Structural Econometrics in Industrial Organization

Dynamics

Marc Rysman

Boston University

February 16, 2009

Many important economic decisions are dynamic

- Consumers:
  - Buy today or wait (learning)
  - Experience goods (experimentation)
  - stockpiling from sales
- Firms:
  - Entry and exit
  - Investment
  - Product introductions
- Note that many of these involve discrete decisions.

Traditional continuous choice with a single agent

- Consumers chooses consumption and savings:

$$\max_{c_t} E\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} U(c_t) \middle| \Omega_t\right] \texttt{s.t.} c_t \leq r_t q_t + y_t; \quad q_{t+1} = r_t q_t + y_t - c_t$$

  - Where q_t is savings, r_t is savings payoff, y_t is income, Ω_t is information.
- The Bellman equation is:

$$V(\Omega_t) = \max_{c_t} U(c_t) + \beta E\left[V(\Omega_{t+1})|\Omega_t\right]$$

- Use derivative based arguments to get rid of V and derive the Euler equation:

$$U'(c_t) = \beta E[U'(c_{t+1})|\Omega_t]$$

Dynamic Discrete Choice

Rust (1987, Econometrica)

- Consider Harold Zurcher, chief mechanic for the public bus system in Madison, WI
- Zurcher decides when to overhaul engines.
- After overhaul, engines are like new.
- This is an “optimal stopping problem”.

Zurcher's problem

- State variable: Number of miles on the bus, s (follows Markov process)
- Infinite, discrete time. Discount rate: β.
- Choice variable: Whether or not to overhaul, $i \in \{0, 1\}$.
- Cost function $C(s, i, \theta) + \epsilon_i$:
  - Overhaul: $C(s, 1, \theta) = \theta_0$
  - No overhaul: $C(s, 0, \theta) = theta_1 + theta_2 s$
- Stationary problem. The Bellman equation is:

$$V(s, \epsilon) = \max_{i \in \{0,1\}} C(s, i, \theta) + \epsilon_i + \beta E\left[V(s', \epsilon')|s, i, \epsilon\right]$$

Simplification

- We want to solve for the value function but ϵ makes the problem very hard.

Assumption of Conditional Independence

$p(s', \epsilon' | s, \epsilon, i) = p^1(\epsilon' | s') p^2(s' | s, i)$

- ϵ has no dynamic content. s and i are sufficient to predict future states.
- In practice, we assume ϵ is *iid*. $\epsilon \sim EV$.
- Study:

$$EV(s) = \int_{\epsilon} EV(s, \epsilon)$$

Empirical approach

- Bellman equation:

$$EV(s) = \ln\left(\sum_i \exp\left(C(s,i,\gamma) + \beta E\left[EV(s')|s,i\right]\right)\right)$$

- Probabilities of i follows logit form:

$$P(i|s) = \frac{\exp\left(C(s,i,\theta) + \beta E\left[EV(s')|s,i\right]\right)}{\sum_{k\in\{0,1\}} \exp\left(C(s,k,\theta) + \beta E\left[EV(s')|s,k\right]\right)}$$

- If we knew EV, we could estimate θ. If we knew θ, we could solve for EV.

Algorithm

The Nested Fixed Point Algorithm

1. Discretize EV. Estimate discrete transition matrix $p(s'|s,i)$.
2. Pick γ.
3. Pick values for EV.
4. Solve

$$EV' = \ln\left(\sum_i \exp\left(C(s,i,\gamma) + \beta E\left[EV(s')|s,i\right]\right)\right)$$

5. If $d(EV', EV) > \texttt{cutoff}$, go to 3.
6. Construct likelihood:

$$L(\gamma) = \sum_{j=1}^{J} \frac{\exp\left(C(s_j,i_j,\gamma) + \beta E\left[EV(s')|s_j,i_j\right]\right)}{\sum_{k\in\{0,1\}} \exp\left(C(s_j,k,\gamma) + \beta E\left[EV(s')|s_j,k\right]\right)}$$

7. Pick γ to raise $L(\gamma)$. Go to 2.

Issues

- Exogenous persistent heterogeneity.
  - Use EM algorithm,
    - Heckman and Singer (Econometrica, 1984), Cameron and Trivedi (Textbook, Sec. 10.3.7)
- Endogenous heterogeneity (learning)
  - Can be solved with simulation
    - Ackerberg (IER, 2003), Crawford and Shum (Econometrica, 2005)
- Large state spaces
  - Randomization
    - Rust (Econometrica, 1997), Hotz and Miller (ReStud 1993), Imai, Jain and Ching (2008 SSRN).
  - Conditional choice probabilities
    - Hotz and Miller (ReStud 1993), Aguirregabiria and Mira (Econometrica, 2002)
- Applications to games
  - Bajari, Benkard and Levin (2007, Econometrica), Aguirregabiria and Mira (2007, Econometrica), Pakes, Ostrovsky and Berry (2007, RAND)

Bajari, Benkard and Levin, 2007

Estimating dynamic games introduces two important problems:

1. Enormous state spaces
   - Benkard (ReStud, 2004) takes a month to solve once!
2. Multiple equilibria
   - Multiple solutions to fixed point algorithms means we cannot construct a likelihood function.

BBL addresses both of these problems.

Basic idea

- Estimate in reduced form choices and state transitions as a function of state variables.
- Use simulation to calculate value function at any given state resulting from possible choices.
- Estimate structural parameters in payoff function taking value function as given.

Model

- J firms, discrete, infinite time
- state space $s_t \in \mathbb{R}^L$
- actions $i \in \mathbb{I}$ (ctns or discrete)
- private shock $nu_{jt} \sim G(\cdot|s_t)$
- flow profit $\pi_j(i_{jt}, s_t, \nu_{jt})$.
- Before realizing ν_{jt}:

$$V(s_t) = E\left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \pi_j(i_\tau, s_\tau, \nu_{j\tau} | s_t)\right]$$

$$\texttt{where} \quad s_{t+1} \sim P(s_{t+1}|i_t, s_t)$$

Model II

- Markov Perfect Equilibrium
- Denote strategies as $\sigma(s, \nu_j)$
- Bellman equation conditioning on strategies:

$$V_j(s, \sigma) = E\left[\pi_j(\sigma(s, \nu_j), s, \nu_j) + \int V_j(s', \sigma) dP(s' | \sigma(s, \nu_j), s) \middle| s\right]$$

First stage of estimation

- Assume data is generated by a single MPE so we can estimate $\sigma(s, \nu_i)$ from observed data.
- Estimate $\sigma(s, \nu_j)$, $P(s'|i_t, s_t)$.
- Now we can calculate the value of being at any given state s by simulation up to a set of parameters.
- Rather than solve for exact value function from equilibrium conditions, we are using observed choices and state transitions to approximately obtain value function.

Calculate value function

1. Draw ν_{jt} for all firms for T periods into the future
2. Calculate choices in period t from $\sigma(s, \nu_{jt})$.
3. Calculate s_{t+1} from $P(s'|i_t, s_t)$.
4. Go to 2.
5. Repeat *ns* times (go to 1).
6. Calculate:

$$\widehat{V}_j(s_t, \sigma) = \frac{1}{ns} \sum_{k=1}^{ns} \sum_{\tau=t}^{T} \beta^{\tau-t} \pi_j(i_t^k, s_t^k, \nu_{jt}^k, \theta)$$

It will be handy if $\pi_j(i_t, s_t, \nu_{jt}, \theta) = \pi_j(i_t, s_t, \nu_{jt})\theta$

Second stage

Method 1

- For each firm in each period, calculate optimal choice from model.
  - For continuous choices, find:

$$i^* \texttt{s.t.} \frac{d\widehat{V}_j(s_t, \sigma(i^*))}{di_{jt}} = 0.$$

  - For discrete choices, find:

$$i^* \texttt{s.t.} \widehat{V}_j(s_t, \sigma(i^*)) \geq \widehat{V}_j(s_t, \sigma(i)) \forall i$$

  - Let $\xi = i^*(\theta) - i^{\texttt{data}}$.
  - Form moments $m = z'\xi$, objective function $\texttt{obj} = m'wm$.
- May be very computationally costly in practice

Second stage

Method 2

- Determine an alternative strategy $\sigma'(s, \nu_{jt})$.
  - For instance, add random normal terms to parameters in σ.
- Use revealed preference to derive moment inequalities.

$$g_j(s, \sigma, \sigma', \theta) = V_j(s, \sigma_j, \sigma_{-j}, \theta) - V_j(s, \sigma'_j, \sigma_{-j}, \theta)$$

- Form objective function:

$$\texttt{obj} = \sum_{s \in \texttt{data}} \left(\min\left\{g(s, \sigma, \sigma', \theta), 0\right\}\right)^2$$

- Pick θ to minimize this function.

Example: Rust 1987

- Estimate choice probabilities using logit model:

$$\sigma(s_{jt}, \nu_{jt}) = 1 \quad \texttt{if} \quad \alpha_0 + \alpha_1 s_{jt} + \nu_{jt} > 1$$

  - $\nu_j \sim EV$

$$\Rightarrow P(i_{jt}|s_{jt}) = \frac{\exp(\alpha_0 + \alpha_1 s_{jt})}{1 + \exp(\alpha_0 + \alpha_1 s_{jt})}$$

- Estimate state transitions:

$$s_{jt+1} = \gamma_0 + \gamma_1 s_{jt} + u_{jt}$$

- Now compute value of state s_{jt+1} up to parameters θ.

Rust Example: Computing V

- Draw u_{jt},ϵ_{jt} for T periods into the future, for each state observed in the data ns times.
- Simulate future realizations of s_{jt}, i_{jt}. Compute:

$$x_{jt}^1 = \frac{1}{ns} \sum_{k=1}^{ns} \sum_{\tau=t}^{T} \beta^{\tau-t} 1\{i_{jt}^k = 1\}$$

$$x_{jt}^2 = \frac{1}{ns} \sum_{k=1}^{ns} \sum_{\tau=t}^{T} \beta^{\tau-t} 1\{i_{jt}^k = 0\}$$

$$x_{jt}^3 = \frac{1}{ns} \sum_{k=1}^{ns} \sum_{\tau=t}^{T} \beta^{\tau-t} s_{jt}^k$$

- Value function is:

$$V(s_{jt}, \sigma, \theta) = \begin{bmatrix} x_{jt}^1 & x_{jt}^2 & x_{jt}^3 \end{bmatrix} \theta$$

Rust Example: Estimate θ

- Add random numbers to α_0, α_1 to create $\sigma'(s_{jt}, \nu_{jt})$.
- Compute $V(s_{jt}, \sigma', \theta)$
- Value of observed strategy minus value of alternative strategy:

$$g(s_{jt}, \sigma, \sigma', \theta) = \left[x_{jt}^1 - x_{jt}^{1\prime} \quad x_{jt}^2 - x_{jt}^{2\prime} \quad x_{jt}^3 - x_{jt}^{3\prime} \right] \theta$$

- Objective function:

$$\texttt{obj} = \sum_{s_{jt} \in \texttt{data}} \left(\min \left\{ g(s_{jt}, \sigma, \sigma', \theta), 0 \right\} \right)^2$$

- In practice, this also includes a summation over many alternative strategies.

Applications

- Typically, these models are followed by simulations of market equilibrium under alternative policy regimes in the spirit of Ericson and Pakes (ReStud 1995) and Pakes and McGuire (RAND, 1994).
- Ryan (2008): Entry and investment by US cement producing plants
- Stahl (2009): Consolidation of US local television industry after deregulation.
- Snider (2009): Predatory pricing by US airlines.

Outstanding issues

- If there is any persistent unobserved heterogeneity, the first stage results is not right for any individual agent.
  - Analog to conditional independence assumption
- Many papers do not sufficiently concern themselves with obtaining causal parameters in the first stage.
  - Just regressing choice variables on state variables does not mean we have estimated a causal effect.