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# News Impact Curve for Stochastic Volatility Models

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**Abstract**

This paper proposes a new method to compute the news impact curve for stochastic volatility (SV) models. The new method incorporates the joint movement of return and volatility, which has been ignored by the extant literature, by simply adding a couple of steps to the Bayesian MCMC estimation procedures for SV models. This simple procedure is versatile and applicable to various SV type models. Contrary to the monotonic news impact functions in the extant literature, the new method gives a U-shaped news impact curve comparable to GARCH models. It also captures the volatility asymmetry for the asymmetric SV models.

## 1 Introduction

Modeling and forecasting financial asset volatility has attracted many researchers and practitioners since the seminal work of Engle (1982) that proposed the autoregressive conditional heteroskedasticity (ARCH) model. Bollerslev (1986) proposed the generalized ARCH (GARCH) model and a number of extensions including asymmetric GARCH models such as the exponential GARCH (EGARCH) model of Nelson (1991) and the GJR model of Glosten, Jagannathan and Runkle (1993) have followed. These models have various specifications on volatility dynamics which imply different impact of past return shocks, or information, on the return volatility.

Engle and Ng (1993) define the news impact curve which measures how the new information affects the return volatility in the context of GARCH models. In GARCH models, today's volatility is a function of observations up to yesterday and therefore today's news shock is a change of today's return not explained by the estimated today's volatility. With today's volatility fixed, typically at unconditional volatility, a plot of tomorrow's volatility against

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today's news shock shows well known U-shaped news impact curve. The news impact curve also reflects the volatility asymmetry or leverage effect (negative shock yields higher volatility than positive shock) for asymmetric GARCH models such as the EGARCH and GJR models.

The news impact curve for stochastic volatility (SV) models has been defined similarly in the extant literature. The news impact function is typically defined as the expectation of tomorrow's volatility conditional on today's return with today's volatility fixed at unconditional volatility. Contrary to the U-shaped news impact curve for GARCH models, this news impact function is a flat line for symmetric SV models and downward-sloping curve for asymmetric SV models.

The monotonic news impact curve, instead of the U-shaped curve, for SV models is due to the different specification of volatility process. Contrary to GARCH models, SV models treat today's volatility as a latent variable and thus a change of today's return can be due to either a change of volatility or news shock, or both. Therefore, it is problematic to define the news impact function with today's volatility fixed as in GARCH models.

Considering the joint move of today's volatility and news shock, this paper proposes a new method to compute the news impact curve for SV models. The new method simply adds a couple of steps to the Markov chain Monte Carlo (MCMC) estimation procedures for SV models. This simple procedure is versatile and applicable to the various SV extensions such as realized SV models recently proposed by Takahashi, Omori and Watanabe (2009) and Koopman and Scharth (2012). An empirical example with Spyder, the S&P 500 exchange-traded fund, shows that the new method gives a U-shaped news impact curve comparable to GARCH models and also captures the asymmetry for the asymmetric SV (ASV) models.

The rest of this paper is organized as follows. Next section illustrates the problem in the traditional method to compute the news impact curve for SV models. Section 3 proposes the new method. Then, we demonstrate the news impact curve with actual daily returns of Spyder in Section 4. The final section concludes.

## 2 News Impact Curve

To illustrate a news impact curve, consider an asset return,

$$r_t = \sigma_t \epsilon_t, \quad \epsilon_t \sim N(0,1), \tag{1}$$

where $r_t$ is the asset return and we call $\sigma_t^2$ volatility in this paper.

Engle and Ng (1993) define the news impact function as a relation between $r_t$ and $\sigma_{t+1}^2$, implied by a volatility specification, with all lagged conditional variances evaluated at the level of the unconditional variance of the asset return, $\sigma^2$. GARCH models specify $\sigma_{t+1}^2$ as a function of the information up to $t$. Since $\sigma_t^2$ is known at $t-1$, a change of $r_t$ is solely due to a change of $\epsilon_t$. This feature of GARCH models justifies the news impact function with lagged conditional variances fixed at $\sigma^2$. For example, the GARCH(1,1) model specifies the volatility as follows.

$$\sigma_{t+1}^2 = \omega + \beta\sigma_t^2 + \alpha r_t^2, \tag{2}$$

where it is assumed that $\omega > 0$, $\beta \geq 0$ and $\alpha \geq 0$ to assure that the volatility $\sigma_t^2$ is always positive and that $|\alpha + \beta| < 1$ to guarantee that the volatility is stationary. The news impact function is then

$$\sigma_{t+1}^2 = \omega + \beta\sigma^2 + \alpha r_t^2. \tag{3}$$

This implies the well known U-shaped news impact curve.

Under SV models, however, $\sigma_t^2$ is a latent variable and hence it is unknown at $t-1$. For example, consider the following standard SV model.

$$h_{t+1} = \mu + \phi(h_t - \mu) + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2), \quad cov[\epsilon_t, \eta_t] = \rho\sigma_\eta^2, \tag{4}$$

where $h_t = \log\sigma_t^2$, $|\phi| < 1$ for a stationary process, and $\rho$ captures the volatility asymmetry. If $\rho = 0$, this model becomes the symmetric SV model. If $\rho < 0$, it is consistent with the volatility asymmetry or leverage effect of stock returns observed in stock markets.

Following Yu (2005), we define the news impact function for SV models as a relation between $r_t$ and $h_{t+1}$ in this section while we also consider the relation between $r_t$ and $\sigma_{t+1}^2$ in the next section. Contrary to GARCH models, a change of $r_t$ is due to a change of either $\epsilon_t$ or $h_t$ or both. This implies a stochastic relation between $r_t$ and $h_{t+1}$ instead of the deterministic relation in GARCH models. This relation can be expressed as a conditional expectation of $h_{t+1}$,

$$\begin{aligned} E[h_{t+1}|r_t] &= \mu + \phi(E[h_t|r_t] - \mu) + E[\eta_t|r_t] \\ &= \mu + \phi(E[h_t|r_t] - \mu) + \rho\sigma_\eta r_t E[\exp(-h_t/2)|r_t]. \end{aligned} \tag{5}$$

Replacing the conditional expectations with the unconditional expectations yields the following news impact function,[1]

$$E[h_{t+1}|r_t] \approx \mu + \rho\sigma_\eta \exp\left\{-\frac{\mu}{2} + \frac{\sigma_\eta^2}{8(1-\phi^2)}\right\} r_t. \tag{6}$$

If $\rho = 0$, this is a flat line. If $\rho < 0$, this is a downward sloping line.

[1]See, e.g., Yu (2005) and Asai and McAleer (2009) for other approximation methods.

Such a monotonic news impact line is due to ignoring the dependence between $r_t$ and $h_t$ and replacing the conditional expectations with the unconditional ones. If $E[h_t|r_t]$ is increasing in the absolute return $|r_t|$, the conditional expectation in (5) implies a non-monotonic news impact curve. Thus, incorporating the joint distribution of $r_t$ and $h_t$ may give the U-shaped news impact curve.

In the next section, we propose a new method which incorporates the joint movement to compute the news impact curve. Instead of directly computing the conditional expectations, $E[h_t|r_t]$ and $E[\exp(-h_t/2)|r_t]$, we take a simulation based approach via the MCMC estimation scheme. The new method does not require possibly complicated conditional distributions but only a stationary distribution of $h$ and hence it is versatile to various SV specifications.

# 3 New Method

We illustrate our new method to compute a news impact curve for the standard SV model given by equations (1) and (4). We assume that $|\phi| < 1$ for a stationary log-volatility process, $h_0 = \mu$, and

$$\eta_0 \sim N\left(0, \frac{\sigma_\eta^2}{1-\phi^2}\right). \tag{7}$$

We incorporate the joint movement of $h_t$ and $r_t$ (or $\epsilon_t$), by adding a couple of steps to the MCMC estimation scheme for SV models. For each parameter sample generated in the MCMC estimation, we implement the following steps.

1. Generate $h$ from its stationary distribution,

$$h \sim N\left(\mu, \frac{\sigma_\eta^2}{1-\phi^2}\right), \tag{8}$$

and $\epsilon$ from the standard normal distribution, $\epsilon \sim N(0,1)$.

2. Compute daily return,

$$r = \epsilon \exp(h/2), \tag{9}$$

and one day ahead log-volatility forecast,

$$\hat{h} = \mu + \phi(h-\mu) + \rho\sigma_\eta\epsilon. \tag{10}$$

With the generated samples of the daily return ($r$) and log-volatility forecast ($\hat{h}$), we can estimate the news impact curve defined as a relation between $r_t$ and $h_{t+1}$ by, for example, the local linear Gaussian kernel regression. We can also estimate the news impact curve defined as a relation between $r_t$ and $\sigma^2_{t+1}$ simply by transforming the generated samples of the log-volatility forecast ($\hat{h}$) to those of the volatility forecast ($\exp(\hat{h})$).

## 4 Empirical Illustration

This section gives an empirical example of the news impact curve proposed in the previous section. We use 1758 samples of daily returns for Spyder, the S&P 500 exchange-traded fund, from February 1, 2001 to January 31, 2007. Table 1 shows descriptive statistics of the daily return, its square, and the logarithm of the squared return.

We estimate the symmetric and asymmetric SV models using the Bayesian MCMC estimation method with the following prior distributions,

$$\mu \sim N(-0.1, 1), \quad \frac{\phi+1}{2} \sim Beta(20, 1.5), \quad \sigma_\eta^{-2} \sim Gamma(2.5, 0.025), \quad \frac{\rho+1}{2} \sim Beta(1, 1).$$

To sample $h$ efficiently, we employ the estimation procedure with the block sampler of Watanabe and Omori (2004) for the SV model and Omori and Watanabe (2008) for the ASV model. Table 2 shows the MCMC estimation results, which are consistent with the extant literature. For example, the posterior mean of $\phi$ is close to one, which implies the high persistence of volatility. Additionally, the posterior mean of $\rho$ is negative and its 95% interval does not contain zero, which indicates the well known volatility asymmetry.

With the MCMC estimation procedures, we compute the news impact curves. Figure 1 shows the news impact curves defined as a relation between $r_t$ and $h_{t+1}$ by the conventional and new methods. Figure 2 shows the news impact curves defined as a relation between $r_t$ and $\sigma^2_{t+1}$. In the both figures, the new method gives the familiar U-shaped news impact curves, comparable to GARCH models, while the conventional method gives the flat or downward-sloping lines. The both figures also show that the new method captures the volatility asymmetry for the asymmetric SV models.

## 5 Conclusion

This paper proposes a new method to compute the news impact curve for SV models. The new method incorporates the joint movement of return and volatility, which has been ignored by the extant literature, by simply adding a couple of steps to the Bayesian MCMC estimation procedures for SV models. Empirical results with Spyder, the S&P 500 exchange-traded fund, show that the new method gives the familiar U-shaped news impact curves and captures the volatility asymmetry.

Although we illustrate the new method with a simple SV model, the method is versatile and easily applicable to various SV models. For example, it is straightforward to compute a news impact curve for the SV model of Nakajima and Omori (2012) where a more general distribution is assumed for $\epsilon_t$ and the realized SV model of Takahashi, Omori and Watanabe (2009) which specifies daily returns and realized volatility measures jointly. The simple computational method also enables us to utilize the news impact curve for volatility predictions as in Chen and Ghysels (2011).

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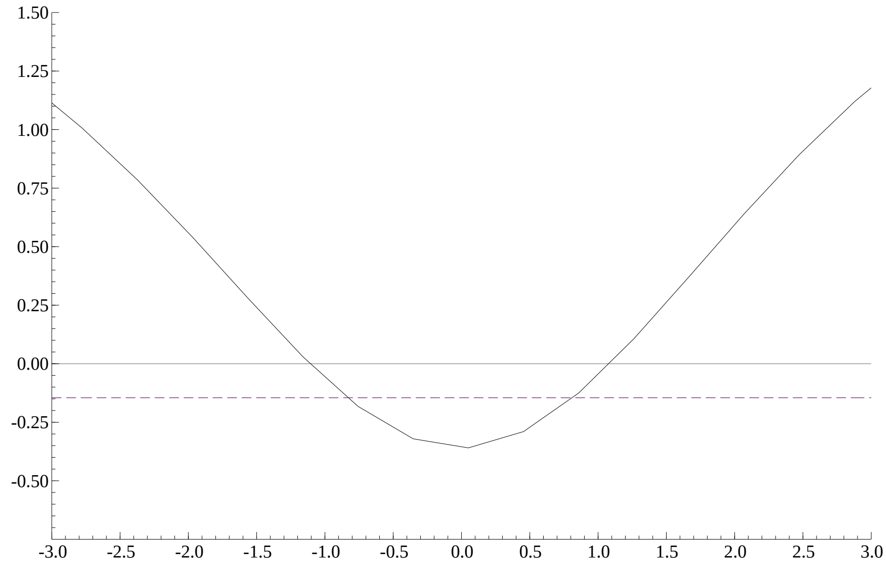
| Variable | Mean | SE | SD | Skew | Kurt | Min | Max | JB | LB(10) |
|---|---|---|---|---|---|---|---|---|---|
| $r$ | 0.0094 | 0.0255 | 1.0693 | 0.0309 | 5.5577 | -5.0995 | 5.6772 | 0.00 | 0.85 |
| $r^2$ | 1.1435 | 0.0582 | 2.4413 | 5.4005 | 45.0918 | 0.0000 | 32.2310 | 0.00 | 0.00 |
| $\log r^2$ | -1.5945 | 0.0594 | 2.4926 | -1.3549 | 6.8798 | -17.9407 | 3.4729 | 0.00 | 0.00 |

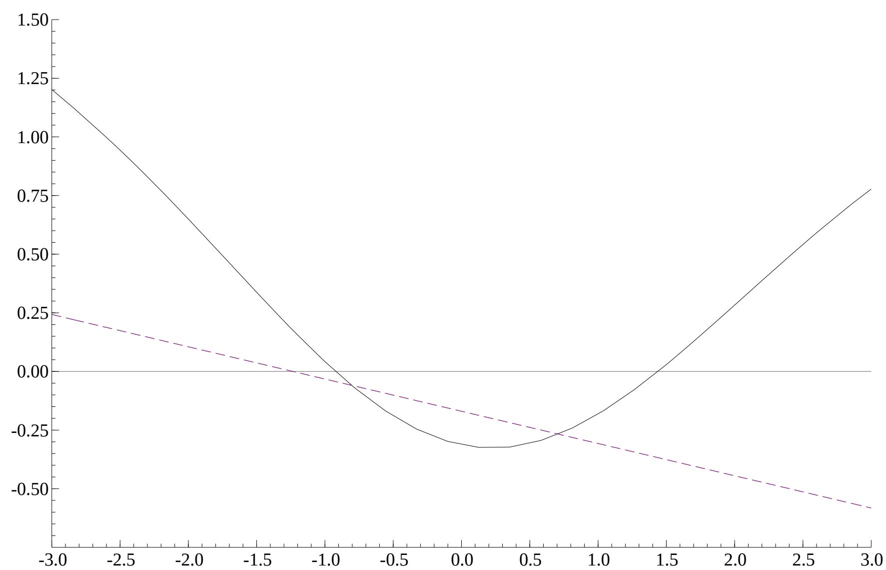
Table 1: Descriptive statistics of daily return ($r$), its square ($r^2$), and loarithm of the squared return ($\log r^2$). There are 1758 samples during the period from February 1, 2001 to January 31, 2007. The standard errors of skewness and kurtosis are 0.0584 and 0.1167, respectively. JB is the $p$-value of the Jaque-Bera statistics to test the null hypothesis of nomality. LB(10) is the $p$-value of the Ljung-Box statistics adjusted for heteroskedasticity following Diebold (1988) to test the null hypothesis of no autocorrelation up to 10 lags.

| Model | RM | Mean | Stdev. | 95%L | Median | 95%U | CD |
|---|---|---|---|---|---|---|---|
| SV | $\phi$ | 0.9727 | 0.0107 | 0.9444 | 0.9744 | 0.9878 | 0.399 |
| | $\sigma$ | 0.2239 | 0.0531 | 0.1701 | 0.2087 | 0.3992 | 0.348 |
| | $\mu$ | -0.1453 | 0.2247 | -0.5780 | -0.1501 | 0.3154 | 0.528 |
| ASV | $\phi$ | 0.9705 | 0.0064 | 0.9570 | 0.9708 | 0.9821 | 0.787 |
| | $\sigma$ | 0.2085 | 0.0175 | 0.1783 | 0.2070 | 0.2455 | 0.586 |
| | $\rho$ | -0.5517 | 0.0580 | -0.6545 | -0.5552 | -0.4295 | 0.047 |
| | $\mu$ | -0.1697 | 0.1553 | -0.4618 | -0.1751 | 0.1510 | 0.290 |

Table 2: MCMC estimation results of symmetric and asymmetric SV (SV and ASV) models. 95%L and 95%U are the lower and upper quantiles of 95% credible interval, respectively. The last column is the $p$-value of the convergence diagnostic test proposed by Geweke (1992).

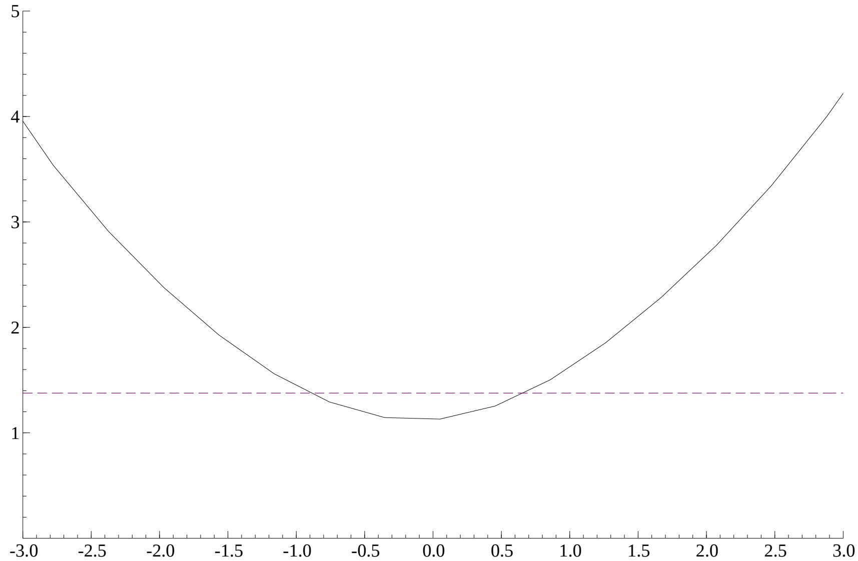



Symmetric SV model

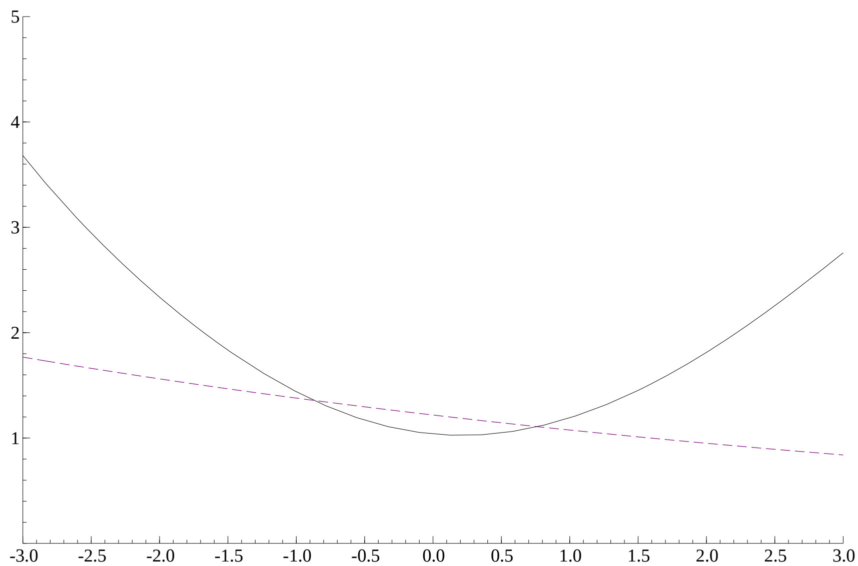



Asymmetric SV model

Figure 1: News impact curves for symmetric (top) and asymmetric SV (bottom) models by the conventional method (dashed line) using equation (6) and the new method (solid line). Horizontal and vertical axes represents today's daily return ($r$) and tomorrow's log-volatility ($\hat{h}$), respectively. Using parameter samples generated in the MCMC estimation for each model, we implement the new method with the local linear Gaussian kernel regression with the bandwidth of 1 and the number of grid points of 100.




Symmetric SV model




Asymmetric SV model

Figure 2: News impact curves for symmetric (top) and asymmetric SV (bottom) models by the conventional method (dashed line) using equation (3.4) in Asai and McAleer (2009) and the new method (solid line). Horizontal and vertical axes represents today's daily return ($r$) and tomorrow's volatility ($\hat{\sigma}^2 = \exp(\hat{h})$), respectively. Using parameter samples generated in the MCMC estimation for each model, we implement the new method with the local linear Gaussian kernel regression with the bandwidth of 1 and the number of grid points of 100.