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Optimal Design of Scoring Auction with Multidimensional Quality

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Abstract

This article studies the optimal design of scoring auction used in public procurement. In this auction, each supplier's offer consists of both price and quality, and a supplier whose offer achieves the highest score wins. The environment we consider has the feature that quality is multi-dimensional. We show that there exists a quasi-linear scoring rule which implements the optimal mechanism for the buyer and is supermodular in quality if the virtual surplus is quasisupermodular in quality. We further investigate how the buyer should classify quality attributes when using a scoring rule which is additively separable in the attributes.

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1 Introduction

Auction rules of public procurement have changed from one-dimensional bidding to multi-dimensional bidding. In contrast with the former traditional rule in which each supplier submits only a price-bid, the latter auction rule requires suppliers to offer not only price but also quality they promise to ensure in a project. For instance, in the EU, Article 53 of Directive 2004/18/EC specifies the "Most Economically Advantageous Tender", in which the procurement authorities award contracts based on various criteria such as price, technical merit, aesthetic characteristics, delivery date, and so on. The design of the multi-dimensional auction is a matter of great concern to the procurement authorities, reflecting the fact that public procurement accounts for about 16% of GDP in OECD member countries (OECD, 2008).

The essential element of the multi-dimensional auction is a scoring rule. The rule, which evaluates suppliers' offers and gives them scores, should be carefully designed because it considerably affects suppliers' decisions what offers to make. There are many examples of scoring rules adopted by state departments of transportation in the US: "A+B bidding" (Arizona, etc.), "weighted criteria" (Delaware, Idaho, Massachusetts, Oregon, Utah, Virginia, etc.), "adjusted bid" (Arizona, Maine, Michigan, North Carolina, South Carolina, South Dakota, etc.), and so on (see Molenaar and Yakowenko, 2007). For instance, the rule of "weighted criteria" puts a weight on each of price and quality attributes (e.g. delivery date, safety level) and evaluates each attribute individually, so that a total score of each offer is a weighted sum of subscores and a supplier with the highest total score wins a contract.

The previous studies have confirmed that the *scoring auction* performs very well from the buyer's point of view. A common feature of the environments is that a buyer procures a single product (e.g. highway) differentiated by its quality from one of suppliers, who have private information about cost parameters. In a seminal article, Che (1993) shows that a scoring auction with a properly designed scoring rule implements the buyer's optimal (i.e. utility-maximizing) mechanism characterized by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). Branco (1997) extends this result to an environment where each supplier's production cost has a common-cost component, so that his cost is correlated with the other suppliers' costs.

In contrast with Che and Branco, who assume that both quality and each supplier's type are one-dimensional, Asker and Cantillon (2008) consider a fully general environment where both elements are multi-dimensional. Although the main results of Asker and Cantillon are the characterization of equilibrium bidding behavior and the expected utility equivalence between some formats of scoring auction, they also show that the scoring auction outperforms some other mechanisms from the buyer's viewpoint. On the other hand, they have not investigated whether a scoring auction can implement the optimal mechanism. The likely reason is that it is extremely difficult to characterize the optimal mechanism when the supplier has multi-dimensional private information. However, Asker and Cantillon (2010) characterize the optimal mechanism in a specific environment where quality is one-dimensional, each supplier's type consists of two parameters (fixed cost and marginal cost) and each parameter is a binary random variable. They show that the scoring auction yields a performance close to that of the optimal mechanism, taking a numerical simulation approach.

All of the above studies focus on "quasi-linear" scoring rules under which a total score of each offer is given by a quality score minus price. A typical example of quasi-linear rule is "weighted criteria". In a recent study, Hanazono et al. (2011) consider "price-quality ratio" scoring rules under which a total score of each offer is given by price divided by a quality score, and analyze the equilibrium bidding behavior.

In addition to these theoretical studies, there is some experimental evidence supporting the high performance of scoring auction compared to that of traditional price-only auction (Bichler, 2000; Chen-Ritzo et al., 2005).

Although many scoring rules used in practice apply multiple quality criteria, there are no theoretical studies investigating whether a scoring auction can implement the optimal mechanism in an environment where quality is multi-dimensional. It is by no means trivial to answer the question and show what form of scoring rule succeeds in the implementation, at least from the Revelation Principle (see, e.g., Myerson, 1981). Moreover, with the model of multi-dimensional quality, it will be anticipated that the interaction among quality attributes significantly affects the optimal form of scoring rule. These observations motivate our current work. We generalize the model of Che (1993) by allowing for multi-dimensional quality. A scoring rule announced by the buyer induces the following auction game. First, all suppliers' (one-dimensional) cost parameters are realized, and each supplier is privately informed about his own parameter respectively. Second, each supplier simultaneously and independently offers both price and quality. A supplier wins if the score of his offer is the highest among suppliers and higher than the predetermined reserve score.[1] The winner's offer becomes a binding contract.

We derive two results. First, we show that there exists a quasi-linear scoring rule which implements the optimal mechanism and is supermodular in quality if the so-called "virtual surplus" is quasisupermodular in quality. These properties of supermodularity and quasisupermodularity, which represent the concept of complementarity between qual-

[1]The introduction of reserve score is also an extension of Che (1993).

ity attributes, are specific to the model of multi-dimensional quality (see Topkis, 1998 for the definitions). In particular, our result implies that the scoring rule should evaluate all quality attributes as a whole (not separately) to give a total score in a complementary way. Second, we derive a necessary condition and a sufficient condition for the existence of a quasi-linear scoring rule which implements the optimal mechanism and is additively separable in some quality attributes. Our results imply that when the buyer establishes some sets of subcriteria to use the additively separable rule, each pair of quality attributes in the distinct sets should be complementary in terms of the production cost.

This article contributes to the literature on scoring auction in two ways. First, we construct the optimal scoring rule in a different manner from Che (1993) and Branco (1997). When quality is one-dimensional, the scoring rule constructed by them provides suppliers with incentives to offer the appropriate quality level. On the other hand, our example shows that when quality is multi-dimensional, a natural extension of their scoring rule may induce unsuitable quality offers from the buyer's viewpoint. To resolve this problem, we construct the optimal scoring rule by applying the method of monotone comparative statics (Topkis, 1998). The constructed rule satisfies supermodularity in quality to deter a supplier's deviation from the desirable quality offer for the buyer. Also, as a by-product of using the monotone comparative statics method, we require no assumptions of concavity (or convexity) and differentiability in quality of the value and cost functions, and allow quality attributes to be discrete. Second, we provide a useful guide to designing scoring rules which are additively separable in some quality attributes. The result has important policy implications because additively separable scoring rules (e.g. "weighted criteria") are widely adopted and it must be easier for procurement authorities to administer those rules.

The article is organized as follows. Section 2 presents the model which generalizes that of Che (1993). Section 3 derives the equilibrium bidding strategy. Section 4 shows how to design a scoring rule to implement the optimal mechanism. Section 5 investigates how the buyer should classify quality attributes when using a scoring rule which is additively separable in the attributes. Section 6 concludes. All proofs are in the Appendix.

2 The model

Consider a buyer who procures a single product from one of N suppliers. A (production) *contract* between the buyer and a supplier $i \in \{1, ..., N\}$ is denoted by $(p_i, \boldsymbol{q_i}) \in \mathbb{R}_+ \times \mathcal{Q}$, under which the supplier i must deliver a product of *quality* $\boldsymbol{q_i} = (q_i^1, ..., q_i^M) \in \mathcal{Q} \subset \mathbb{R}^M$ in exchange for price $p_i \in \mathbb{R}_+$.[2] For each $m \in \{1, ..., M\}$, q_i^m represents a level of quality (non-monetary) attribute. The supplier i's cost parameter is given by $\theta_i \in [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$. The suppliers' types $(\theta_1, ..., \theta_N)$ are random variables which are independent across suppliers. The cumulative distribution function of θ_i is given by F, with a density function f that is strictly positive everywhere. Each supplier has private information about his realized type respectively, but the prior probability distribution is common knowledge.

A supplier i of type θ_i earns profits $p - c(\boldsymbol{q}, \theta_i)$ from a contract $(p, \boldsymbol{q})$, where $c(\boldsymbol{q}, \theta_i) > 0$ is his production cost. The buyer's utility from a contract $(p, \boldsymbol{q})$ with a supplier of type θ_i is a weighted sum of consumers' surplus and profits, i.e. $v(\boldsymbol{q}) - p + \alpha(p - c(\boldsymbol{q}, \theta_i))$, where $v(\boldsymbol{q})$ is the valuation for a product of quality $\boldsymbol{q}$ and $\alpha \in [0, 1]$ is a weight on profits. We make the following assumptions.

Assumption 1. $\mathcal{Q} = \times_{m=1}^{M} \mathcal{Q}^m$, where $\mathcal{Q}^m \subset \mathbb{R}$ is a closed interval or a finite set.

[2] In this article, bold letters denote some vectors: $\boldsymbol{q} \geq \hat{\boldsymbol{q}}$ means $q^m \geq \hat{q}^m$ for each m; $\boldsymbol{q} > \hat{\boldsymbol{q}}$ means $\boldsymbol{q} \geq \hat{\boldsymbol{q}}$ and $\boldsymbol{q} \neq \hat{\boldsymbol{q}}$; $\boldsymbol{q} \gg \hat{\boldsymbol{q}}$ means $q^m > \hat{q}^m$ for each m.

Assumption 2. v is continuous in $\boldsymbol{q}$.

Assumption 3. c is continuous in $(\boldsymbol{q}, \theta)$ and strictly increasing in $\boldsymbol{q}$. c is differentiable and strictly increasing in θ. $c_\theta \equiv \partial c/\partial\theta$ is continuous in $(\boldsymbol{q}, \theta)$ and increasing in θ.

Assumption 4. c has strictly increasing differences in $(\boldsymbol{q}, \theta)$, i.e. $c(\boldsymbol{q'}, \theta) - c(\boldsymbol{q}, \theta)$ is strictly increasing in θ for each $\boldsymbol{q'} > \boldsymbol{q}$ in $\mathcal{Q}$. c_θ has increasing differences in $(\boldsymbol{q}, \theta)$, i.e. $c_\theta(\boldsymbol{q'}, \theta) - c_\theta(\boldsymbol{q}, \theta)$ is increasing in θ for each $\boldsymbol{q'} > \boldsymbol{q}$ in $\mathcal{Q}$.

Assumption 5. $\frac{F}{f}$ is increasing in θ.

We will apply the monotone comparative statics method. See Topkis (1998) for some notions. Assumption 1 ensures that $\mathcal{Q}$ is a compact lattice. The notions of increasing differences express the concept of complementarity between quality attributes and a cost parameter. Assumptions 4 and 5 ensure that the "virtual surplus" defined later has strictly decreasing differences in $(\boldsymbol{q}, \theta)$.

There is an auction rule (mechanism) that is feasible for the buyer: a *scoring auction*. We first define a *scoring rule* as $S : \mathbb{R}_+ \times \mathcal{Q} \to \mathbb{R}$. In a scoring auction, each supplier offers both price and quality, and the scoring rule S assigns a score $S(p, \boldsymbol{q})$ to each offer $(p, \boldsymbol{q})$. With a *reserve score* which is normalized to zero, a supplier i wins only if his score is nonnegative and the highest among suppliers.[3] We consider a *first-score (sealed-bid)* format, in which a winner i is awarded a binding contract $(p_i, \boldsymbol{q_i})$ he offered in the auction; this format corresponds to the first-price format in the standard auction. We focus on a *quasi-linear* scoring rule, which takes a form of $S(p, \boldsymbol{q}) = s(\boldsymbol{q}) - p$. We also call the function $s : \mathcal{Q} \to \mathbb{R}$ a (quasi-linear) scoring rule. We assume that s is upper semicontinuous in $\boldsymbol{q}$, and s has a cost parameter $\bar{\theta}^s \in (\underline{\theta}, \bar{\theta}]$ which satisfies $\max_{\boldsymbol{q} \in \mathcal{Q}}[s(\boldsymbol{q}) - c(\boldsymbol{q}, \bar{\theta}^s)] = 0$.

[3]We assume that if there is a nonnegative tie score, then each supplier achieving the highest score wins with equal probability. All results hold for any other tie-breaking rule.

A scoring rule s publicly announced by the buyer induces the following auction game. First, all suppliers' types $\boldsymbol{\theta} = (\theta_1, ..., \theta_N)$ are realized, and each supplier is privately informed about his own type respectively. Second, each supplier i simultaneously and independently submits an offer $(p_i, \boldsymbol{q_i})$. Then, the game ends. If a supplier i of type θ_i who offers $(p_i, \boldsymbol{q_i})$ such that $s(\boldsymbol{q_i}) - p_i = \max_j[s(\boldsymbol{q_j}) - p_j] \geq 0$ wins, then he receives $p_i - c(\boldsymbol{q_i}, \theta_i)$, the other suppliers receive zero payoffs, and the buyer receives $v(\boldsymbol{q_i}) - p_i + \alpha(p_i - c(\boldsymbol{q_i}, \theta_i))$. If no supplier wins, then the buyer and all suppliers receive zero payoffs.

In the next section, we explore the Bayesian Nash equilibrium of the auction game. With a slight abuse of notation, we denote a supplier i's bidding strategy by $(p_i, \boldsymbol{q_i}) : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}_+ \times \mathcal{Q}$. We assume that no supplier uses weakly dominated strategies.

3 Equilibrium bidding strategy

In this section, we derive the equilibrium bidding strategy. The following lemma characterizes a symmetric equilibrium, where all suppliers use the same bidding strategy. Although our environment is substantially more general than Che (1993) and there are slight technical difficulties, we can apply his technique to prove the lemma.

Lemma 1. *The auction game induced by a quasi-linear scoring rule s has a symmetric equilibrium in which (i) the bidding strategy $(p^*, \boldsymbol{q^*})$ is such that for each $\theta \in [\underline{\theta}, \bar{\theta}^s]$,*

$$\boldsymbol{q^*}(\theta) \in \arg\max_{\boldsymbol{q} \in \mathcal{Q}} \, [s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)] \tag{1}$$

$$p^*(\theta) = c(\boldsymbol{q^*}(\theta), \theta) + \int_{\theta}^{\bar{\theta}^s} c_\theta(\boldsymbol{q^*}(z), z) \left(\frac{1 - F(z)}{1 - F(\theta)} \right)^{N-1} dz, \tag{2}$$

and for each $\theta \in (\bar{\theta}^s, \bar{\theta}]$, $(p^*(\theta), \boldsymbol{q^*}(\theta))$ is an arbitrary offer which satisfies $s(\boldsymbol{q^*}(\theta)) - p^*(\theta) < 0$, and (ii) a supplier of type θ_i wins only if $\theta_i = \min\{\theta_1, ..., \theta_N, \bar{\theta}^s\}$. Moreover, for each $\theta \in [\underline{\theta}, \bar{\theta}^s]$, any offer $(p', \boldsymbol{q'})$ such that $\boldsymbol{q'} \notin \arg\max_{\boldsymbol{q} \in \mathcal{Q}} \, [s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ is weakly *dominated by* $(p, \boldsymbol{q}^*(\theta))$ *such that* $s(\boldsymbol{q}^*(\theta)) - p = s(\boldsymbol{q}') - p'$.

It is important to note that in equilibrium any supplier who wins with positive probability chooses quality so as to maximize a quality score minus his production cost, as in (1). Actually, any offer which does not maximize $s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)$ is weakly dominated by the quality offer $\boldsymbol{q}^*(\theta)$ with some price offer. Given the optimal quality offer $\boldsymbol{q}^*(\theta)$, the optimal price offer $p^*(\theta)$ is determined by (2). The price offer is greater than the production cost due to the "shading behavior".

Lemma 1 also implies that in equilibrium the most efficient supplier wins provided that his type is lower than $\bar{\theta}^s$. Let $\theta_{(N)} \equiv \min\{\theta_1, ..., \theta_N\}$ be the lowest cost parameter among suppliers, which is also a random variable. We denote the cumulative distribution function of $\theta_{(N)}$ by $F_{(N)}(\cdot) = 1 - (1 - F(\cdot))^N$. Then, the buyer's expected utility from announcing a quasi-linear scoring rule s is

$$F_{(N)}(\bar{\theta}^s) E\left[v(\boldsymbol{q}^*(\theta_{(N)})) - p^*(\theta_{(N)}) + \alpha[p^*(\theta_{(N)}) - c(\boldsymbol{q}^*(\theta_{(N)}), \theta_{(N)})] \mid \theta_{(N)} \leq \bar{\theta}^s\right]$$
$$= \int_{\underline{\theta}}^{\bar{\theta}^s} \left[v(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta) - (1-\alpha) c_\theta(\boldsymbol{q}^*(\theta), \theta) \frac{F(\theta)}{f(\theta)}\right] dF_{(N)}(\theta),$$

where the equality follows from the substitution of $p^*(\theta_{(N)})$ and the interchange of the order of integration. We now define the *virtual surplus* as the function $\Phi \equiv v - c - (1 - \alpha) c_\theta \frac{F}{f}$. Its value $\Phi(\boldsymbol{q}, \theta) = v(\boldsymbol{q}) - c(\boldsymbol{q}, \theta) - (1-\alpha) c_\theta(\boldsymbol{q}, \theta) \frac{F(\theta)}{f(\theta)}$ times the density $f(\theta)$ is the social surplus generated by trading a product of quality $\boldsymbol{q}$ between the buyer and a supplier of type θ, minus the sum of information rents paid to the more efficient supplier than θ. Using this virtual surplus, the buyer's expected utility can be rewritten as

$$\int_{\underline{\theta}}^{\bar{\theta}^s} \Phi(\boldsymbol{q}^*(\theta), \theta) dF_{(N)}(\theta).$$

4 Optimal scoring rule

In this section, we study the implementation problem. The analysis proceeds in two steps. First, we characterize an optimal mechanism, following the standard mechanism-design approach with the envelope theorem of Milgrom and Segal (2002). Second, we find the condition under which a scoring auction implements the optimal mechanism, and investigate how the scoring rule should be designed.

In a first step, we use the Revelation Principle to focus on "incentive compatible direct mechanisms". A *direct mechanism* is an $3N$-tuple of measurable functions $(P_i, \boldsymbol{Q_i}, X_i)_{i\in\{1,...,N\}}$ where $(P_i, \boldsymbol{Q_i}, X_i) : [\underline{\theta}, \bar{\theta}]^N \to \mathbb{R} \times \mathcal{Q} \times [0,1]$. For each profile of types $\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, ..., \hat{\theta}_N)$ reported by suppliers, a payment schedule $P_i(\hat{\boldsymbol{\theta}})$ specifies the expected payment from the buyer to the supplier i, a quality schedule $\boldsymbol{Q_i}(\hat{\boldsymbol{\theta}})$ specifies quality the supplier i must ensure when delivering the product, and $X_i(\hat{\boldsymbol{\theta}})$ with $\sum_{i=1}^N X_i(\hat{\boldsymbol{\theta}}) \le 1$ specifies the trading probability between the buyer and the supplier i. A direct mechanism $(P_i^*, \boldsymbol{Q_i^*}, X_i^*)_{i\in\{1,...,N\}}$ is *optimal* (for the buyer) if it solves the following problem:

$$\max_{\rho=(P_i,\boldsymbol{Q_i},X_i)_{i\in\{1,...,N\}}} \sum_{i=1}^N E\left[X_i(\boldsymbol{\theta})v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - P_i(\boldsymbol{\theta}) + \alpha[P_i(\boldsymbol{\theta}) - X_i(\boldsymbol{\theta})c(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i)]\right]$$

$$\text{s.t. } \Pi_i^\rho(\theta_i \mid \theta_i) \ge \Pi_i^\rho(\hat{\theta}_i \mid \theta_i) \quad \text{for each } \theta_i, \hat{\theta}_i \in [\underline{\theta}, \bar{\theta}], i \in \{1, ...N\} \qquad (3)$$

$$\Pi_i^\rho(\theta_i \mid \theta_i) \ge 0 \qquad \text{for each } \theta_i \in [\underline{\theta}, \bar{\theta}], i \in \{1, ...N\} \qquad (4)$$

where $\Pi_i^\rho(\hat{\theta}_i \mid \theta_i) \equiv E_{\boldsymbol{\theta_{-i}}}[P_i(\hat{\theta}_i, \boldsymbol{\theta_{-i}}) - X_i(\hat{\theta}_i, \boldsymbol{\theta}_{-i})c(\boldsymbol{Q_i}(\hat{\theta}_i, \boldsymbol{\theta}_{-i}), \theta_i)]$. The first constraint is the interim incentive compatibility (IC) constraint for each supplier, and the second one is the interim individual rationality (IR) constraint for each supplier.

The next lemma characterizes an optimal mechanism. To explain the hypothesis of the lemma and the related notions, we introduce some definitions. Denote $\boldsymbol{q} \wedge \hat{\boldsymbol{q}} \equiv (\min\{q^1, \hat{q}^1\}, ..., \min\{q^M, \hat{q}^M\})$ and $\boldsymbol{q} \vee \hat{\boldsymbol{q}} \equiv (\max\{q^1, \hat{q}^1\}, ..., \max\{q^M, \hat{q}^M\})$. Notice that

$\boldsymbol{q}, \hat{\boldsymbol{q}} \in \mathcal{Q}$ implies $\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \boldsymbol{q} \vee \hat{\boldsymbol{q}} \in \mathcal{Q}$ because $\mathcal{Q}$ is a lattice. The virtual surplus Φ is *quasisupermodular* in quality if, for each $\boldsymbol{q}, \hat{\boldsymbol{q}} \in \mathcal{Q}$ and each $\theta \in [\underline{\theta}, \bar{\theta}]$, $\Phi(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta) \leq \Phi(\boldsymbol{q}, \theta)$ implies $\Phi(\hat{\boldsymbol{q}}, \theta) \leq \Phi(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta)$, and $\Phi(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta) < \Phi(\boldsymbol{q}, \theta)$ implies $\Phi(\hat{\boldsymbol{q}}, \theta) < \Phi(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta)$. The virtual surplus Φ is *supermodular* in quality if, for each $\boldsymbol{q}, \hat{\boldsymbol{q}} \in \mathcal{Q}$ and each $\theta \in [\underline{\theta}, \bar{\theta}]$, $\Phi(\boldsymbol{q}, \theta) - \Phi(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta) \leq \Phi(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta) - \Phi(\hat{\boldsymbol{q}}, \theta)$; we apply the same definition to a scoring rule s. It is easy to show that if Φ is supermodular in quality, then Φ is quasisupermodular in quality. Thus, quasisupermodularity is a weaker notion than supermodularity. These notions express the concept of complementarity between non-monetary attributes.

Lemma 2. *Suppose that the virtual surplus Φ is quasisupermodular in quality. Then, the following direct mechanism $(P_i^*, \boldsymbol{Q_i^*}, X_i^*)_{i \in \{1,...,N\}}$ is optimal for the buyer:*

$$X_i^*(\boldsymbol{\theta}) = \begin{cases} 1 & \text{if } \theta_i < \min\{\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_N, \bar{\theta}^*\} \\ \frac{1}{\sharp\{j | \theta_j = \theta_i\}} & \text{if } \theta_i = \min\{\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_N, \bar{\theta}^*\} \\ 0 & \text{if } \theta_i > \min\{\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_N, \bar{\theta}^*\} \end{cases} \tag{5}$$

$$\boldsymbol{Q_i^*}(\boldsymbol{\theta}) = \boldsymbol{Q^*}(\theta_i) \in \arg\max_{\boldsymbol{q} \in \mathcal{Q}} \Phi(\boldsymbol{q}, \theta_i) \tag{6}$$

$$P_i^*(\boldsymbol{\theta}) = X_i^*(\boldsymbol{\theta}) \left[c(\boldsymbol{Q^*}(\theta_i), \theta_i) + \int_{\theta_i}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q^*}(z), z) \left(\frac{1 - F(z)}{1 - F(\theta_i)} \right)^{N-1} dz \right], \tag{7}$$

where $\bar{\theta}^* \in [\underline{\theta}, \bar{\theta}]$ is a cost parameter such that $\Phi(\boldsymbol{Q^*}(\theta), \theta) \geq 0$ iff $\theta \in [\underline{\theta}, \bar{\theta}^*]$. Moreover, $\boldsymbol{Q^*}(\theta_i)$ is decreasing in θ_i.

The quasisupermodularity of the virtual surplus, which is trivially satisfied if quality is one-dimensional, plays a key role in determining the property of the optimal quality schedule $\boldsymbol{Q_i^*}$. The proof shows that a necessary and sufficient condition for the IC constraints (3) is given by the two conditions. With these conditions, we can rewrite the buyer's maximization problem as follows:

$$\max_{(\boldsymbol{Q_i},X_i)_{i\in\{1,\ldots,N\}}} \sum_{i=1}^{N} E\left[X_i(\boldsymbol{\theta})\Phi(\boldsymbol{Q_i}(\boldsymbol{\theta}),\theta_i)\right]$$

s.t. For each $\theta, \hat{\theta} \in [\underline{\theta}, \bar{\theta}]$ with $\theta < \hat{\theta}$ and each $i \in \{1, ...N\}$,

$$\int_{\theta}^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z,\boldsymbol{\theta_{-i}})c_\theta(\boldsymbol{Q_i}(z,\boldsymbol{\theta_{-i}}),z) - X_i(\hat{\theta},\boldsymbol{\theta_{-i}})c_\theta(\boldsymbol{Q_i}(\hat{\theta},\boldsymbol{\theta_{-i}}),z)]dz \geq 0. \quad (8)$$

The condition (8), which we call the *monotonicity condition*, is automatically satisfied if both $\boldsymbol{Q_i}$ and X_i are decreasing in θ_i. Using the result of Milgrom and Shannon (1994), the proof shows that the quasisupermodularity of the virtual surplus in quality together with Assumptions 4 and 5 imply that a quality schedule $\boldsymbol{Q_i}$ which maximizes the virtual surplus $\Phi(\boldsymbol{q}, \theta_i)$ is decreasing in θ_i; notice that this quality schedule $\boldsymbol{Q}^*$ depends only on a supplier's type report because all suppliers are *ex ante* symmetric. The proof also shows that the maximized virtual surplus $\Phi(\boldsymbol{Q}^*(\theta_i), \theta_i)$ is decreasing in θ_i. As a result, the quality schedule $\boldsymbol{Q}^*$ and the trading probability X_i^* defined in Lemma 2 satisfy the monotonicity condition (8) and constitute the optimal mechanism with the payment schedule P_i^*. Thus, the more efficient a supplier is, the higher levels of *all* non-monetary attributes he is required to achieve. On the other hand, without the quasisupermodularity, the optimal quality schedule $\boldsymbol{Q}^*$ may be *nonmonotonic* in θ_i. This is because even if Q^{m*} is not decreasing in θ_i for some m, the monotonicity condition (8) can be satisfied when $c_\theta(\boldsymbol{Q}(\theta_i), z)$ is decreasing in θ_i. The following example shows that this is the case.[4]

Example 1. Assume that $M = 2$, $\mathcal{Q}^1 = \mathcal{Q}^2 = \{1, 2\}$, $v(q^1, q^2) = 0$ if $q^1 = q^2 = 1$ and $v(q^1, q^2) = 100$ otherwise, $c(q^1, q^2, \theta) = 6\theta q^1 + (3\theta + 9)q^2 - q^1 q^2$, θ is uniformly distributed on $[1, 3]$, and $\alpha = 0$. Then, one can show that $\Phi(q^1, q^2, \theta) = v(q^1, q^2) - (12\theta - 6)q^1 - (6\theta + 6)q^2 + q^1 q^2$, and Φ is *not* quasisupermodular in quality. Actually, Φ is submodular

[4] This can occur even in a single-agent screening model. See Laffont and Martimort (2002).

in quality, i.e. $-\Phi$ is supermodular in quality. The following levels of non-monetary attributes maximizes $\Phi(q^1, q^2, \theta)$ for each θ:

$$\boldsymbol{Q}^*(\theta) = (Q^{1*}(\theta), Q^{2*}(\theta)) = \begin{cases} (2,1) & \text{if } \theta \in [1,2) \\ (1,2) & \text{if } \theta \in [2,3]. \end{cases}$$

The quality schedule $\boldsymbol{Q}^*$ together with X_i^* defined in Lemma 2 and $\bar{\theta}^* = \bar{\theta} = 3$ satisfy the monotonicity condition (8) because $c_\theta(\boldsymbol{Q}^*(\theta), z) = 15$ if $\theta \in [1,2)$ and $c_\theta(\boldsymbol{Q}^*(\theta), z) = 12$ if $\theta \in [2,3]$, and thus $c_\theta(\boldsymbol{Q}^*(\theta), z)$ is decreasing in θ. Therefore, the optimal quality schedule $\boldsymbol{Q}^*$ is *nonmonotonic* in θ because Q^{1*} is decreasing whereas Q^{2*} is increasing.

In a second step, we discuss the implementation of the optimal mechanism via a scoring auction. We say that a (quasi-linear) scoring rule s *implements the optimal mechanism* if the auction game induced by s has a Bayesian Nash equilibrium which yields the same outcome as $(P_i^*, \boldsymbol{Q}^*, X_i^*)_{i\in\{1,\dots,N\}}$ for each realization of $\boldsymbol{\theta}$. Lemmas 1 and 2 imply that the buyer's goal is to construct a scoring rule s so that each supplier's equilibrium offer $\boldsymbol{q}^*(\theta)$ is equal to $\boldsymbol{Q}^*(\theta)$ for each $\theta \in [\underline{\theta}, \bar{\theta}^*]$ and the inefficient suppliers ($\theta \in (\bar{\theta}^*, \bar{\theta}]$) are excluded by the reserve score. The next proposition demonstrates how the scoring rule should be constructed under the same condition as Lemma 2.

Proposition 1. *Suppose that the virtual surplus Φ is quasisupermodular in quality. Then, there exists a quasi-linear scoring rule s^* which implements the optimal mechanism and is supermodular in quality.*

In the construction of the optimal scoring rule s^*, we fully utilize the monotonicity of the optimal quality schedule $\boldsymbol{Q}^*$, which is guaranteed by the quasisupermodularity of the virtual surplus in quality. Comparing (1) with (6), we see that if a weight α on profits is equal to one so that the buyer does not care about information rents, then the scoring rule s which is equal to her valuation v succeeds in the implementation. In general, however, because $\alpha < 1$, we must carefully devise the optimal scoring rule. Figure 1 shows how the rule s^* is constructed. Suppose that quality is two-dimensional. In the left figure, the solid curve represents the qualities which the optimal quality schedule $\boldsymbol{Q}^*$ requires suppliers to ensure for each type $\theta \in [\underline{\theta}, \bar{\theta}^*]$. Lemma 2 implies that this curve is upward sloping due to the monotonicity of $\boldsymbol{Q}^*$. The rule s^* is constructed so that a score remains constant even if, starting from any point on the curve, the level of only one attribute increases, and a score falls to zero if the level of at least one attribute is lower than $Q^{m*}(\bar{\theta}^*)$. Thus, any supplier who wishes to win has no incentive to make a quality offer other than offers on the solid curve. In the right figure, the three solid curves below the dotted curve represent the production costs for some cost parameters. These production costs have the "single-crossing property" due to the monotonicity of $\boldsymbol{Q}^*$ with the assumption that the incremental cost is increasing in a cost parameter. Thus, there exists a "lower envelope" of the cost curves shifted up, which is described by the dotted curve and given by the following function of $\hat{\theta}$:

$$c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz.$$

This is used as a score of the quality offer $\boldsymbol{Q}^*(\hat{\theta})$ with $\hat{\theta} \in [\underline{\theta}, \bar{\theta}^*]$. The second term is a score added to the production cost $c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta})$ to facilitate the separation of types. Then, each supplier of type θ optimally makes the quality offer $\boldsymbol{Q}^*(\hat{\theta}) = \boldsymbol{Q}^*(\theta)$ so as to maximize a quality score $s^*(\boldsymbol{Q}^*(\hat{\theta}))$ minus his production cost $c(\boldsymbol{Q}^*(\hat{\theta}), \theta)$.

Proposition 1 states that the quasi-linear scoring rule s^*, which is supermodular in quality, implements the optimal mechanism. The supermodularity is derived from the Leontief-like shape of s^*. With the quasisupermodularity of the virtual surplus in quality, the buyer desires a more efficient supplier to achieve higher levels of *all* non-monetary attributes, so that a scoring rule which gives a score in a complementary way works well.

On the other hand, without the quasisupermodularity, a supermodular scoring rule s may fail in the implementation. This problem arises from the nonmonotonicity of the optimal quality schedule, as shown in Example 1. The next example shows that this is the case.

Example 2. We make the same assumptions as Example 1. Suppose that a quasi-linear scoring rule s which is supermodular in quality implements the optimal mechanism. Because the optimal quality schedule satisfies $\boldsymbol{Q}^*(\theta) = (1, 2)$ for $\theta = 2$, Lemma 1 implies that $s(2, 2) - s(1, 2) \leq c(2, 2, \theta = 2) - c(1, 2, \theta = 2) = 10$. Now, $c(2, 1, \theta) - c(1, 1, \theta) = 6\theta - 1 > 10$ for each $\theta > 11/6$. Because s is supermodular in quality, $s(2, 1) - s(1, 1) \leq s(2, 2) - s(1, 2)$. Hence, $c(2, 1, \theta) - c(1, 1, \theta) > s(2, 1) - s(1, 1)$ for each $\theta > 11/6$, and thus a supplier of type $\theta \in (11/6, 2)$ never chooses $\boldsymbol{Q}^*(\theta) = (2, 1)$ in any equilibrium. This contradicts the hypothesis that s implements the optimal mechanism.

We construct the scoring rule s^* in a different way from Che (1993) (and Branco (1997)). We now show that a scoring rule *a la* Che (1993) which is naturally extended to an environment where quality is multi-dimensional may fail in the implementation. Suppose first that quality is one-dimensional and a continuous variable, and the optimal quality schedule Q^* is strictly decreasing in θ. Then, a quasi-linear scoring rule s constructed by Che is

$$s(q) = v(q) - (1 - \alpha) \int_0^q \frac{\partial^2 c}{\partial q \partial \theta}(y, (Q^*)^{-1}(y)) \frac{F((Q^*)^{-1}(y))}{f((Q^*)^{-1}(y))} dy,$$

where $(Q^*)^{-1}$ is the inverse function of Q^*. The rule underrewards quality relative to the valuation because the quality level which maximizes the social surplus $v(q) - c(q, \theta)$ is excessive from the buyer's viewpoint. With some differentiability assumptions and Inada conditions, the first-order condition of the problem $\max_q [s(q) - c(q, \theta)]$ is given by

$$\frac{dv}{dq}(q) - (1 - \alpha) \frac{\partial^2 c}{\partial q \partial \theta}(q, (Q^*)^{-1}(q)) \frac{F((Q^*)^{-1}(q))}{f((Q^*)^{-1}(q))} - \frac{\partial c}{\partial q}(q, \theta) = 0,$$

which is satisfied if $q = Q^*(\theta)$. Moreover, together with the assumptions in this article, the assumption that v, $-c$ and $-c_\theta$ are concave in quality imply that s is also concave, as shown by Che (1993). Thus, $q = Q^*(\theta)$ is a global optimal solution to $\max_q[s(q)-c(q,\theta)]$.

Suppose now that quality is multi-dimensional. Consider the following scoring rule:

$$s(\boldsymbol{q}) = v(\boldsymbol{q}) - (1-\alpha)\sum_{m=1}^{M}\int_0^{q^m}\frac{\partial^2 c}{\partial q^m \partial\theta}(y, \boldsymbol{Q^{-m*}}(\theta^m(y)), \theta^m(y))\frac{F(\theta^m(y))}{f(\theta^m(y))}dy, \tag{9}$$

where θ^m is the inverse function of Q^{m*}. With some differentiability assumptions and Inada conditions, the first-order conditions of $\max_{\boldsymbol{q}}[s(\boldsymbol{q}) - c(\boldsymbol{q},\theta)]$ are given by

$$\frac{\partial v}{\partial q^m}(\boldsymbol{q}) - (1-\alpha)\frac{\partial^2 c}{\partial q^m\partial\theta}(q^m, \boldsymbol{Q^{-m*}}(\theta^m(q^m)), \theta^m(q^m))\frac{F(\theta^m(q^m))}{f(\theta^m(q^m))} - \frac{\partial c}{\partial q^m}(\boldsymbol{q},\theta) = 0$$

for each m, which are satisfied if $\boldsymbol{q} = \boldsymbol{Q^*}(\theta)$. However, even if v, $-c$ and $-c_\theta$ are concave in quality, the scoring rule s defined by (9) may *not* be concave and cannot implement the optimal mechanism. The next example shows that this is the case.

Example 3. Assume that $M = 2$, $\mathcal{Q}^1 = \mathcal{Q}^2 = [0,1]$, $v(\boldsymbol{q}) = q^1 + q^2 + 100$, $c(\boldsymbol{q},\theta) = \theta((q^1)^2 + (q^2)^2) + q^1q^2$, θ is uniformly distributed on $[1,2]$, and $\alpha = 0$. Note that because v is linear in $\boldsymbol{q}$ and both $-c$ and $-c_\theta$ are strictly concave in $\boldsymbol{q}$, Φ is strictly concave in $\boldsymbol{q}$. One can show that $\Phi(\boldsymbol{q},\theta) = (q^1+q^2) + 100 - (2\theta-1)((q^1)^2 + (q^2)^2) - q^1q^2$, $\boldsymbol{Q^*}(\theta) = (1/(4\theta-1), 1/(4\theta-1))$, and $\bar{\theta}^* = \bar{\theta} = 2$. The scoring rule s defined by (9) is

$$s(\boldsymbol{q}) = v(\boldsymbol{q}) - \sum_{m=1}^{2}\int_0^{q^m} 2y\left(\frac{1}{4y} - \frac{3}{4}\right)dy = \frac{1}{2}(q^1+q^2) + \frac{3}{4}((q^1)^2 + (q^2)^2) + 100.$$

Then, for some θ, $s - c$ is *not* concave in $\boldsymbol{q}$, so that the quality offer $\boldsymbol{Q^*}(\theta) = (1/(4\theta - 1), 1/(4\theta-1))$ may not be a global optimal solution to $\max_{\boldsymbol{q}}[s(\boldsymbol{q}) - c(\boldsymbol{q},\theta)]$. Actually, for $\theta = 1$, $s(\boldsymbol{Q^*}(\theta)) - c(\boldsymbol{Q^*}(\theta),\theta) = 100 + 1/6 < 100 + 1/4 = s(1,0) - c(1,0,\theta)$.

5 Additively separable scoring rule

In this section, we investigate whether a quasi-linear scoring rule which is additively separable in some quality attributes can implement the optimal mechanism.

The scoring rule s^* constructed in Proposition 1 is supermodular in quality. Then, a question arises: Can an additively separable scoring rule such as "weighted criteria" mentioned in the Introduction implement the optimal mechanism? To answer this question, we introduce a definition. We assume that the set of quality attributes $\{1, ..., M\}$ is partitioned into two nonempty subsets $\mathcal{M}^1$ and $\mathcal{M}^2$ of criteria; our analysis can be extended to the case of more than two subsets at the expense of notational complexity. A quasi-linear scoring rule s is *additively separable* if the rule takes a form of $s(\boldsymbol{q}) = s^1(\boldsymbol{q^1}) + s^2(\boldsymbol{q^2})$ for each $\boldsymbol{q} = (\boldsymbol{q^1}, \boldsymbol{q^2})$ with $\boldsymbol{q^1} \in \times_{m \in \mathcal{M}^1} \mathcal{Q}^m$ and $\boldsymbol{q^2} \in \times_{m \in \mathcal{M}^2} \mathcal{Q}^m$. In the next example, an additively separable scoring rule can *never* implement the optimal mechanism.

Example 4. Assume that $M = 2$, $\mathcal{Q}^1 = \mathcal{Q}^2 = \{1, 2\}$, $v(q^1, q^2) = 9(q^1 + q^2) + 100$, $c(q^1, q^2, \theta) = 3\theta(q^1 + q^2) + (3 - \theta)q^1 q^2$, θ is uniformly distributed on $[1, 3]$, and $\alpha = 0$. Then, one can show that $\Phi(q^1, q^2, \theta) = (12 - 6\theta)(q^1 + q^2) - (4 - 2\theta)q^1 q^2 + 100$, and Φ is quasisupermodular in quality. The optimal quality schedule is given by

$$\boldsymbol{Q}^*(\theta) = (Q^{1*}(\theta), Q^{2*}(\theta)) = \begin{cases} (2, 2) & \text{if } \theta \in [1, 2) \\ (1, 1) & \text{if } \theta \in [2, 3] \end{cases}$$

and $\bar{\theta}^* = \bar{\theta} = 3$. Suppose that a quasi-linear scoring rule s which is additively separable implements the optimal mechanism. Because $\boldsymbol{Q}^*(\theta) = (1, 1)$ for $\theta = 2$, Lemma 1 implies that $s(1, 2) - s(1, 1) \leq c(1, 2, \theta = 2) - c(1, 1, \theta = 2) = 7$. Now, $c(2, 2, \theta) - c(2, 1, \theta) = 6 + \theta > 7$ for each $\theta > 1$. Hence, $c(2, 2, \theta) - c(2, 1, \theta) > 7 \geq s(1, 2) - s(1, 1) = [s^1(1) - s^1(1)] + [s^2(2) - s^2(1)] = [s^1(2) - s^1(2)] + [s^2(2) - s^2(1)] = s(2, 2) - s(2, 1)$ for each $\theta > 1$, and thus a supplier of type $\theta \in (1, 2)$ never chooses $\boldsymbol{Q}^*(\theta) = (2, 2)$ in any equilibrium.

This contradicts the hypothesis that s implements the optimal mechanism.

What is the cause of the failure? One can find the answer in the functional form of the production cost. For each type except $\theta = 3$, the incremental cost for each attribute is strictly increasing in the level of the other attribute. That is, the two quality attributes are substitutable in terms of the production cost. Now, the set of types is partitioned into two groups: From the buyer's point of view, each type in the efficient group $[1, 2)$ should offer the greatest quality $(2, 2)$, and each type in the inefficient group $[2, 3]$ should offer the least quality $(1, 1)$. Then, given any additively separable rule which deters the inefficient suppliers' deviations, an incentive for a supplier of type $\theta = 2$ to deviate from the quality offer $(1, 1)$ to $(1, 2)$ is weaker than that for a supplier of type $\theta \in (1, 2)$ to deviate from the quality offer $(2, 2)$ to $(2, 1)$ because of the substitutability. This means that any additively separable rule causes deviations of some types in either group. As one would expect, if the production cost has decreasing differences in (q^1, q^2), then we can construct an additively separable scoring rule which is immune to the deviation.

We can obtain the more general results although the underlying structure is the same as Example 4. Here, the notion of increasing (decreasing) differences expresses the concept of complementarity (substitutability) between some quality attributes in terms of the production cost. The following proposition gives a necessary condition and a sufficient condition for the existence of an additively separable scoring rule which implements the optimal mechanism.

Proposition 2. *Suppose that the virtual surplus Φ is quasisupermodular. Then: (i) If c has increasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ and there exists $(m, m') \in \mathcal{M}^1 \times \mathcal{M}^2$ such that c has strictly increasing differences in $(q^m, q^{m'})$, $Q^{m*}(\theta-) > Q^{m*}(\theta+)$ and $Q^{m'*}(\theta-) > Q^{m'*}(\theta+)$ for some $\theta \in (\underline{\theta}, \bar{\theta}^*)$, then there is no additively separable scoring rule which* *implements the optimal mechanism. (ii) If* c *has decreasing differences in* $(\boldsymbol{q^1}, \boldsymbol{q^2})$ *and* $\mathcal{Q}^m$ *is finite for each* m*, then there is an additively separable scoring rule which implements the optimal mechanism.*

This proposition has several implications. First, the implementation possibility via an additively separable scoring rule heavily depends on whether the cost function has increasing differences or decreasing differences in quality attributes in the distinct sets of subcriteria, rather than the property of the value function. In particular, Example 4 shows that an additively separable scoring rule cannot implement the optimal mechanism even if the value function is additively separable in quality.

Second, the results of Proposition 2 provide a useful guide to designing additively separable scoring rules. Consider an example of the highway construction. Suppose that the quality attributes represent delivery date ($m = 1$), durability of the highway ($m = 2$), maintenance service after delivery ($m = 3$), respectively. Moreover, we consider the following plausible scenario: as the delivery date is earlier, it is more costly to increase the durability level; as the durability level is higher, it is less costly to increase the level of maintenance service; the incremental cost for the maintenance service is independent of the delivery date. That is, the production cost has strictly increasing differences in (q^1, q^2), decreasing differences in (q^2, q^3), and is additively separable in (q^1, q^3). Thus, the cost function has decreasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with $\mathcal{M}^1 = \{1, 2\}$ and $\mathcal{M}^2 = \{3\}$ whereas the cost function has increasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with $\mathcal{M}^1 = \{1\}$ and $\mathcal{M}^2 = \{2, 3\}$. Proposition 2 then implies that the buyer should classify quality attributes so that $s(\boldsymbol{q}) = s^1(q^1, q^2) + s^2(q^3)$ not $s(\boldsymbol{q}) = s^1(q^1) + s^2(q^2, q^3)$. When the buyer establishes some sets of subcriteria to use the additively separable rule, each pair of quality attributes in the distinct sets should be complementary in terms of the production cost.

6 Concluding remarks

We have studied the optimal design of scoring auction in an environment where quality is multi-dimensional. Our main result shows that if the virtual surplus is quasisupermodular in quality, then there exists a quasi-linear scoring rule which implements the optimal mechanism and is supermodular in quality. Thus, when the virtual surplus exhibits a kind of complementarity between quality attributes, a scoring rule which gives a quality score in a complementary way works well. This in turn implies that the buyer should carefully design scoring rules which are additively separable in some quality attributes.

One may wonder why each supplier should offer *all* quality attributes in a scoring auction. Alternatively, the buyer can require each supplier to offer only one quality attribute (with price). Then, with a scoring rule *a la* Che (1993), the most efficient supplier achieves the highest score, and the winner's type is revealed to the buyer. If the levels of the remaining quality attributes are properly specified based on the winner's type, then this mechanism can implement the optimal mechanism. The mechanism or procedure, however, requires the buyer to prespecify the levels of the remaining quality attributes for each cost parameter. In practice, it may be prohibitively costly for the buyer to do so. For instance, the buyer may not be able to specify aesthetic and functional characteristics of highway although suppliers with expertise can offer these characteristics. Therefore, when there are at least two such quality attributes, this study is of significance.

Finally, we should point out a potential limitation of this analysis. In our model, a more efficient supplier (i.e. a supplier with less cost parameter) has a superior technology of increasing the levels of *all* quality attributes. Thus, our model does not cover the case in which one type of supplier has a superior technology of increasing the level of one quality attribute to another type of supplier whereas the latter type has a superior technology of increasing the level of another quality attribute to the former type. To cover the case, we need to allow each supplier's type to be multi-dimensional. It is interesting and challenging to study the optimal design of scoring auction in an environment where both quality and a supplier's type are multi-dimensional. This is left for future research.

Appendix

Proof of Lemma 1. (i) First, note that $\arg\max_{\boldsymbol{q}\in\mathcal{Q}}[s(\boldsymbol{q}) - c(\boldsymbol{q},\theta)]$ is nonempty for each θ because $\mathcal{Q}$ is compact and $s - c$ is upper semicontinuous in $\boldsymbol{q}$. (See, for example, Kolmogorov and Fomin (1975).) We now show that in equilibrium a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^s]$ never offers $(p', \boldsymbol{q'})$ such that $\boldsymbol{q'} \notin \arg\max_{\hat{\boldsymbol{q}}\in\mathcal{Q}}[s(\hat{\boldsymbol{q}}) - c(\hat{\boldsymbol{q}},\theta)]$. Suppose, on the contrary, that a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^s]$ makes such an offer $(p', \boldsymbol{q'})$. Consider another offer $(p, \boldsymbol{q})$ such that $\boldsymbol{q} \in \arg\max_{\hat{\boldsymbol{q}}\in\mathcal{Q}}[s(\hat{\boldsymbol{q}}) - c(\hat{\boldsymbol{q}},\theta)]$ and $s(\boldsymbol{q}) - p = s(\boldsymbol{q'}) - p'$. The score of $(p, \boldsymbol{q})$ is equal to that of $(p', \boldsymbol{q'})$, so that both offers yield the same winning probability given the other suppliers' strategies. The supplier's expected profit from $(p', \boldsymbol{q'})$ is not higher than his expected profit from $(p, \boldsymbol{q})$ because

$$
\begin{aligned}
&[p' - c(\boldsymbol{q'},\theta)]Prob[win \mid S(p', \boldsymbol{q'})] \\
&\leq [p' - c(\boldsymbol{q'},\theta) + (s(\boldsymbol{q}) - c(\boldsymbol{q},\theta) - (s(\boldsymbol{q'}) - c(\boldsymbol{q'},\theta)))]Prob[win \mid S(p', \boldsymbol{q'})] \\
&= [p - c(\boldsymbol{q},\theta)]Prob[win \mid S(p, \boldsymbol{q})],
\end{aligned}
$$

where the inequality follows from the hypothesis that $\boldsymbol{q'} \notin \arg\max_{\hat{\boldsymbol{q}}\in\mathcal{Q}}[s(\hat{\boldsymbol{q}}) - c(\hat{\boldsymbol{q}},\theta)] \ni \boldsymbol{q}$, and the equality follows from the construction of $(p, \boldsymbol{q})$. The inequality is strict if $Prob[win \mid S(p, \boldsymbol{q})] > 0$, which occurs for some strategies of the other suppliers. This contradicts the assumption that no supplier uses weakly dominated strategies. Thus, the latter statement holds, and we can assume that a symmetric equilibrium bidding strategy

$(p, \boldsymbol{q}^*)$ satisfies $\boldsymbol{q}^*(\theta) \in \arg\max_{\boldsymbol{q}\in\mathcal{Q}}[s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ for each $\theta \in [\underline{\theta}, \bar{\theta}^s]$.

Second, consider the following change of variable: $k(\theta) \equiv s(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta)$ for each $\theta \in [\underline{\theta}, \bar{\theta}^s]$. Because c_θ is continuous and thus bounded on $[\underline{\theta}, \bar{\theta}]$, it follows from the integral form envelope theorem of Milgrom and Segal (2002) (see also Theorem 3.1 of Milgrom (2004)) that k is absolutely continuous, and is given by

$$k(\theta) = k(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} c_\theta(\boldsymbol{q}^*(z), z)dz.$$

Finally, we show that the bidding strategy $(p^*, \boldsymbol{q}^*)$ in the lemma constitutes a symmetric equilibrium. For each $\theta \in [\underline{\theta}, \bar{\theta}^s]$, the score $s(\boldsymbol{q}^*(\theta)) - p^*(\theta)$ is given by

$$\begin{aligned} s(\boldsymbol{q}^*(\theta)) - p^*(\theta) &= s(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta) - [p^*(\theta) - c(\boldsymbol{q}^*(\theta), \theta)] \\ &= k(\theta) - \int_{\theta}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta)}dz. \end{aligned}$$

Note that $s(\boldsymbol{q}^*(\bar{\theta}^s)) - p^*(\bar{\theta}^s) = s(\boldsymbol{q}^*(\bar{\theta}^s)) - c(\boldsymbol{q}^*(\bar{\theta}^s), \bar{\theta}^s) = 0$ by assumption. Because k is continuous in $\theta \in [\underline{\theta}, \bar{\theta}^s]$, so is $s(\boldsymbol{q}^*(\theta)) - p^*(\theta)$. Moreover, the score $s(\boldsymbol{q}^*(\theta)) - p^*(\theta)$ is strictly decreasing in $\theta \in [\underline{\theta}, \bar{\theta}^s]$ because for each $\theta, \theta' \in [\underline{\theta}, \bar{\theta}^s]$ with $\theta < \theta'$,

$$\begin{aligned} &[s(\boldsymbol{q}^*(\theta)) - p^*(\theta)] - [s(\boldsymbol{q}^*(\theta')) - p^*(\theta')] \\ &= (k(\theta) - k(\theta')) - \left[\int_{\theta}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta)}dz - \int_{\theta'}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta')}dz\right] \\ &> (k(\theta) - k(\theta')) - \int_{\theta}^{\theta'} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta)}dz \\ &> (k(\theta) - k(\theta')) - \int_{\theta}^{\theta'} c_\theta(\boldsymbol{q}^*(z), z)dz \\ &= \int_{\theta}^{\theta'} [c_\theta(\boldsymbol{q}^*(z), z) - c_\theta(\boldsymbol{q}^*(z), z)]\, dz = 0. \end{aligned}$$

When the other suppliers follow the strategy $(p^*, \boldsymbol{q}^*)$, the expected profit of a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^s]$ from offering $(p, \boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta)) - p = s(\boldsymbol{q}^*(\hat{\theta})) - p^*(\hat{\theta})$ for some

$\hat{\theta} \in [\underline{\theta}, \bar{\theta}^s]$ can be written as

$$
\begin{aligned}
&[p - c(\boldsymbol{q}^*(\theta), \theta)]Prob[win \mid S(p, \boldsymbol{q}^*(\theta))] \\
&= [s(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta) - s(\boldsymbol{q}^*(\hat{\theta})) + p^*(\hat{\theta})]Prob[win \mid S(p^*(\hat{\theta}), \boldsymbol{q}^*(\hat{\theta}))] \\
&= [k(\theta) - s(\boldsymbol{q}^*(\hat{\theta})) + p^*(\hat{\theta})](1 - F_{(N-1)}(\hat{\theta})) \\
&= (k(\theta) - k(\hat{\theta}))(1 - F_{(N-1)}(\hat{\theta})) + \int_{\hat{\theta}}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)(1 - F_{(N-1)}(z))dz,
\end{aligned}
$$

where the second equality follows from the observation that the score $S(p^*(\hat{\theta}), \boldsymbol{q}^*(\hat{\theta})) = s(\boldsymbol{q}^*(\hat{\theta})) - p^*(\hat{\theta})$ is strictly decreasing in $\hat{\theta}$. The supplier cannot obtain a higher expected profit by deviating from $(p^*(\theta), \boldsymbol{q}^*(\theta))$ to $(p, \boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta)) - p = s(\boldsymbol{q}^*(\hat{\theta})) - p^*(\hat{\theta})$ because the difference between the expected profits is given by

$$
\begin{aligned}
&- (k(\theta) - k(\hat{\theta}))(1 - F_{(N-1)}(\hat{\theta})) + \int_{\theta}^{\hat{\theta}} c_\theta(\boldsymbol{q}^*(z), z)(1 - F_{(N-1)}(z))dz \\
&= \int_{\theta}^{\hat{\theta}} \left[-c_\theta(\boldsymbol{q}^*(z), z)(1 - F_{(N-1)}(\hat{\theta})) + c_\theta(\boldsymbol{q}^*(z), z)(1 - F_{(N-1)}(z)) \right] dz \\
&= \int_{\theta}^{\hat{\theta}} c_\theta(\boldsymbol{q}^*(z), z)(F_{(N-1)}(\hat{\theta}) - F_{(N-1)}(z))dz \geq 0.
\end{aligned}
$$

It is easy to show that the supplier cannot obtain a higher expected profit by deviating from $(p^*(\theta), \boldsymbol{q}^*(\theta))$ to $(p, \boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta)) - p \notin [0, s(\boldsymbol{q}^*(\underline{\theta})) - p^*(\underline{\theta})]$. Also, a supplier of type $\theta \in (\bar{\theta}^s, \bar{\theta}]$ obtains a negative expected profit if he offers $(p, \boldsymbol{q})$ such that $s(\boldsymbol{q}) - p \geq 0$ whereas he obtains zero profit by offering $(p^*(\theta), \boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta)) - p^*(\theta) < 0$. This completes the proof of part (i).

(ii) In the above equilibrium, the score $s(\boldsymbol{q}^*(\theta_i)) - p^*(\theta_i)$ is strictly decreasing in $\theta_i \in [\underline{\theta}, \bar{\theta}^s]$ with $s(\boldsymbol{q}^*(\bar{\theta}^s)) - p^*(\bar{\theta}^s) = 0$, and is negative for each $\theta_i \in (\bar{\theta}^s, \bar{\theta}]$. Thus, a supplier of type θ_i wins only if $\theta_i = \min\{\theta_1, ..., \theta_N, \bar{\theta}^s\}$. □

Proof of Lemma 2. (i) We show that a necessary and sufficient condition for the IC constraints (3) is given by the two conditions: *envelope condition* and *monotonicity condition.*

We say that a direct mechanism ρ satisfies the envelope condition if for each i and θ,

$$\Pi_i^\rho(\theta) = \Pi_i^\rho(\bar{\theta}) + \int_\theta^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] dz,$$

where $\Pi_i^\rho(\theta) \equiv \Pi_i^\rho(\theta \mid \theta)$. We say that a direct mechanism ρ satisfies the monotonicity condition if for each i, θ and $\hat{\theta}$ with $\hat{\theta} > \theta$,

$$\int_\theta^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z) - X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), z)] dz \geq 0.$$

Because c has (strictly) increasing differences in $(\boldsymbol{q}, \theta)$, it must hold that for each $\boldsymbol{q}, \boldsymbol{q'} \in \mathcal{Q}$ with $\boldsymbol{q} \leq \boldsymbol{q'}$ and each $z, z' \in [\underline{\theta}, \bar{\theta}]$ with $z \leq z'$, $c(\boldsymbol{q'}, z) - c(\boldsymbol{q}, z) \leq c(\boldsymbol{q'}, z') - c(\boldsymbol{q}, z')$, and thus $c_\theta(\boldsymbol{q'}, z) \geq c_\theta(\boldsymbol{q}, z) > 0$. Therefore, if both $X_i(\theta_i, \boldsymbol{\theta_{-i}})$ and $\boldsymbol{Q_i}(\theta_i, \boldsymbol{\theta_{-i}})$ are decreasing in θ_i for each $\boldsymbol{\theta_{-i}}$, then the monotonicity condition is automatically satisfied.

First, we prove sufficiency. Suppose that a supplier i's IC constraint is not satisfied. Then, there exist θ and $\hat{\theta}$ such that $\Pi_i^\rho(\hat{\theta} \mid \theta) > \Pi_i^\rho(\theta)$. Hence, $E_{\boldsymbol{\theta_{-i}}}[X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \hat{\theta}) - X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \theta)] > \Pi_i^\rho(\theta) - \Pi_i^\rho(\hat{\theta})$ by definition of Π_i^ρ. Rewriting the left-hand side as the definite integral and applying the envelope condition to the right-hand side, we obtain

$$\int_\theta^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), z)] dz > \int_\theta^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] dz.$$

This contradicts the monotonicity condition.

Next, we prove necessity. Using the integral form envelope theorem of Milgrom and Segal (2002), the IC constraints (3) imply that for each i and θ, $\Pi_i^\rho(\theta) = \max_{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[P_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) - X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \theta)]$ is given by

$$\Pi_i^\rho(\theta) = \Pi_i^\rho(\bar{\theta}) - \int_\theta^{\bar{\theta}} \frac{\partial \Pi_i^\rho}{\partial \theta}(z \mid z) dz = \Pi_i^\rho(\bar{\theta}) + \int_\theta^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] dz.$$

We thus obtain the envelope condition. Also, the IC constraints (3) imply that for each i, θ and $\hat{\theta}$, $E_{\boldsymbol{\theta_{-i}}}[X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \hat{\theta}) - X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \theta)] \leq \Pi_i^\rho(\theta) - \Pi_i^\rho(\hat{\theta})$.

Rewriting the left-hand side as the definite integral and applying the envelope condition to the right-hand side, we obtain the monotonicity condition.

(ii) We solve the optimization problem. The IC constraints (3) imply that for each i, $\Pi_i^\rho(\theta)$ is decreasing in θ because $\Pi_i^\rho(\theta) \geq \Pi_i^\rho(\theta' \mid \theta) \geq \Pi_i^\rho(\theta')$ for each $\theta < \theta'$; the second inequality follows from the assumption that c is (strictly) increasing in θ_i. Hence, the IR constraints (4) are replaced by $\Pi_i^\rho(\bar{\theta}) \geq 0$ for each i. Using the result (i), the IC constraints (3) are replaced by the envelope and monotonicity conditions. By the envelope condition and the interchange of the order of integration, $E[\Pi_i^\rho(\theta_i)]$ is given by

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} \Pi_i^\rho(\theta_i) f(\theta_i) d\theta_i &= \Pi_i^\rho(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta_i}^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] dz f(\theta_i) d\theta_i \\ &= \Pi_i^\rho(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] \frac{F(z)}{f(z)} f(z) dz.\end{aligned}$$

Hence, the buyer's objective function is rewritten as

$$\begin{aligned}&\sum_{i=1}^N E\left[X_i(\boldsymbol{\theta}) v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - P_i(\boldsymbol{\theta}) + \alpha \Pi_i^\rho(\theta_i)\right] \\ &= \sum_{i=1}^N E\left[X_i(\boldsymbol{\theta})[v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - c(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i)] - (1-\alpha)\Pi_i^\rho(\theta_i)\right] \\ &= \sum_{i=1}^N E\left[X_i(\boldsymbol{\theta}) \left[v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - c(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i) - (1-\alpha) c_\theta(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i) \frac{F(\theta_i)}{f(\theta_i)}\right] - (1-\alpha)\Pi_i^\rho(\bar{\theta})\right] \\ &= \sum_{i=1}^N E\left[X_i(\boldsymbol{\theta}) \Phi(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i) - (1-\alpha)\Pi_i^\rho(\bar{\theta})\right].\end{aligned}$$

Note that $\arg\max_{\boldsymbol{q} \in \mathcal{Q}} \Phi(\boldsymbol{q}, \theta_i)$ is nonempty for each θ_i because $\mathcal{Q}$ is compact and Φ is continuous in $\boldsymbol{q}$. The above objective function is maximized when $\Pi_i^\rho(\bar{\theta}) = 0$, and $\boldsymbol{Q_i}(\boldsymbol{\theta})$ and $X_i(\boldsymbol{\theta})$ are respectively given by $\boldsymbol{Q}^*(\theta_i)$ and $X_i^*(\boldsymbol{\theta})$ in the lemma. This is because $\boldsymbol{Q}^*(\theta_i)$ maximizes $\Phi(\boldsymbol{q}, \theta_i)$ and the maximized value $\Phi(\boldsymbol{Q}^*(\theta_i), \theta_i)$ is strictly decreasing in θ_i. The latter fact follows from $\Phi(\boldsymbol{Q}^*(\theta), \theta) \geq \Phi(\boldsymbol{Q}^*(\theta'), \theta) > \Phi(\boldsymbol{Q}^*(\theta'), \theta')$ for each $\theta < \theta'$; the second inequality follows from the assumptions that c is strictly increasing in θ_i, and both c_θ and F/f are increasing in θ_i.

Finally, we show that the direct mechanism $\rho^* = (P_i^*, \boldsymbol{Q_i^*}, X_i^*)_{i\in\{1,\ldots,N\}}$ satisfies the ignored monotonicity condition. Now, Φ is quasisupermodular in $\boldsymbol{q}$ by hypothesis, and has strictly increasing differences in $(\boldsymbol{q}, -\theta_i)$ from Assumptions 4 and 5. It then follows from Theorem 2.8.7 of Topkis (1998) that $\boldsymbol{Q^*}(\theta_i) \geq \boldsymbol{Q^*}(\theta_i')$ for each $\theta_i < \theta_i'$. Also, X_i^* is decreasing in θ_i. These facts imply that ρ^* satisfies the monotonicity condition. □

Proof of Proposition 1. (i) We first show that there exists a quasi-linear scoring rule which implements the optimal mechanism. From Lemmas 1 and 2, it suffices to show that there exists an upper semicontinuous function $s^* : \mathcal{Q} \to \mathbb{R}$ which satisfies $\boldsymbol{Q^*}(\theta) \in \arg\max_{\boldsymbol{q}\in\mathcal{Q}}[s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ for each $\theta \in [\underline{\theta}, \bar{\theta}^*]$ and $s^*(\boldsymbol{Q^*}(\bar{\theta}^*)) - c(\boldsymbol{Q^*}(\bar{\theta}^*), \bar{\theta}^*) = 0$. Lemma 2 implies that Q^{m*} is decreasing in θ. Hence, $Q^{m*} : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}$ can have no more than countably many points of discontinuity. Let $\{\theta^1, \theta^2, ..., \theta^l, ...\}$ be the discontinuous points of $\boldsymbol{Q^*}$ in $[\underline{\theta}, \bar{\theta}^*]$. For each l with $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$, there exist some m such that $Q^{m*}(\theta^l+) \equiv \lim_{\theta\to\theta^l+} Q^{m*}(\theta) \leq Q^{m*}(\theta^l) \leq \lim_{\theta\to\theta^l-} Q^{m*}(\theta) \equiv Q^{m*}(\theta^l-)$ with either or both inequalities being strict.

First, we define a function $\sigma : [\underline{\theta}, \bar{\theta}^*] \to \mathbb{R}$: For each $\theta \in [\underline{\theta}, \bar{\theta}^*]$,

$$\sigma(\theta) = c(\boldsymbol{Q^*}(\theta), \theta) + \int_\theta^{\bar{\theta}^*} c_\theta(\boldsymbol{Q^*}(z), z)dz.$$

Note that if $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta')$, and thus $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta'')$ for each $\theta'' \in [\theta, \theta']$, then $\sigma(\theta) = \sigma(\theta')$. Using the function σ, we construct s^* in the following way. (a) If $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\bar{\theta}^*)$, then let $s^*(\boldsymbol{q}) = 0$. (b) If there exists $\theta' \in (\underline{\theta}, \bar{\theta}^*) \setminus \{\theta^1, \theta^2, ..., \theta^l, ...\}$ or $\theta' \in \{\underline{\theta}, \bar{\theta}^*\}$ such that $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta')$ and there exists no $\theta < \theta'$ which satisfies $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta) > \boldsymbol{Q}^*(\theta')$, then let $s^*(\boldsymbol{q}) = \sigma(\theta')$.[5] (c) If there exists l such that $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$, $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta^l+)$ and $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\theta^l)$, then let $s^*(\boldsymbol{q}) = \sigma(\theta^l+)$. (d) If there exists l such that $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$, $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta^l)$ and

[5] For some $\boldsymbol{q}$, there may exist another $\theta(\neq \theta')$ which satisfies the condition. However, because it then follows that $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta')$, the equality $s^*(\boldsymbol{q}) = \sigma(\theta) = \sigma(\theta')$ holds.

$\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\theta^l-)$, then let $s^*(\boldsymbol{q}) = \sigma(\theta^l)$. (e) If there exists l such that $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$ and there exists no $\theta < \theta^l$ which satisfies $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta) > \boldsymbol{Q}^*(\theta^l-)$, then let $s^*(\boldsymbol{q}) = \sigma(\theta^l-)$.

Second, we show that $s^*(\boldsymbol{Q}^*(\bar{\theta}^*)) - c(\boldsymbol{Q}^*(\bar{\theta}^*), \bar{\theta}^*) = 0$ and s^* is upper semicontinuous on $\mathcal{Q}$. The former is trivial because $s^*(\boldsymbol{Q}^*(\bar{\theta}^*)) = \sigma(\bar{\theta}^*) = c(\boldsymbol{Q}^*(\bar{\theta}^*), \bar{\theta}^*)$. By construction, the function σ is continuous in $[\underline{\theta}, \bar{\theta}^*] \setminus \{\theta^1, \theta^2, ..., \theta^l, ...\}$. Also, $\sigma(\theta) = \sigma(\theta')$ for each $\theta, \theta' \in [\underline{\theta}, \bar{\theta}^*]$ with $\theta < \theta'$ if $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta')$ and thus $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta'') = \boldsymbol{Q}^*(\theta')$ for each $\theta'' \in [\theta, \theta']$. Hence, for each $\boldsymbol{q} \in \mathcal{Q}$ and $\epsilon > 0$, there exists a neighborhood of $\boldsymbol{q}$ in which $s^*(\boldsymbol{q'}) \leq s^*(\boldsymbol{q}) + \epsilon$. This means that the function s^* is upper semicontinuous on $\mathcal{Q}$.

Finally, we show that $\boldsymbol{Q}^*(\theta) \in \arg\max_{\boldsymbol{q} \in \mathcal{Q}}[s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$. By construction of s^*, $\boldsymbol{q} \notin \{\boldsymbol{Q}^*(\theta) \mid \theta \in [\underline{\theta}, \bar{\theta}^*]\} \cup \{\boldsymbol{Q}^*(\theta^l+) \mid l = 1, 2, ...\} \cup \{\boldsymbol{Q}^*(\theta^l-) \mid l = 1, 2, ...\}$ cannot maximize $s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)$ for each $\theta \in [\underline{\theta}, \bar{\theta}^*]$ because $s^*(\boldsymbol{Q}^*(\theta)) - c(\boldsymbol{Q}^*(\theta), \theta) \geq 0$ whereas $s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta) < 0$ for each $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\bar{\theta}^*)$, and c is strictly increasing in $\boldsymbol{q}$. Hence, it suffices to show that for each $\theta, \hat{\theta} \in [\underline{\theta}, \bar{\theta}^*]$, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\hat{\theta}) - c(\boldsymbol{Q}^*(\hat{\theta}), \theta)$, and for each l, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\theta^l+) - c(\boldsymbol{Q}^*(\theta^l+), \theta)$, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\theta^l-) - c(\boldsymbol{Q}^*(\theta^l-), \theta)$. First, for each $\theta \in [\underline{\theta}, \bar{\theta}^*]$, $\hat{\theta} = \theta$ maximizes $\sigma(\hat{\theta}) - c(\boldsymbol{Q}^*(\hat{\theta}), \theta)$ because for each $\hat{\theta} \neq \theta$,

$$
\begin{aligned}
&[\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta)] - [\sigma(\hat{\theta}) - c(\boldsymbol{Q}^*(\hat{\theta}), \theta)] \\
&= \int_{\theta}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz - \left[c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz - c(\boldsymbol{Q}^*(\hat{\theta}), \theta)\right] \\
&= \int_{\theta}^{\hat{\theta}} [c_\theta(\boldsymbol{Q}^*(z), z) - c_\theta(\boldsymbol{Q}^*(\hat{\theta}), z)]dz \geq 0.
\end{aligned}
$$

The inequality holds because Lemma 2 implies that $\boldsymbol{Q}^*(z) \geq \boldsymbol{Q}^*(\hat{\theta})$ for each $z < \hat{\theta}$, and the assumption that c has (strictly) increasing differences in $(\boldsymbol{q}, \theta)$ implies that $c_\theta(\boldsymbol{Q}^*(z), z) \geq c_\theta(\boldsymbol{Q}^*(\hat{\theta}), z)$. Second, for each $\theta \in [\underline{\theta}, \bar{\theta}^*]$ and l, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq$

$\sigma(\theta^l+) - c(\boldsymbol{Q}^*(\theta^l+), \theta)$ because

$$\begin{aligned}
&[\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta)] - [\sigma(\theta^l+) - c(\boldsymbol{Q}^*(\theta^l+), \theta)] \\
&= \int_\theta^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz - \left[\lim_{\hat{\theta}\to\theta^l+} c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta}) + \lim_{\hat{\theta}\to\theta^l+} \int_{\hat{\theta}}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz - c(\boldsymbol{Q}^*(\theta^l+), \theta)\right] \\
&= \int_\theta^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz - \left[c(\boldsymbol{Q}^*(\theta^l+), \theta^l) + \int_{\theta^l}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz - c(\boldsymbol{Q}^*(\theta^l+), \theta)\right] \\
&= \int_\theta^{\theta^l} [c_\theta(\boldsymbol{Q}^*(z), z) - c_\theta(\boldsymbol{Q}^*(\theta^l+), z)]dz \geq 0.
\end{aligned}$$

The first equality follows from the construction of σ, the second equality follows from the continuity of c in $(\boldsymbol{q}, \theta)$, and the inequality holds because Lemma 2 and Assumption 3 imply that $c_\theta(\boldsymbol{Q}^*(z), z) \geq c_\theta(\boldsymbol{Q}^*(\theta^l+), z)$ for each $z \leq \theta^l$. Similarly, we can show that $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\theta^l-) - c(\boldsymbol{Q}^*(\theta^l-), \theta)$.

(ii) We now show that the function s^* is supermodular in quality. First, we show that s^* is increasing in $\boldsymbol{q}$. Note that σ is decreasing in θ because for each θ, θ' with $\theta < \theta'$,

$$\begin{aligned}
\sigma(\theta) - \sigma(\theta') &= c(\boldsymbol{Q}^*(\theta), \theta) + \int_\theta^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz - c(\boldsymbol{Q}^*(\theta'), \theta') - \int_{\theta'}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz \\
&\geq c(\boldsymbol{Q}^*(\theta'), \theta) - c(\boldsymbol{Q}^*(\theta'), \theta') + \int_\theta^{\theta'} c_\theta(\boldsymbol{Q}^*(z), z)dz \\
&= \int_\theta^{\theta'} [c_\theta(\boldsymbol{Q}^*(z), z) - c_\theta(\boldsymbol{Q}^*(\theta'), z)]dz \geq 0.
\end{aligned}$$

Consider any $\boldsymbol{q}, \boldsymbol{q'} \in \mathcal{Q}$ with $\boldsymbol{q'} > \boldsymbol{q}$. If $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\bar{\theta}^*)$, then $s^*(\boldsymbol{q'}) \geq s^*(\boldsymbol{q}) = 0$. If $\boldsymbol{q} \geq \boldsymbol{Q}^*(\bar{\theta}^*)$ and $\boldsymbol{q'} \not\geq \boldsymbol{Q}^*(\underline{\theta})$, then $s^*(\boldsymbol{q'}) \geq s^*(\boldsymbol{q})$ because there exist θ and θ' with $\theta \geq \theta'$ such that $s^*(\boldsymbol{q})$ is equal to $\sigma(\theta-)$, $\sigma(\theta)$ or $\sigma(\theta+)$ and $s^*(\boldsymbol{q'})$ is equal to $\sigma(\theta'-)$, $\sigma(\theta')$ or $\sigma(\theta'+)$; note that $s^*(\boldsymbol{q}) = \sigma(\theta'-)$ implies that $s^*(\boldsymbol{q'}) \geq \sigma(\theta'-)$, and $s^*(\boldsymbol{q}) = \sigma(\theta')$ implies that $s^*(\boldsymbol{q'}) \geq \sigma(\theta')$. If $\boldsymbol{q'} \geq \boldsymbol{Q}^*(\underline{\theta})$, then $s^*(\boldsymbol{q'}) = \sigma(\underline{\theta}) \geq s^*(\boldsymbol{q})$.

Second, we show that the function s^* is supermodular in $\boldsymbol{q}$. Fix any unordered pair $\boldsymbol{q}$ and $\hat{\boldsymbol{q}}$ in $\mathcal{Q}$, so that $q^m > \hat{q}^m$ and $q^{\hat{m}} < \hat{q}^{\hat{m}}$ for some $m, \hat{m}$. Because s^* is increasing in $\boldsymbol{q}$, $s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) \geq s^*(\hat{\boldsymbol{q}})$ and $s^*(\boldsymbol{q}) \geq s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. Thus, if $s^*(\boldsymbol{q}) = s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$, then $s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s^*(\hat{\boldsymbol{q}}) \geq 0 = s^*(\boldsymbol{q}) - s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. We claim that if $s^*(\boldsymbol{q}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$, then $s^*(\hat{\boldsymbol{q}}) = s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. Suppose, on the contrary, that $s^*(\boldsymbol{q}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$ and $s^*(\hat{\boldsymbol{q}}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. Then, by construction of s^*, there exist $m, \hat{m}, \theta, \hat{\theta}$ such that $q^m \geq Q^{m*}(\theta) > \min\{q^m, \hat{q}^m\}$ with $q^{\hat{m}} \geq Q^{\hat{m}*}(\theta)$, and $\hat{q}^{\hat{m}} \geq Q^{\hat{m}*}(\hat{\theta}) > \min\{q^{\hat{m}}, \hat{q}^{\hat{m}}\}$ with $\hat{q}^m \geq Q^{m*}(\hat{\theta})$. This contradicts the result that $\boldsymbol{Q^*}$ is increasing in θ because $Q^{m*}(\theta) > Q^{m*}(\hat{\theta})$ and $Q^{\hat{m}*}(\theta) < Q^{\hat{m}*}(\hat{\theta})$, so that $\boldsymbol{Q^*}(\theta)$ and $\boldsymbol{Q^*}(\hat{\theta})$ are unordered. Thus, if $s^*(\boldsymbol{q}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$, then $s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s^*(\hat{\boldsymbol{q}}) = s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) \geq s^*(\boldsymbol{q}) - s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. This completes the proof. □

Proof of Proposition 2. (i) Suppose that $Q^{m*}(\theta-) > Q^{m*}(\theta+)$ and $Q^{m'*}(\theta-) > Q^{m'*}(\theta+)$ with $(m, m') \in \mathcal{M}^1 \times \mathcal{M}^2$ and $\theta \in (\underline{\theta}, \bar{\theta}^*)$. Then, $(Q^{m*}(\theta-))_{m\in\mathcal{M}^1} > (Q^{m*}(\theta+))_{m\in\mathcal{M}^1}$ and $(Q^{m*}(\theta-))_{m\in\mathcal{M}^2} > (Q^{m*}(\theta+))_{m\in\mathcal{M}^2}$. Because c has increasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ and strictly increasing differences in $(q^m, q^{m'})$, we obtain $c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{Q}}, \theta) > c(\boldsymbol{Q}, \theta) - c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta)$, where $\boldsymbol{Q} = ((Q^{m*}(\theta+))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta-))_{m\in\mathcal{M}^2})$ and $\hat{\boldsymbol{Q}} = ((Q^{m*}(\theta-))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta+))_{m\in\mathcal{M}^2})$, so that $\boldsymbol{Q} \vee \hat{\boldsymbol{Q}} = \boldsymbol{Q^*}(\theta-)$ and $\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}} = \boldsymbol{Q^*}(\theta+)$. Let ϵ be a positive real number such that $2\epsilon < [c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{Q}}, \theta)] - [c(\boldsymbol{Q}, \theta) - c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta)]$. Fix any θ', θ'' with $\theta'' < \theta < \theta'$ such that $[c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta) - c(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta')] - [c(\boldsymbol{Q}, \theta) - c(\boldsymbol{q}, \theta')] < \epsilon$ and $[c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta'')] - [c(\hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{q}}, \theta'')] < \epsilon$, where $\boldsymbol{q} = ((Q^{m*}(\theta'))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta''))_{m\in\mathcal{M}^2})$ and $\hat{\boldsymbol{q}} = ((Q^{m*}(\theta''))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta'))_{m\in\mathcal{M}^2})$, so that $\boldsymbol{q} \vee \hat{\boldsymbol{q}} = \boldsymbol{Q^*}(\theta'')$ and $\boldsymbol{q} \wedge \hat{\boldsymbol{q}} = \boldsymbol{Q^*}(\theta')$. The existence of such θ' and θ'' is guaranteed by the continuity of c in $(\boldsymbol{q}, \theta)$.

Suppose that a scoring rule s implements the optimal mechanism. Then, it follows from Lemma 1 that the inequality $s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta') \leq s(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) - c(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta')$ must hold because $\boldsymbol{q} \wedge \hat{\boldsymbol{q}} = \boldsymbol{Q}^*(\theta')$ whereas $\boldsymbol{q} \neq \boldsymbol{Q}^*(\theta')$. Hence, we obtain

$$\begin{aligned} s(\boldsymbol{q}) - s(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) &\leq c(\boldsymbol{q}, \theta') - c(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta') \\ &< c(\boldsymbol{Q}, \theta) - c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta) + \epsilon \\ &< c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{Q}}, \theta) - \epsilon \\ &< c(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta'') - c(\hat{\boldsymbol{q}}, \theta''). \end{aligned}$$

Now, if the scoring rule s is additively separable, then $s(\boldsymbol{q}) - s(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) = s^2(\boldsymbol{q^2}) - s^2(\hat{\boldsymbol{q}}^{\boldsymbol{2}}) = s(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s(\hat{\boldsymbol{q}})$ because $\boldsymbol{q^1} = \boldsymbol{q^1} \wedge \hat{\boldsymbol{q}}^{\boldsymbol{1}} = (Q^{m*}(\theta'))_{m \in \mathcal{M}^1}$ and $\hat{\boldsymbol{q}}^{\boldsymbol{1}} = \boldsymbol{q^1} \vee \hat{\boldsymbol{q}}^{\boldsymbol{1}} = (Q^{m*}(\theta''))_{m \in \mathcal{M}^1}$. Then, $s(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s(\hat{\boldsymbol{q}}) < c(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta'') - c(\hat{\boldsymbol{q}}, \theta'')$, and thus a supplier of type θ'' never chooses $\boldsymbol{q} \vee \hat{\boldsymbol{q}} = \boldsymbol{Q}^*(\theta'')$ from Lemma 1. Therefore, there is no additively separable scoring rule which implements the optimal mechanism.

(ii) It follows from Lemma 1 that Q^{m*} is decreasing in θ. Because $\mathcal{Q}^m$ is finite, $Q^{m*} : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}$ can have no more than finitely many points of discontinuity. Suppose, without loss of generality, that the discontinuous points of $\boldsymbol{Q}^*$ in $[\underline{\theta}, \bar{\theta}^*]$ are indexed in decreasing order, i.e. $\theta^1 > \theta^2 > ... > \theta^L$. Note that $\boldsymbol{Q}^*(\theta^{l+1}+) = \boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta^l-)$ for each $\theta \in (\theta^{l+1}, \theta^l)$, and $Q^{m*}(\theta^l-) \geq Q^{m*}(\theta^l+)$. Assume for simplicity that $\bar{\theta} > \theta^1$ and $\theta^L > \underline{\theta}$; without this assumption, the proof proceeds with some notational complexity.

First, we define functions $\sigma^1 : [\underline{\theta}, \bar{\theta}^*] \to \mathbb{R}$ and $\sigma^2 : [\underline{\theta}, \bar{\theta}^*] \to \mathbb{R}$: For each $\theta \in [\underline{\theta}, \bar{\theta}^*]$ such that $\theta \in (\theta^{l+1}, \theta^l]$ for some l or $\theta \in [\underline{\theta}, \theta^l]$ for $l = L$,

$$\begin{aligned} \sigma^1(\theta) = \frac{1}{2} c(\boldsymbol{Q}^*(\bar{\theta}^*), \bar{\theta}^*) &+ \sum_{r=1}^{l-1} [c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta^r) - c(\boldsymbol{Q}^*(\theta^r+), \theta^r)] \\ &+ [c((Q^{m*}(\theta))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^l+))_{m \in \mathcal{M}^2}, \theta^l) - c((Q^{m*}(\theta^l+))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^l+))_{m \in \mathcal{M}^2}, \theta^l)], \\ \sigma^2(\theta) = \frac{1}{2} c(\boldsymbol{Q}^*(\bar{\theta}^*), \bar{\theta}^*) &+ \sum_{r=1}^{l-1} [c(\boldsymbol{Q}^*(\theta^r-), \theta^r) - c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta^r)] \\ &+ [c((Q^{m*}(\theta^l-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta^l) - c((Q^{m*}(\theta^l-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^l+))_{m \in \mathcal{M}^2}, \theta^l)], \end{aligned}$$

and for each $\theta \in (\theta^1, \bar{\theta}^*]$, $\sigma^1(\theta) = \sigma^2(\theta) = \frac{1}{2}c(\boldsymbol{Q^*}(\bar{\theta}^*), \bar{\theta}^*)$. Using these functions, we construct a function $s^a : \times_{m\in\mathcal{M}^a}\mathcal{Q}^m \to \mathbb{R}$ for each $a = 1, 2$ in the following way. (a) If $\boldsymbol{q^a} \not\geq (Q^{m*}(\bar{\theta}^*))_{m\in\mathcal{M}^a}$, then let $s^a(\boldsymbol{q^a}) = -\max_{\boldsymbol{q}\in\mathcal{Q}} c(\boldsymbol{q}, \bar{\theta})$. (b) If there exists $\theta' \in [\underline{\theta}, \bar{\theta}^*]$ such that $\boldsymbol{q^a} \geq (Q^{m*}(\theta'))_{m\in\mathcal{M}^a}$ and there exists no $\theta < \theta'$ which satisfies $\boldsymbol{q^a} \geq (Q^{m*}(\theta))_{m\in\mathcal{M}^a} > (Q^{m*}(\theta'))_{m\in\mathcal{M}^a}$, then let $s^a(\boldsymbol{q^a}) = \sigma^a(\theta')$. It is easy to show that $s^1((Q^{m*}(\bar{\theta}^*))_{m\in\mathcal{M}^1}) + s^2((Q^{m*}(\bar{\theta}^*))_{m\in\mathcal{M}^2}) - c(\boldsymbol{Q^*}(\bar{\theta}^*), \bar{\theta}^*) = 0$. Moreover, the function $\underline{s} + \bar{s}$ is trivially upper semicontinuous on $\mathcal{Q}$ because $\mathcal{Q}$ is finite.

Next, we show that $\boldsymbol{Q^*}(\theta) \in \arg\max_{\boldsymbol{q}\in\mathcal{Q}}[s^1(\boldsymbol{q^1}) + s^2(\boldsymbol{q^2}) - c(\boldsymbol{q}, \theta)]$. By construction of s^1 and s^2, $\boldsymbol{q} \notin \{(Q^{m*}(\theta))_{m\in\mathcal{M}^1} \mid \theta \in [\underline{\theta}, \bar{\theta}^*]\} \times \{(Q^{m*}(\theta))_{m\in\mathcal{M}^2} \mid \theta \in [\underline{\theta}, \bar{\theta}^*]\}$ cannot be the maximizer. Hence, it suffices to show that for each $\theta, \theta', \theta'' \in [\underline{\theta}, \bar{\theta}^*]$, $\sigma^1(\theta) + \sigma^2(\theta) - c(\boldsymbol{Q^*}(\theta), \theta) \geq \sigma^1(\theta') + \sigma^2(\theta'') - c((Q^{m*}(\theta'))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta''))_{m\in\mathcal{M}^2}, \theta)$. Suppose first that $\theta \in (\theta^{l+1}, \theta^l)$ and $\theta' \in (\theta^{l'+1}, \theta^{l'})$ with $l \geq l'$. Then, we obtain

$$
\begin{aligned}
&\sigma^1(\theta) - \sigma^1(\theta') \\
&= \sum_{r=l'+1}^{l} [c((Q^{m*}(\theta^r-))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m\in\mathcal{M}^2}, \theta^r) - c(\boldsymbol{Q^*}(\theta^r+), \theta^r)] \\
&\geq \sum_{r=l'+1}^{l} [c((Q^{m*}(\theta^r-))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta))_{m\in\mathcal{M}^2}, \theta^r) - c((Q^{m*}(\theta^r+))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta))_{m\in\mathcal{M}^2}, \theta^r)] \\
&\geq \sum_{r=l'+1}^{l} [c((Q^{m*}(\theta^r-))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta))_{m\in\mathcal{M}^2}, \theta) - c((Q^{m*}(\theta^r+))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta))_{m\in\mathcal{M}^2}, \theta)] \\
&= c(\boldsymbol{Q^*}(\theta), \theta) - c((Q^{m*}(\theta'))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta))_{m\in\mathcal{M}^2}, \theta),
\end{aligned}
$$

where the first equality follows from the observation that $Q^{m*}(\theta^l-) = Q^{m*}(\theta)$ and $Q^{m*}(\theta^{l'}-) = Q^{m*}(\theta')$, the first inequality follows from the hypothesis that c has decreasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with the observation that $Q^{m*}(\theta^r+) \leq Q^{m*}(\theta)$ for each $r \in \{l'+1, ..., l\}$, the second inequality follows from the assumption that c has (strictly) increasing differences in $(\boldsymbol{q}, \theta)$, the last equality follows from the observation that $Q^{m*}(\theta^r-) =$

$Q^{m*}(\theta^{r+1}+)$, $Q^{m*}(\theta^{l}-) = Q^{m*}(\theta)$ and $Q^{m*}(\theta^{l'+1}+) = Q^{m*}(\theta')$. Suppose next that $\theta'' \in (\theta^{l''+1}, \theta^{l''})$ with $l \leq l''$. Then, the same argument as above yields

$$\begin{aligned}
&\sigma^2(\theta) - \sigma^2(\theta'') \\
&= -\sum_{r=l+1}^{l''} [c(\boldsymbol{Q^*}(\theta^r-), \theta^r) - c((Q^{m*}(\theta^r-))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m\in\mathcal{M}^2}, \theta^r)] \\
&\geq -\sum_{r=l+1}^{l''} [c((Q^{m*}(\theta))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta^r-))_{m\in\mathcal{M}^2}, \theta^r) - c((Q^{m*}(\theta))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m\in\mathcal{M}^2}, \theta^r)] \\
&\geq -\sum_{r=l+1}^{l''} [c((Q^{m*}(\theta))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta^r-))_{m\in\mathcal{M}^2}, \theta) - c((Q^{m*}(\theta))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m\in\mathcal{M}^2}, \theta)] \\
&= -[c((Q^{m*}(\theta))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta''))_{m\in\mathcal{M}^2}, \theta) - c(\boldsymbol{Q^*}(\theta), \theta)] \\
&\geq -[c((Q^{m*}(\theta'))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta''))_{m\in\mathcal{M}^2}, \theta) - c((Q^{m*}(\theta'))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta))_{m\in\mathcal{M}^2}, \theta)],
\end{aligned}$$

where the last inequality follows from the hypothesis that c has decreasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with the observation that $Q^{m*}(\theta') \leq Q^{m*}(\theta)$. Therefore, we obtain

$$[\sigma^1(\theta) - \sigma^1(\theta')] + [\sigma^2(\theta) - \sigma^2(\theta'')] \geq c(\boldsymbol{Q^*}(\theta), \theta) - c((Q^{m*}(\theta'))_{m\in\mathcal{M}^1}, (Q^{m*}(\theta''))_{m\in\mathcal{M}^2}, \theta),$$

for each $\theta, \theta', \theta'' \in [\underline{\theta}, \bar{\theta}^*]$ such that $\theta \in (\theta^{l+1}, \theta^{l})$, $\theta' \in (\theta^{l'+1}, \theta^{l'})$ and $\theta'' \in (\theta^{l''+1}, \theta^{l''})$ with $l'' \geq l \geq l'$. A similar argument applies to the other combinations of θ, θ' and θ''. □

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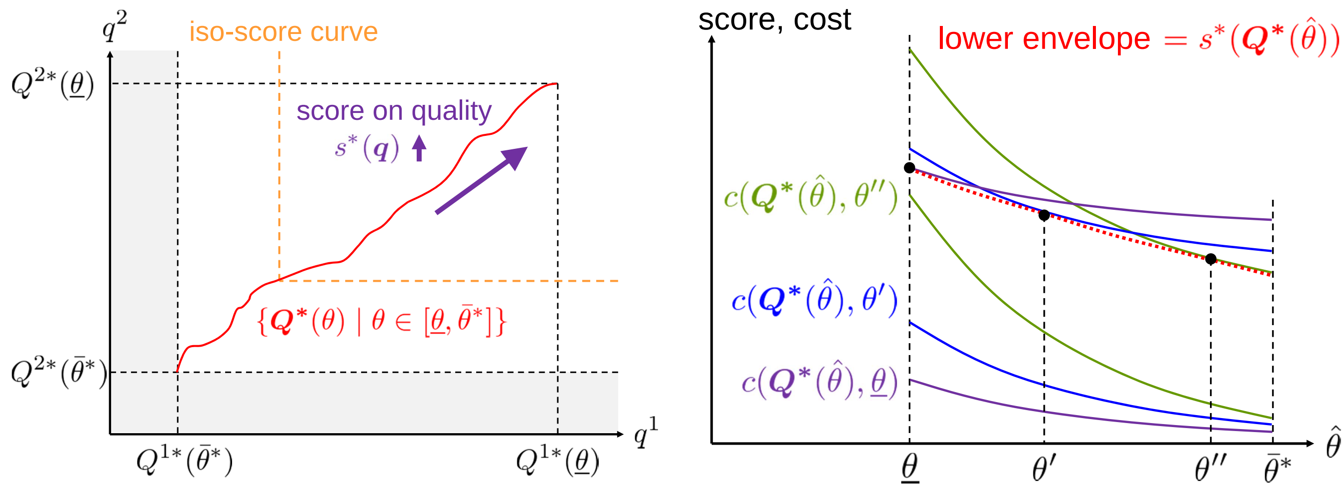

Figure 1: Illustration of Proposition 1