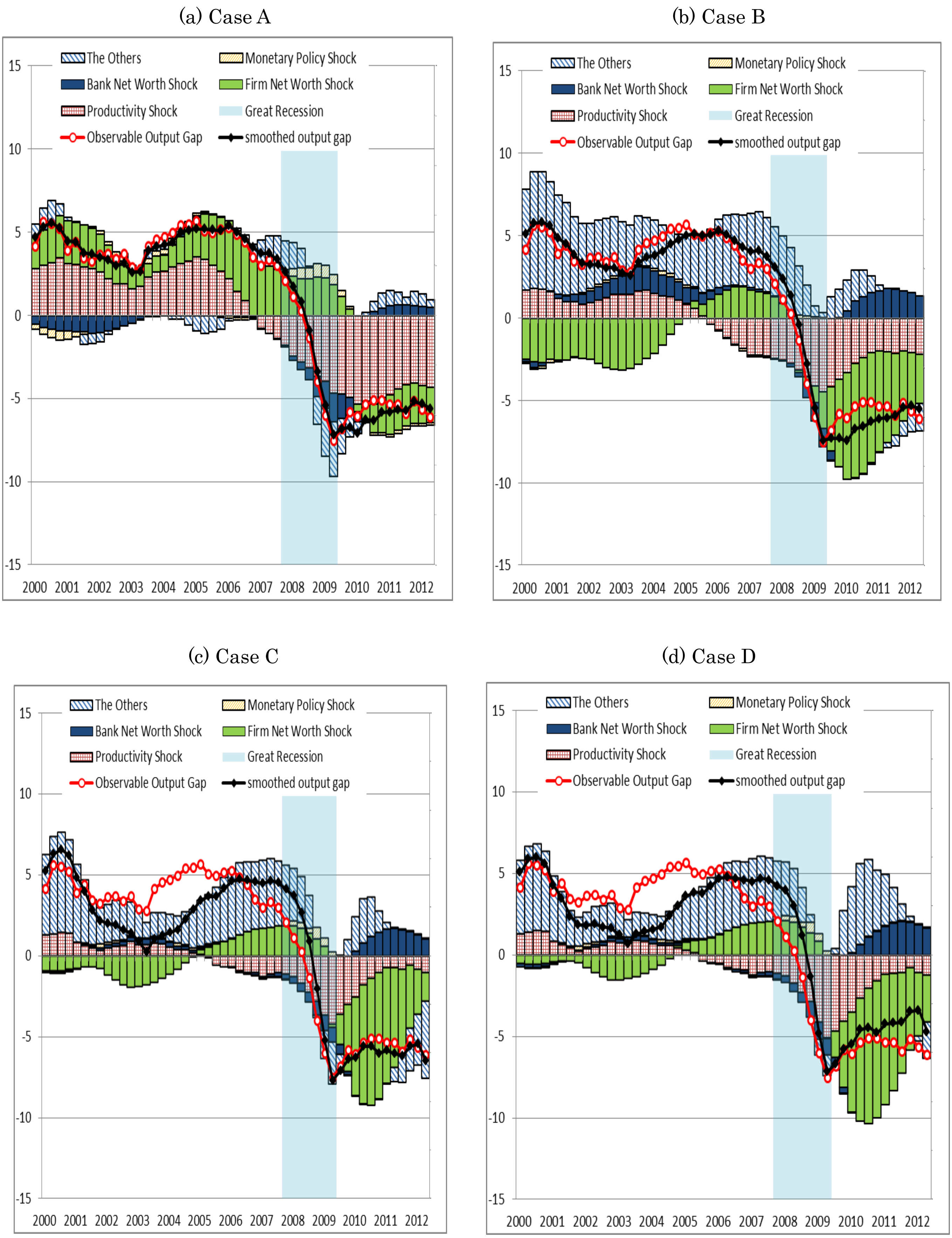


Figure 4: Historical Decomposition of Real GDP

Note: Case A; 11 observable variables and structural shocks with i.i.d. Case B: 41 observable variables and structural shocks with i.i.d. Case C: 11 observable variables and structural shocks with SV. Case D: 40 observable variables and structural shocks with SV.

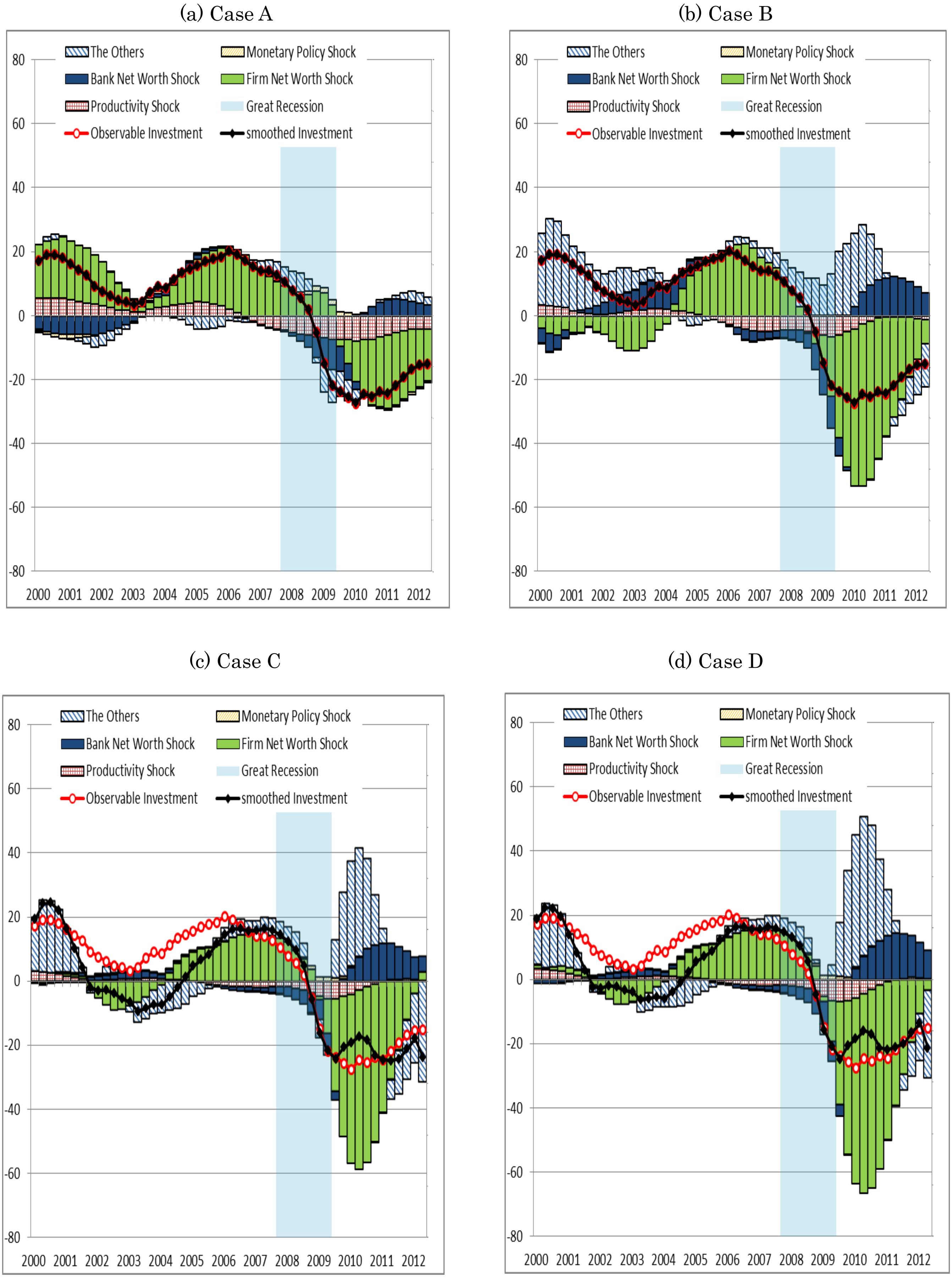


Figure 5: Historical Decomposition of Gross Private Domestic Investment

Note: Case A; 11 observable variables and structural shocks with i.i.d. Case B: 41 observable variables and structural shocks with i.i.d. Case C: 11 observable variables and structural shocks with SV. Case D: 40 observable variables and structural shocks with SV.

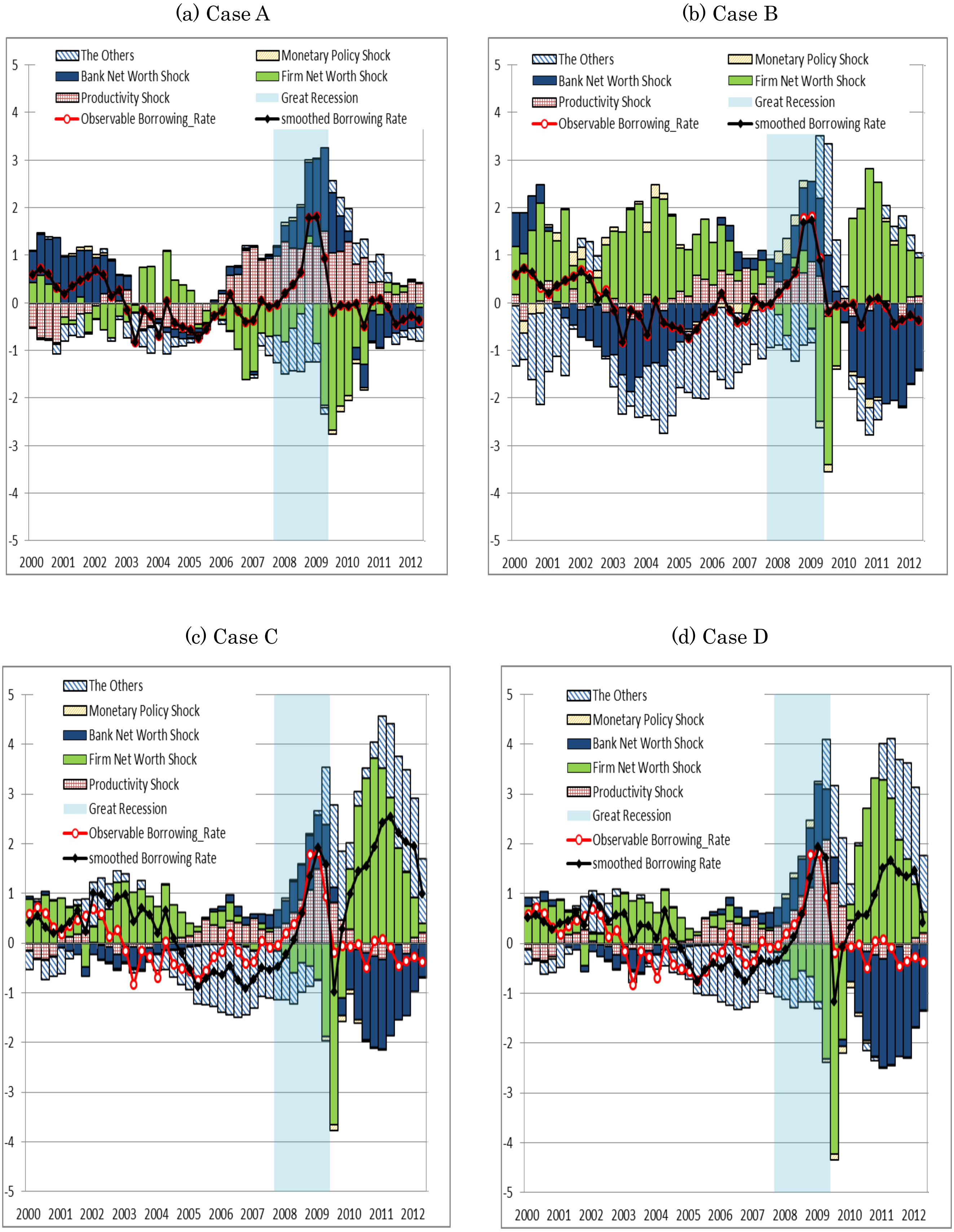
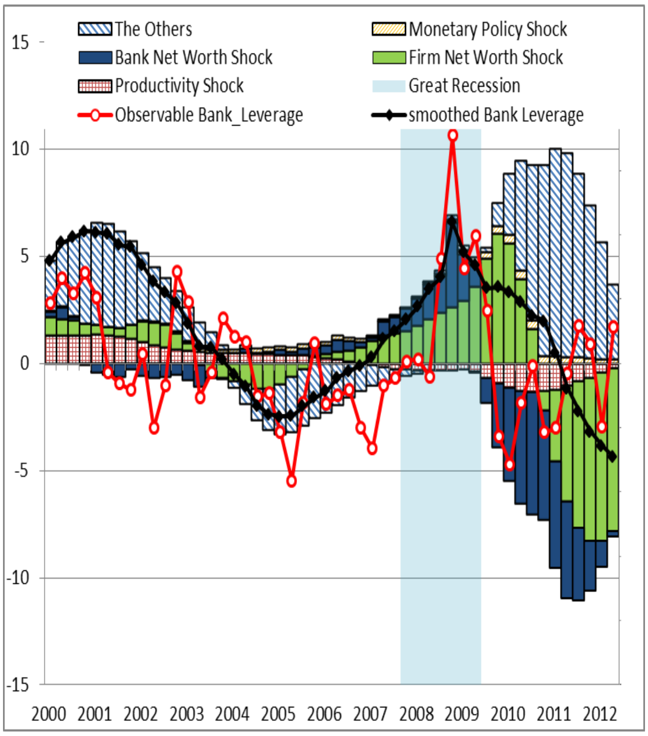


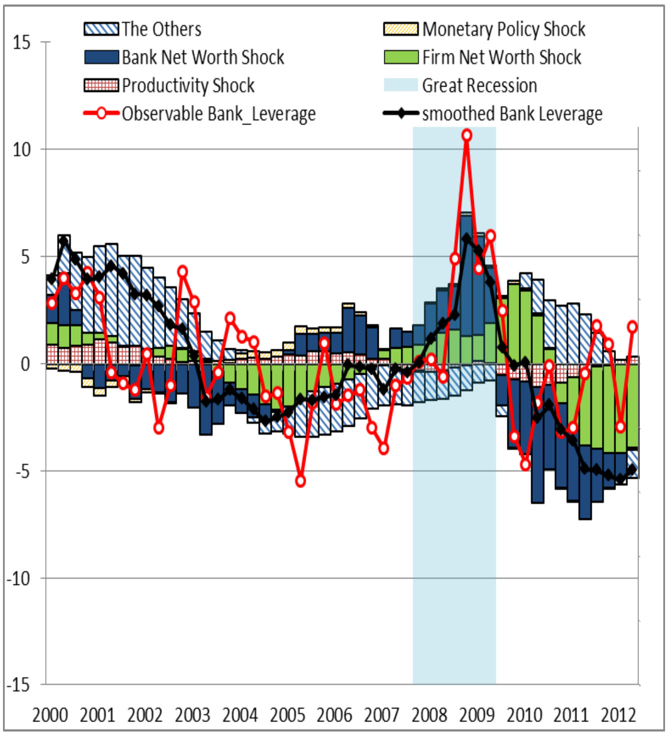
Figure 6: Historical Decomposition of Moody's Bond Index (Corporate Baa)

Note: Case A; 11 observable variables and structural shocks with i.i.d. Case B: 41 observable variables and structural shocks with i.i.d. Case C: 11 observable variables and structural shocks with SV. Case D: 40 observable variables and structural shocks with SV.

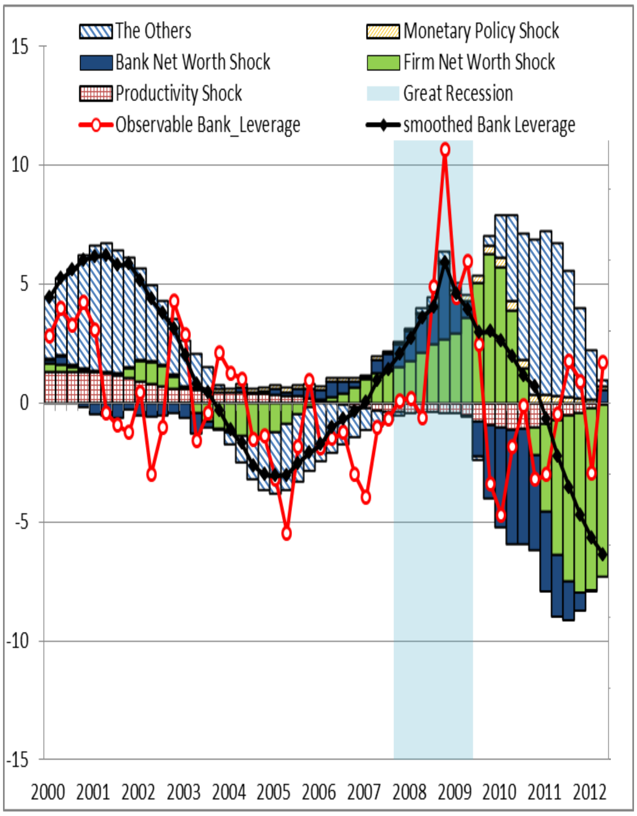
(a) Case A



(b) Case B



(c) Case C



(d) Case D

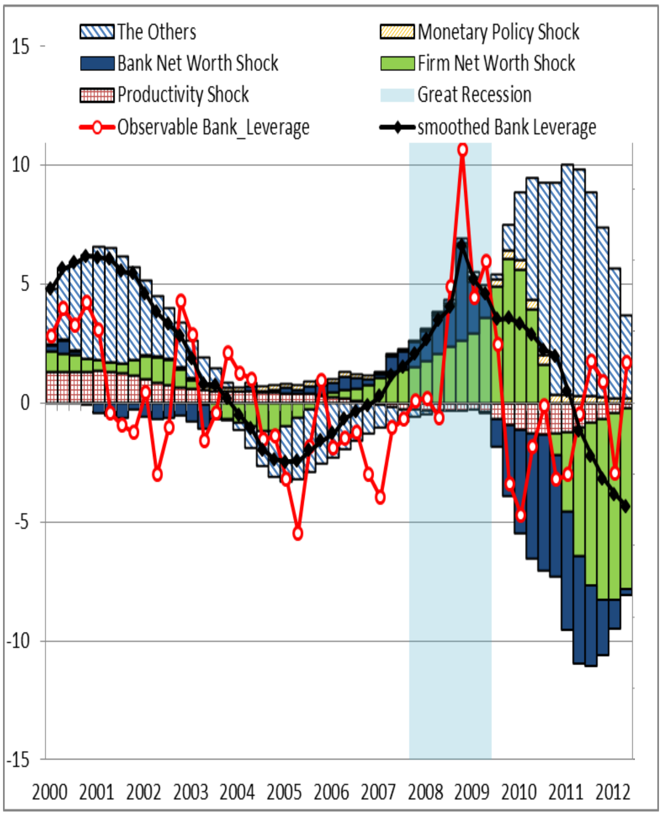


Figure 7: Historical Decomposition of Commercial Bank Leverage Ratio

Note: Case A; 11 observable variables and structural shocks with i.i.d. Case B: 41 observable variables and structural shocks with i.i.d. Case C: 11 observable variables and structural shocks with SV. Case D: 40 observable variables and structural shocks with SV.